Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic"

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a^2+b+2\,a\,x^2+x^4}\,dx$$

Optimal (type 3, 299 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right]}{2\,\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{a+\sqrt{a^2+b}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right]}{2\,\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{a+\sqrt{a^2+b}}}-\frac{\text{Log}\left[\sqrt{a^2+b}-\sqrt{2}\,\sqrt{-a+\sqrt{a^2+b}}\,x+x^2\right]}{4\,\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{-a+\sqrt{a^2+b}}}+\frac{\text{Log}\left[\sqrt{a^2+b}+\sqrt{2}\,\sqrt{-a+\sqrt{a^2+b}}\,x+x^2\right]}{4\,\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{-a+\sqrt{a^2+b}}}$$

Result (type 3, 81 leaves):

$$-\frac{i\left(\frac{\text{ArcTan}\left[\frac{x}{\sqrt{a-i\sqrt{b}}}\right]}{\sqrt{a-i\sqrt{b}}}-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{a+i\sqrt{b}}}\right]}{\sqrt{a+i\sqrt{b}}}\right)}{2\,\sqrt{b}}$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+a^2+2\,a\,x^2+x^4}\,dx$$

Optimal (type 3, 299 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right]}{2\sqrt{2}\,\sqrt{1+a^2}\,\sqrt{a+\sqrt{1+a^2}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right]}{2\sqrt{2}\,\sqrt{1+a^2}\,\sqrt{a+\sqrt{1+a^2}}}-$$

$$\frac{\text{Log}\left[\sqrt{1+a^2}-\sqrt{2}\,\sqrt{-a+\sqrt{1+a^2}}\,x+x^2\right]}{4\sqrt{2}\,\sqrt{1+a^2}\,\sqrt{-a+\sqrt{1+a^2}}}+\frac{\text{Log}\left[\sqrt{1+a^2}+\sqrt{2}\,\sqrt{-a+\sqrt{1+a^2}}\,x+x^2\right]}{4\sqrt{2}\,\sqrt{1+a^2}\,\sqrt{-a+\sqrt{1+a^2}}}$$

Result (type 3, 52 leaves):

$$-\frac{1}{2}\,i\left(\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-i+a}}\right]}{\sqrt{-i+a}}-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{i+a}}\right]}{\sqrt{i+a}}\right)$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{4-5\,x^2+x^4}\,dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\frac{1}{6}\,\text{ArcTanh}\left[\frac{x}{2}\right]+\frac{\text{ArcTanh}[x]}{3}$$

Result (type 3, 37 leaves):

$$-\frac{1}{6}\,\text{Log}[1-x]+\frac{1}{12}\,\text{Log}[2-x]+\frac{1}{6}\,\text{Log}[1+x]-\frac{1}{12}\,\text{Log}[2+x]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{9+5\,x^2+x^4}\,dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2\,x}{\sqrt{11}}\right]}{6\sqrt{11}}+\frac{\text{ArcTan}\left[\frac{1+2\,x}{\sqrt{11}}\right]}{6\sqrt{11}}-\frac{1}{12}\,\text{Log}\left[3-x+x^2\right]+\frac{1}{12}\,\text{Log}\left[3+x+x^2\right]$$

Result (type 3, 91 leaves):

$$-\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right]}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}+\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right]}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1-x^2+x^4}\,dx$$

Optimal (type 3, 74 leaves, 9 steps):

$$-\frac{1}{2}\text{ArcTan}\left[\sqrt{3}-2x\right]+\frac{1}{2}\text{ArcTan}\left[\sqrt{3}+2x\right]-\frac{\text{Log}\left[1-\sqrt{3}\,x+x^2\right]}{4\sqrt{3}}+\frac{\text{Log}\left[1+\sqrt{3}\,x+x^2\right]}{4\sqrt{3}}$$

Result (type 3, 77 leaves):

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\text{ArcTan}\left[\frac{1}{2}\left(1-i\sqrt{3}\right)x\right]-\sqrt{-1+i\sqrt{3}}\,\text{ArcTan}\left[\frac{1}{2}\left(1+i\sqrt{3}\right)x\right]\right)}{\sqrt{6}}$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2+2x^2+x^4}\,dx$$

Optimal (type 3, 176 leaves, 9 steps):

$$-\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right]+\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right]-$$

$$\frac{\text{Log}\left[\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2\right]}{8\sqrt{-1+\sqrt{2}}}+\frac{\text{Log}\left[\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2\right]}{8\sqrt{-1+\sqrt{2}}}$$

Result (type 3, 41 leaves):

$$\frac{1}{4}\left(\left(1-i\right)^{3/2}\text{ArcTan}\left[\frac{x}{\sqrt{1-i}}\right]+\left(1+i\right)^{3/2}\text{ArcTan}\left[\frac{x}{\sqrt{1+i}}\right]\right)$$

Problem 16: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-6\right]$$

Result (type 4, 65 leaves):

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\,\sqrt{1+3x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{3}\,x\right],-\frac{1}{6}\right]}{\sqrt{3}\,\sqrt{2+5x^2-3x^4}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\,x\right],\frac{1}{3}\left(-7-2\sqrt{10}\right)\right]$$

Result (type 4, 49 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1+\sqrt{\frac{5}{2}}}\,x\right],\frac{1}{3}\left(-7+2\sqrt{10}\right)\right]}{\sqrt{2+\sqrt{10}}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-3+\sqrt{33}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{6}{3+\sqrt{33}}}\ x\right],\ \frac{1}{4}\left(-7-\sqrt{33}\right)\right]$$

Result (type 4, 53 leaves):

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{6}{-3+\sqrt{33}}}\ x\right],\ \frac{1}{4}\left(-7+\sqrt{33}\right)\right]$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{1+\sqrt{7}}}\ x\right],\ \frac{1}{3}\left(-4-\sqrt{7}\right)\right]}{\sqrt{-1+\sqrt{7}}}$$

Result (type 4, 49 leaves):

$$-\frac{i\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{3}{-1+\sqrt{7}}}\ x\right],\ \frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}}$$

Problem 20: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{2+x^2-3x^4}}\,dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x],\ -\frac{3}{2}\right]}{\sqrt{2}}$$

Result (type 4, 63 leaves):

$$-\frac{i\sqrt{1-x^2}\ \sqrt{2+3x^2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right],\ -\frac{2}{3}\right]}{\sqrt{3}\ \sqrt{2+x^2-3x^4}}$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right],\frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}}$$

Result (type 4, 51 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right],-\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$$

Optimal (type 4, 46 leaves, 2 steps):

$$\sqrt{\frac{2}{3+\sqrt{33}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right],\frac{1}{4}\left(-7+\sqrt{33}\right)\right]$$

Result (type 4, 55 leaves):

$$-i\sqrt{\frac{2}{-3+\sqrt{33}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right],-\frac{7}{4}-\frac{\sqrt{33}}{4}\right]$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{1}{6}\left(-2+\sqrt{10}\right)}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\;x\right],\frac{1}{3}\left(-7+2\sqrt{10}\right)\right]$$

Result (type 4, 49 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{2}}}\;x\right],\frac{1}{3}\left(-7-2\sqrt{10}\right)\right]}{\sqrt{-2+\sqrt{10}}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{2-5x^2-3x^4}}\,dx$$

Optimal (type 4, 18 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{3}\;x\right],-\frac{1}{6}\right]}{\sqrt{6}}$$

Result (type 4, 54 leaves):

$$\frac{\sqrt{1-3x^2}\;\sqrt{2+x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{3}\;x\right],-\frac{1}{6}\right]}{\sqrt{6}\;\sqrt{2-5x^2-3x^4}}$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{3+7x^2-2x^4}}\,dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-7+\sqrt{73}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{7+\sqrt{73}}}\right],\frac{1}{12}\left(-61-7\sqrt{73}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{7+\sqrt{73}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{2x}{\sqrt{-7+\sqrt{73}}}\right],\frac{1}{12}\left(-61+7\sqrt{73}\right)\right]$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\ x\right],\ -4-\sqrt{15}\right]$$

Result (type 4, 43 leaves):

$$-\frac{i\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{1+\sqrt{\frac{5}{3}}}\ x\right],\ -4+\sqrt{15}\right]}{\sqrt{3+\sqrt{15}}}$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{3}}\right],\ -6\right]$$

Result (type 4, 65 leaves):

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\ \sqrt{1+2x^2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{2}\ x\right],\ -\frac{1}{6}\right]}{\sqrt{2}\ \sqrt{3+5x^2-2x^4}}$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{2+\sqrt{10}}}\ x\right],\ \frac{1}{3}\left(-7-2\sqrt{10}\right)\right]}{\sqrt{-2+\sqrt{10}}}$$

Result (type 4, 51 leaves):

$$-\frac{\mathrm{i}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\sqrt{\frac{2}{-2+\sqrt{10}}}\ x\right],\ -\frac{7}{3}+\frac{2\sqrt{10}}{3}\right]}{\sqrt{2+\sqrt{10}}}$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-3+\sqrt{33}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{3+\sqrt{33}}}\right],\ \frac{1}{4}\left(-7-\sqrt{33}\right)\right]$$

Result (type 4, 50 leaves):

$$-\mathrm{i}\sqrt{\frac{2}{3+\sqrt{33}}}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\frac{2x}{\sqrt{-3+\sqrt{33}}}\right],\ \frac{1}{4}\left(-7+\sqrt{33}\right)\right]$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{7}}}\ x\right],\ \frac{1}{3}\left(-4-\sqrt{7}\right)\right]}{\sqrt{-1+\sqrt{7}}}$$

Result (type 4, 49 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right],\frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}}$$

Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x^2-2\,x^4}}\,dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x],-\frac{2}{3}\right]}{\sqrt{3}}$$

Result (type 4, 65 leaves):

$$-\frac{i\,\sqrt{1-x^2}\,\sqrt{3+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3}}\,x\right],-\frac{3}{2}\right]}{\sqrt{2}\,\sqrt{3-x^2-2\,x^4}}$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2\,x^2-2\,x^4}}\,dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right],\frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}}$$

Result (type 4, 51 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right],-\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}}$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$$

Optimal (type 4, 43 leaves, 2 steps):

$$\sqrt{\frac{2}{3+\sqrt{33}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{-3+\sqrt{33}}}\right],\frac{1}{4}\left(-7+\sqrt{33}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{-3+\sqrt{33}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{2x}{\sqrt{3+\sqrt{33}}}\right],-\frac{7}{4}-\frac{\sqrt{33}}{4}\right]$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-2+\sqrt{10}}}\ x\right],\frac{1}{3}\left(-7+2\sqrt{10}\right)\right]}{\sqrt{2+\sqrt{10}}}$$

Result (type 4, 51 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{2+\sqrt{10}}}\ x\right],-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right]}{\sqrt{-2+\sqrt{10}}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx$$

Optimal (type 4, 18 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}\ x\right],-\frac{1}{6}\right]}{\sqrt{6}}$$

Result (type 4, 54 leaves):

$$\frac{\sqrt{1-2\ x^2}\ \sqrt{3+x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}\ x\right],-\frac{1}{6}\right]}{\sqrt{6}\ \sqrt{3-5\ x^2-2\ x^4}}$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-6\ x^2-2\ x^4}}\,dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\sqrt{\frac{1}{6}\left(-3+\sqrt{15}\right)}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\ x\right],-4+\sqrt{15}\right]$$

Result (type 4, 45 leaves):

$$-\frac{i\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{3}}}\ x\right],-4-\sqrt{15}\right]}{\sqrt{-3+\sqrt{15}}}$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-7\ x^2-2\ x^4}}\,dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{7+\sqrt{73}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2\ x}{\sqrt{-7+\sqrt{73}}}\right],\frac{1}{12}\left(-61+7\sqrt{73}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{-7+\sqrt{73}}}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\frac{2\ x}{\sqrt{7+\sqrt{73}}}\right],\frac{1}{12}\left(-61-7\sqrt{73}\right)\right]$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$$

Optimal (type 4, 141 leaves, 1 step):

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\ \sqrt{-2+\left(2+\sqrt{10}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4}\times 5^{1/4}\,x}{\sqrt{-2+\left(2+\sqrt{10}\right)x^2}}\right],\ \frac{1}{10}\left(5+\sqrt{10}\right)\right]}{2\times 10^{1/4}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\ \sqrt{-2+4x^2+3x^4}}$$

Result (type 4, 81 leaves):

$$-\frac{i\sqrt{2-4x^2-3x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{2}}}\ x\right],\ \frac{1}{3}\left(-7-2\sqrt{10}\right)\right]}{\sqrt{-2+\sqrt{10}}\ \sqrt{-2+4x^2+3x^4}}$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$$

Optimal (type 4, 146 leaves, 1 step):

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\ \sqrt{-4+\left(3+\sqrt{33}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 33^{1/4}\,x}{\sqrt{-4+\left(3+\sqrt{33}\right)x^2}}\right],\ \frac{1}{22}\left(11+\sqrt{33}\right)\right]}{2\sqrt{2}\ 33^{1/4}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\ \sqrt{-2+3x^2+3x^4}}$$

Result (type 4, 83 leaves):

$$-\frac{i\sqrt{4-6x^2-6x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{6}{3+\sqrt{33}}}\ x\right],\ -\frac{7}{4}-\frac{\sqrt{33}}{4}\right]}{\sqrt{-3+\sqrt{33}}\ \sqrt{-2+3x^2+3x^4}}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$$

Optimal (type 4, 141 leaves, 1 step):

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+\left(1+\sqrt{7}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 7^{1/4}\ x}{\sqrt{-2+\left(1+\sqrt{7}\right)x^2}}\right],\frac{1}{14}\left(7+\sqrt{7}\right)\right]}{2\times 7^{1/4}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+2x^2+3x^4}}$$

Result (type 4, 83 leaves):

$$-\frac{i\sqrt{2-2x^2-3x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{1+\sqrt{7}}}\ x\right],-\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}\sqrt{-2+2x^2+3x^4}}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$$

Optimal (type 4, 65 leaves, 1 step):

$$\frac{\sqrt{-1+x^2}\sqrt{2+3x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{5}{2}}\ x}{\sqrt{-1+x^2}}\right],\frac{2}{5}\right]}{\sqrt{5}\sqrt{-2-x^2+3x^4}}$$

Result (type 4, 60 leaves):

$$-\frac{i\sqrt{1-x^2}\sqrt{2+3x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right],-\frac{2}{3}\right]}{\sqrt{-6-3x^2+9x^4}}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\frac{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}\,\sqrt{\frac{2+\left(1+\sqrt{7}\right)x^2}{2+\left(1-\sqrt{7}\right)x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,7^{1/4}\,x}{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}}\right],\frac{1}{14}\left(7-\sqrt{7}\right)\right]}{2\times 7^{1/4}\,\sqrt{\frac{1}{2+\left(1-\sqrt{7}\right)x^2}}\,\sqrt{-2-2x^2+3x^4}}$$

Result (type 4, 81 leaves):

$$-\frac{i\,\sqrt{2+2x^2-3x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right],\frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}\,\sqrt{-2-2x^2+3x^4}}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$$

Optimal (type 4, 153 leaves, 1 step):

$$\frac{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}\,\sqrt{\frac{4+\left(3+\sqrt{33}\right)x^2}{4+\left(3-\sqrt{33}\right)x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,33^{1/4}\,x}{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}}\right],\frac{1}{22}\left(11-\sqrt{33}\right)\right]}{2\sqrt{2}\,33^{1/4}\,\sqrt{\frac{1}{4+\left(3-\sqrt{33}\right)x^2}}\,\sqrt{-2-3x^2+3x^4}}$$

Result (type 4, 81 leaves):

$$-\frac{i\,\sqrt{4+6x^2-6x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right],\frac{1}{4}\left(-7+\sqrt{33}\right)\right]}{\sqrt{3+\sqrt{33}}\,\sqrt{-2-3x^2+3x^4}}$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\frac{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}\ \sqrt{\frac{2+\left(2+\sqrt{10}\right)x^2}{2+\left(2-\sqrt{10}\right)x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4}\times 5^{1/4}\,x}{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}}\right],\ \frac{1}{10}\left(5-\sqrt{10}\right)\right]}{2\times 10^{1/4}\ \sqrt{\frac{1}{2+\left(2-\sqrt{10}\right)x^2}}\ \sqrt{-2-4x^2+3x^4}}$$

Result (type 4, 81 leaves) :

$$-\frac{i\ \sqrt{2+4x^2-3x^4}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{1+\sqrt{\frac{5}{2}}}\ x\right],\ \frac{1}{3}\left(-7+2\sqrt{10}\right)\right]}{\sqrt{2+\sqrt{10}}\ \sqrt{-2-4x^2+3x^4}}$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$$

Optimal (type 4, 63 leaves, 1 step) :

$$\frac{\sqrt{-2+x^2}\ \sqrt{1+3x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{7}\ x}{\sqrt{-2+x^2}}\right],\ \frac{1}{7}\right]}{\sqrt{7}\ \sqrt{-2-5x^2+3x^4}}$$

Result (type 4, 65 leaves) :

$$-\frac{i\ \sqrt{1-\frac{x^2}{2}}\ \sqrt{1+3x^2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{3}\ x\right],\ -\frac{1}{6}\right]}{\sqrt{3}\ \sqrt{-2-5x^2+3x^4}}$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$$

Optimal (type 4, 148 leaves, 1 step) :

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\ \sqrt{-6+\left(7+\sqrt{73}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 73^{1/4}\ x}{\sqrt{-6+\left(7+\sqrt{73}\right)x^2}}\right],\ \frac{1}{146}\left(73+7\sqrt{73}\right)\right]}{2\sqrt{3}\ 73^{1/4}\ \sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\ \sqrt{-3+7x^2+2x^4}}$$

Result (type 4, 80 leaves):

$$-\frac{i\sqrt{6-14x^2-4x^4}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\frac{2x}{\sqrt{7+\sqrt{73}}}\right],\ \frac{1}{12}\left(-61-7\sqrt{73}\right)\right]}{\sqrt{-7+\sqrt{73}}\ \sqrt{-3+7x^2+2x^4}}$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\ \sqrt{-3+\left(3+\sqrt{15}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 15^{1/4}\ x}{\sqrt{-3+\left(3+\sqrt{15}\right)x^2}}\right],\ \frac{1}{10}\left(5+\sqrt{15}\right)\right]}{\sqrt{2}\ 3^{3/4}\times 5^{1/4}\ \sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\ \sqrt{-3+6x^2+2x^4}}$$

Result (type 4, 77 leaves):

$$-\frac{i\sqrt{3-6x^2-2x^4}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{3}}}\ x\right],\ -4-\sqrt{15}\right]}{\sqrt{-3+\sqrt{15}}\ \sqrt{-3+6x^2+2x^4}}$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+\left(2+\sqrt{10}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4}\,5^{1/4}\,x}{\sqrt{-3+\left(2+\sqrt{10}\right)x^2}}\right],\frac{1}{10}\left(5+\sqrt{10}\right)\right]}{2^{3/4}\sqrt{3}\,5^{1/4}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+4x^2+2x^4}}$$

Result (type 4, 83 leaves):

$$-\frac{i\sqrt{3-4x^2-2x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right],-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right]}{\sqrt{-2+\sqrt{10}}\sqrt{-3+4x^2+2x^4}}$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$$

Optimal (type 4, 146 leaves, 1 step):

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-6+\left(3+\sqrt{33}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,33^{1/4}\,x}{\sqrt{-6+\left(3+\sqrt{33}\right)x^2}}\right],\frac{1}{22}\left(11+\sqrt{33}\right)\right]}{2\times 3^{3/4}\times 11^{1/4}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-3+3x^2+2x^4}}$$

Result (type 4, 80 leaves):

$$-\frac{i\sqrt{6-6x^2-4x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{2x}{\sqrt{3+\sqrt{33}}}\right],-\frac{7}{4}-\frac{\sqrt{33}}{4}\right]}{\sqrt{-3+\sqrt{33}}\sqrt{-3+3x^2+2x^4}}$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx$$

Optimal (type 4, 143 leaves, 1 step):

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\ \sqrt{-3+\left(1+\sqrt{7}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 7^{1/4}\ x}{\sqrt{-3+\left(1+\sqrt{7}\right)x^2}}\right],\ \frac{1}{14}\left(7+\sqrt{7}\right)\right]}{\sqrt{6}\ 7^{1/4}\ \sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\ \sqrt{-3+2x^2+2x^4}}$$

Result (type 4, 83 leaves):

$$-\frac{i\sqrt{3-2x^2-2x^4}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{7}}}\ x\right],\ -\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}\ \sqrt{-3+2x^2+2x^4}}$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\, dx$$

Optimal (type 4, 63 leaves, 1 step):

$$\frac{\sqrt{-1+x^2}\ \sqrt{3+2x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{5}{3}}\ x}{\sqrt{-1+x^2}}\right],\ \frac{3}{5}\right]}{\sqrt{5}\ \sqrt{-3+x^2+2x^4}}$$

Result (type 4, 63 leaves):

$$-\frac{i\sqrt{1-x^2}\ \sqrt{3+2x^2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{2}{3}}\ x\right],\ -\frac{3}{2}\right]}{\sqrt{2}\ \sqrt{-3+x^2+2x^4}}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\, dx$$

Optimal (type 4, 150 leaves, 1 step):

$$\frac{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}\ \sqrt{\frac{3+\left(1+\sqrt{7}\right)x^2}{3+\left(1-\sqrt{7}\right)x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 7^{1/4}\ x}{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}}\right],\ \frac{1}{14}\left(7-\sqrt{7}\right)\right]}{\sqrt{6}\ 7^{1/4}\ \sqrt{\frac{1}{3+\left(1-\sqrt{7}\right)x^2}}\ \sqrt{-3-2\,x^2+2\,x^4}}$$

Result (type 4, 81 leaves):

$$-\frac{\mathrm{i}\ \sqrt{3+2\,x^2-2\,x^4}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{7}}}\ x\right],\ \frac{1}{3}\left(-4+\sqrt{7}\right)\right]}{\sqrt{1+\sqrt{7}}\ \sqrt{-3-2\,x^2+2\,x^4}}$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-3\,x^2+2\,x^4}}\,dx$$

Optimal (type 4, 153 leaves, 1 step):

$$\frac{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}\ \sqrt{\frac{6+\left(3+\sqrt{33}\right)x^2}{6+\left(3-\sqrt{33}\right)x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\ 33^{1/4}\ x}{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}}\right],\ \frac{1}{22}\left(11-\sqrt{33}\right)\right]}{2\times 3^{3/4}\times 11^{1/4}\ \sqrt{\frac{1}{6+\left(3-\sqrt{33}\right)x^2}}\ \sqrt{-3-3\,x^2+2\,x^4}}$$

Result (type 4, 78 leaves):

$$-\frac{\mathrm{i}\ \sqrt{6+6\,x^2-4\,x^4}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\frac{2\,x}{\sqrt{-3+\sqrt{33}}}\right],\ \frac{1}{4}\left(-7+\sqrt{33}\right)\right]}{\sqrt{3+\sqrt{33}}\ \sqrt{-3-3\,x^2+2\,x^4}}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-4\,x^2+2\,x^4}}\,dx$$

Optimal (type 4, 155 leaves, 1 step):

$$\frac{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}\ \sqrt{\frac{3+\left(2+\sqrt{10}\right)x^2}{3+\left(2-\sqrt{10}\right)x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4}\,5^{1/4}\,x}{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}}\right],\ \frac{1}{10}\left(5-\sqrt{10}\right)\right]}{2^{3/4}\sqrt{3}\ 5^{1/4}\sqrt{\frac{1}{3+\left(2-\sqrt{10}\right)x^2}}\ \sqrt{-3-4x^2+2x^4}}$$

Result (type 4, 83 leaves) :

$$-\frac{i\sqrt{3+4x^2-2x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-2+\sqrt{10}}}\ x\right],\ -\frac{7}{3}+\frac{2\sqrt{10}}{3}\right]}{\sqrt{2+\sqrt{10}}\ \sqrt{-3-4x^2+2x^4}}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$$

Optimal (type 4, 63 leaves, 1 step) :

$$\frac{\sqrt{-3+x^2}\ \sqrt{1+2x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{7}\ x}{\sqrt{-3+x^2}}\right],\ \frac{1}{7}\right]}{\sqrt{7}\ \sqrt{-3-5x^2+2x^4}}$$

Result (type 4, 65 leaves) :

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\ \sqrt{1+2x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\ x\right],\ -\frac{1}{6}\right]}{\sqrt{2}\ \sqrt{-3-5x^2+2x^4}}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$$

Optimal (type 4, 52 leaves, 1 step) :

$$\frac{\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\ -\frac{1}{2}\right]}{\sqrt{2}\ \sqrt{2+5x^2+3x^4}}$$

Result (type 4, 58 leaves) :

$$-\frac{i \sqrt{1+x^2} \sqrt{2+3 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{2}} x\right], \frac{2}{3}\right]}{\sqrt{6+15 x^2+9 x^4}}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+4 x^2+3 x^4}} \, dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+4 x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{2+4 x^2+3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{-2-i \sqrt{2}}} \sqrt{1-\frac{3 x^2}{-2+i \sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-2-i \sqrt{2}}} x\right], \frac{-2-i \sqrt{2}}{-2+i \sqrt{2}}\right]}{\sqrt{3} \sqrt{-\frac{1}{-2-i \sqrt{2}}} \sqrt{2+4 x^2+3 x^4}}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3 x^2+3 x^4}} \, dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+3 x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8} \left(4-\sqrt{6}\right)\right]}{2 \times 6^{1/4} \sqrt{2+3 x^2+3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{6 x^2}{-3-i \sqrt{15}}} \sqrt{1-\frac{6 x^2}{-3+i \sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-3-i \sqrt{15}}} x\right], \frac{-3-i \sqrt{15}}{-3+i \sqrt{15}}\right]}{\sqrt{6} \sqrt{-\frac{1}{-3-i \sqrt{15}}} \sqrt{2+3 x^2+3 x^4}}$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+2 x^2+3 x^4}} \, dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+2 x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{12} \left(6-\sqrt{6}\right)\right]}{2\times 6^{1/4} \sqrt{2+2 x^2+3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{-1-i \sqrt{5}}} \sqrt{1-\frac{3 x^2}{-1+i \sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-1-i \sqrt{5}}} x\right], \frac{-1-i \sqrt{5}}{-1+i \sqrt{5}}\right]}{\sqrt{3} \sqrt{-\frac{1}{-1-i \sqrt{5}}} \sqrt{2+2 x^2+3 x^4}}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+x^2+3 x^4}} \, dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{24} \left(12-\sqrt{6}\right)\right]}{2\times 6^{1/4} \sqrt{2+x^2+3 x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i \sqrt{1-\frac{6 x^2}{-1-i \sqrt{23}}} \sqrt{1-\frac{6 x^2}{-1+i \sqrt{23}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-1-i \sqrt{23}}} x\right], \frac{-1-i \sqrt{23}}{-1+i \sqrt{23}}\right]}{\sqrt{6} \sqrt{-\frac{1}{-1-i \sqrt{23}}} \sqrt{2+x^2+3 x^4}}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3 x^4}} \, dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3\,x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{2}\right]}{2\times 6^{1/4}\sqrt{2+3\,x^4}}$$

Result (type 4, 25 leaves):

$$-\left(-\frac{1}{6}\right)^{1/4}\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{3}{2}\right)^{1/4}x\right],-1\right]$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-x^2+3\,x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3\,x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{24}\left(12+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{2-x^2+3\,x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{6\,x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6\,x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{6}{1-i\sqrt{23}}}\,x\right],\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right]}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2-x^2+3\,x^4}}$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-2\,x^2+3\,x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2\,x^2+3\,x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{12}\left(6+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{2-2\,x^2+3\,x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{1-i \sqrt{5}}} \sqrt{1-\frac{3 x^2}{1+i \sqrt{5}}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{-\frac{3}{1-i \sqrt{5}}} \, x\right], \frac{1-i \sqrt{5}}{1+i \sqrt{5}}\right]}{\sqrt{3} \sqrt{-\frac{1}{1-i \sqrt{5}}} \sqrt{2-2 x^2+3 x^4}}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-3 x^2+3 x^4}} \, dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} \, x^2\right) \sqrt{\frac{2-3 x^2+3 x^4}{\left(2+\sqrt{6} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8} \left(4+\sqrt{6}\right)\right]}{2 \times 6^{1/4} \sqrt{2-3 x^2+3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{6 x^2}{3-i \sqrt{15}}} \sqrt{1-\frac{6 x^2}{3+i \sqrt{15}}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{-\frac{6}{3-i \sqrt{15}}} \, x\right], \frac{3-i \sqrt{15}}{3+i \sqrt{15}}\right]}{\sqrt{6} \sqrt{-\frac{1}{3-i \sqrt{15}}} \sqrt{2-3 x^2+3 x^4}}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-4 x^2+3 x^4}} \, dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} \, x^2\right) \sqrt{\frac{2-4 x^2+3 x^4}{\left(2+\sqrt{6} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2}+\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{2-4 x^2+3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{2-i \sqrt{2}}} \sqrt{1-\frac{3 x^2}{2+i \sqrt{2}}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{-\frac{3}{2-i \sqrt{2}}} \, x\right], \frac{2-i \sqrt{2}}{2+i \sqrt{2}}\right]}{\sqrt{3} \sqrt{-\frac{1}{2-i \sqrt{2}}} \sqrt{2-4 x^2+3 x^4}}$$

Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$$

Optimal (type 4, 110 leaves, 1 step):

$$\frac{\sqrt{\frac{6+\left(9-\sqrt{57}\right)x^2}{6+\left(9+\sqrt{57}\right)x^2}}\left(6+\left(9+\sqrt{57}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}\,x\right],\frac{1}{4}\left(-19+3\sqrt{57}\right)\right]}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3+9x^2+2x^4}}$$

Result (type 4, 97 leaves):

$$-\frac{i\sqrt{\frac{-9+\sqrt{57}-4x^2}{-9+\sqrt{57}}}\sqrt{9+\sqrt{57}+4x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{2x}{\sqrt{9+\sqrt{57}}}\right],\frac{23}{4}+\frac{3\sqrt{57}}{4}\right]}{2\sqrt{3+9x^2+2x^4}}$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$$

Optimal (type 4, 110 leaves, 1 step):

$$\frac{\sqrt{\frac{3+\left(4-\sqrt{10}\right)x^2}{3+\left(4+\sqrt{10}\right)x^2}}\left(3+\left(4+\sqrt{10}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}\,x\right],-\frac{2}{3}\left(5-2\sqrt{10}\right)\right]}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

Result (type 4, 98 leaves):

$$-\frac{i\sqrt{\frac{-4+\sqrt{10}-2x^2}{-4+\sqrt{10}}}\sqrt{4+\sqrt{10}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{4+\sqrt{10}}}\,x\right],\frac{13}{3}+\frac{4\sqrt{10}}{3}\right]}{\sqrt{6+16x^2+4x^4}}$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$$

Optimal (type 4, 60 leaves, 1 step):

$$\frac{\sqrt{\frac{3+x^2}{1+2x^2}}\,\left(1+2x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{2}\,x\right],\frac{5}{6}\right]}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

Result (type 4, 61 leaves):

$$-\frac{i\,\sqrt{3+x^2}\,\sqrt{1+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,x\right],\frac{1}{6}\right]}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx$$

Optimal (type 4, 104 leaves, 1 step):

$$\frac{\sqrt{\frac{3+\left(3-\sqrt{3}\right)x^2}{3+\left(3+\sqrt{3}\right)x^2}}\,\left(3+\left(3+\sqrt{3}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\,x\right],-1+\sqrt{3}\right]}{\sqrt{3\left(3+\sqrt{3}\right)}\,\sqrt{3+6x^2+2x^4}}$$

Result (type 4, 90 leaves):

$$-\frac{i\,\sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}}\,\sqrt{3+\sqrt{3}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-\frac{1}{\sqrt{3}}}\,x\right],2+\sqrt{3}\right]}{\sqrt{6+12x^2+4x^4}}$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$$

Optimal (type 4, 52 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{3+2x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{3}\right]}{\sqrt{3}\,\sqrt{3+5x^2+2x^4}}$$

Result (type 4, 58 leaves):

$$-\frac{i\sqrt{1+x^2}\,\sqrt{3+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3}}\,x\right],\frac{3}{2}\right]}{\sqrt{6+10x^2+4x^4}}$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2\times 6^{1/4}\sqrt{3+4x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\,\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-2-i\sqrt{2}}}\,x\right],\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right]}{\sqrt{2}\,\sqrt{-\frac{1}{-2-i\sqrt{2}}}\,\sqrt{3+4x^2+2x^4}}$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{8}\left(4-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{3+3x^2+2x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\,x\right],\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right]}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3+3x^2+2x^4}}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{12}\left(6-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{3+2x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-1-i\sqrt{5}}}\,x\right],\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right]}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3+2x^2+2x^4}}$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{3+x^2+2x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{24}\left(12-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{3+x^2+2x^4}}$$

Result (type 4, 140 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\,x\right],\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right]}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3+x^2+2x^4}}$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+2 x^4}} \, dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\, x^2\right) \sqrt{\frac{3+2 x^4}{\left(3+\sqrt{6}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2}\right]}{2 \times 6^{1/4} \sqrt{3+2 x^4}}$$

Result (type 4, 25 leaves):

$$-\left(-\frac{1}{6}\right)^{1/4} \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{2}{3}\right)^{1/4} x\right], -1\right]$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-x^2+2 x^4}} \, dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\, x^2\right) \sqrt{\frac{3-x^2+2 x^4}{\left(3+\sqrt{6}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{24}\left(12+\sqrt{6}\right)\right]}{2 \times 6^{1/4} \sqrt{3-x^2+2 x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i \sqrt{1-\frac{4 x^2}{1-i \sqrt{23}}} \sqrt{1-\frac{4 x^2}{1+i \sqrt{23}}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{-\frac{1}{1-i \sqrt{23}}}\, x\right], \frac{1-i \sqrt{23}}{1+i \sqrt{23}}\right]}{2 \sqrt{-\frac{1}{1-i \sqrt{23}}} \sqrt{3-x^2+2 x^4}}$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2 x^2+2 x^4}} \, dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{12}\left(6+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{3-2x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{1-i\sqrt{5}}}\,x\right],\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right]}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3-2x^2+2x^4}}$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{8}\left(4+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{3-3x^2+2x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{3-i\sqrt{15}}}\,x\right],\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right]}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3-3x^2+2x^4}}$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{3-4x^2+2x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{2}+\frac{1}{\sqrt{6}}\right]}{2\times 6^{1/4}\sqrt{3-4x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{2x^2}{2-i\sqrt{2}}} \sqrt{1-\frac{2x^2}{2+i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2}{2-i\sqrt{2}}} x\right], \frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right]}{\sqrt{2} \sqrt{-\frac{1}{2-i\sqrt{2}}} \sqrt{3-4x^2+2x^4}}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}} \, dx$$

Optimal (type 4, 19 leaves, 2 steps):

$$-\frac{\text{EllipticF}\left[\text{ArcCos}\left[\frac{x}{\sqrt{3}}\right], \frac{6}{5}\right]}{\sqrt{5}}$$

Result (type 4, 58 leaves):

$$\frac{\sqrt{1-2x^2} \sqrt{1-\frac{x^2}{3}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2} x\right], \frac{1}{6}\right]}{\sqrt{2} \sqrt{-3+7x^2-2x^4}}$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}} \, dx$$

Optimal (type 4, 14 leaves, 2 steps):

$$-\text{EllipticF}\left[\text{ArcCos}\left[\sqrt{\frac{2}{3}} x\right], 3\right]$$

Result (type 4, 53 leaves):

$$\frac{\sqrt{3-2x^2} \sqrt{1-x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3}} x\right], \frac{3}{2}\right]}{\sqrt{-6+10x^2-4x^4}}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{2}+\frac{1}{\sqrt{6}}\right]}{2\times 6^{1/4}\sqrt{-3+4x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\ \sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{2-i\sqrt{2}}}\ x\right],\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right]}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\ \sqrt{-3+4x^2-2x^4}}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{8}\left(4+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-3+3x^2-2x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\ \sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{3-i\sqrt{15}}}\ x\right],\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right]}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\ \sqrt{-3+3x^2-2x^4}}$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{12}\left(6+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-3+2x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{1-i\sqrt{5}}}\,x\right],\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right]}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3+2x^2-2x^4}}$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3+x^2-2x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{24}\left(12+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-3+x^2-2x^4}}$$

Result (type 4, 140 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{1-i\sqrt{23}}}\,x\right],\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right]}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3+x^2-2x^4}}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-2x^4}}\,dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{2}\right]}{2\times 6^{1/4}\sqrt{-3-2x^4}}$$

Result (type 4, 47 leaves):

$$-\frac{\left(-\frac{1}{6}\right)^{1/4}\sqrt{3+2x^4}\ \text{EllipticF}\left[\text{i}\ \text{ArcSinh}\left[\left(-\frac{2}{3}\right)^{1/4}x\right],-1\right]}{\sqrt{-3-2x^4}}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-x^2-2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{24}\left(12-\sqrt{6}\right)\right]}{2\times6^{1/4}\sqrt{-3-x^2-2x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{\text{i}\sqrt{1-\frac{4x^2}{-1-\text{i}\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+\text{i}\sqrt{23}}}\ \text{EllipticF}\left[\text{i}\ \text{ArcSinh}\left[2\sqrt{-\frac{1}{-1-\text{i}\sqrt{23}}}\,x\right],\frac{-1-\text{i}\sqrt{23}}{-1+\text{i}\sqrt{23}}\right]}{2\sqrt{-\frac{1}{-1-\text{i}\sqrt{23}}}\sqrt{-3-x^2-2x^4}}$$

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\frac{1}{12}\left(6-\sqrt{6}\right)\right]}{2\times6^{1/4}\sqrt{-3-2x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{\text{i}\sqrt{1-\frac{2x^2}{-1-\text{i}\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+\text{i}\sqrt{5}}}\ \text{EllipticF}\left[\text{i}\ \text{ArcSinh}\left[\sqrt{-\frac{2}{-1-\text{i}\sqrt{5}}}\,x\right],\frac{-1-\text{i}\sqrt{5}}{-1+\text{i}\sqrt{5}}\right]}{\sqrt{2}\sqrt{-\frac{1}{-1-\text{i}\sqrt{5}}}\sqrt{-3-2x^2-2x^4}}$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\ \frac{1}{8}\left(4-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-3-3x^2-2x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\,x\right],\ \frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right]}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3-3x^2-2x^4}}$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right],\ \frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2\times 6^{1/4}\sqrt{-3-4x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-2-i\sqrt{2}}}\,x\right],\ \frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right]}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3-4x^2-2x^4}}$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{\sqrt{3+2x^2}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{3}\right]}{\sqrt{3}\ \sqrt{-1-x^2}\ \sqrt{\frac{3+2x^2}{1+x^2}}}$$

Result (type 4, 63 leaves):

$$-\frac{i\sqrt{1+x^2}\ \sqrt{3+2x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3}}\ x\right],\frac{3}{2}\right]}{\sqrt{2}\ \sqrt{-3-5x^2-2x^4}}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$-\frac{\text{EllipticF}\left[\text{ArcCos}\left[\sqrt{\frac{3}{3+\sqrt{3}}}\ x\right],\frac{1}{2}\left(1+\sqrt{3}\right)\right]}{\sqrt{2}\ 3^{1/4}}$$

Result (type 4, 85 leaves):

$$\frac{\sqrt{3-\sqrt{3}-3x^2}\ \sqrt{2+\left(-3+\sqrt{3}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\ x\right],2-\sqrt{3}\right]}{\sqrt{6}\ \sqrt{-2+6x^2-3x^4}}$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx$$

Optimal (type 4, 6 leaves, 2 steps):

$$-\text{EllipticF}[\text{ArcCos}[x],3]$$

Result (type 4, 53 leaves):

$$\frac{\sqrt{2-3x^2}\,\sqrt{1-x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{2}}\,x\right],\frac{2}{3}\right]}{\sqrt{-6+15x^2-9x^4}}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{2}+\frac{1}{\sqrt{6}}\right]}{2\times 6^{1/4}\sqrt{-2+4x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\,\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{3}{2-i\sqrt{2}}}\,x\right],\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right]}{\sqrt{3}\,\sqrt{-\frac{1}{2-i\sqrt{2}}}\,\sqrt{-2+4x^2-3x^4}}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{8}\left(4+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-2+3x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\,\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{6}{3-i\sqrt{15}}}\,x\right],\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right]}{\sqrt{6}\,\sqrt{-\frac{1}{3-i\sqrt{15}}}\,\sqrt{-2+3x^2-3x^4}}$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{12}\left(6+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-2+2x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{3}{1-i\sqrt{5}}}\,x\right],\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right]}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2+2x^2-3x^4}}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\frac{1}{24}\left(12+\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-2+x^2-3x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{6}{1-i\sqrt{23}}}\,x\right],\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right]}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2+x^2-3x^4}}$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\ \frac{1}{2}\right]}{2\times 6^{1/4}\sqrt{-2-3x^4}}$$

Result (type 4, 47 leaves):

$$-\frac{\left(-\frac{1}{6}\right)^{1/4}\sqrt{2+3x^4}\ \text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\left(-\frac{3}{2}\right)^{1/4}x\right],\ -1\right]}{\sqrt{-2-3x^4}}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\ \frac{1}{24}\left(12-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-2-x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{\text{i}\sqrt{1-\frac{6x^2}{-1-\text{i}\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+\text{i}\sqrt{23}}}\ \text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{6}{-1-\text{i}\sqrt{23}}}\,x\right],\ \frac{-1-\text{i}\sqrt{23}}{-1+\text{i}\sqrt{23}}\right]}{\sqrt{6}\sqrt{-\frac{1}{-1-\text{i}\sqrt{23}}}\sqrt{-2-x^2-3x^4}}$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4}x\right],\ \frac{1}{12}\left(6-\sqrt{6}\right)\right]}{2\times 6^{1/4}\sqrt{-2-2x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{-1-i \sqrt{5}}} \sqrt{1-\frac{3 x^2}{-1+i \sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-1-i \sqrt{5}}} x\right], \frac{-1-i \sqrt{5}}{-1+i \sqrt{5}}\right]}{\sqrt{3} \sqrt{-\frac{1}{-1-i \sqrt{5}}} \sqrt{-2-2 x^2-3 x^4}}$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-3 x^2-3 x^4}} \, dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+3 x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8} \left(4-\sqrt{6}\right)\right]}{2 \times 6^{1/4} \sqrt{-2-3 x^2-3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{6 x^2}{-3-i \sqrt{15}}} \sqrt{1-\frac{6 x^2}{-3+i \sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-3-i \sqrt{15}}} x\right], \frac{-3-i \sqrt{15}}{-3+i \sqrt{15}}\right]}{\sqrt{6} \sqrt{-\frac{1}{-3-i \sqrt{15}}} \sqrt{-2-3 x^2-3 x^4}}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-4 x^2-3 x^4}} \, dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(2+\sqrt{6} x^2\right) \sqrt{\frac{2+4 x^2+3 x^4}{\left(2+\sqrt{6} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{-2-4 x^2-3 x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i \sqrt{1-\frac{3 x^2}{-2-i \sqrt{2}}} \sqrt{1-\frac{3 x^2}{-2+i \sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-2-i \sqrt{2}}} x\right], \frac{-2-i \sqrt{2}}{-2+i \sqrt{2}}\right]}{\sqrt{3} \sqrt{-\frac{1}{-2-i \sqrt{2}}} \sqrt{-2-4 x^2-3 x^4}}$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx$$

Optimal (type 4, 52 leaves, 2 steps):

$$-\frac{\sqrt{-2-3x^2}\ \text{EllipticF}\left[\text{ArcTan}[x],-\frac{1}{2}\right]}{\sqrt{2}\ \sqrt{1+x^2}\ \sqrt{\frac{2+3x^2}{1+x^2}}}$$

Result (type 4, 63 leaves):

$$-\frac{i\ \sqrt{1+x^2}\ \sqrt{2+3x^2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right],\frac{2}{3}\right]}{\sqrt{3}\ \sqrt{-2-5x^2-3x^4}}$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left(2+\sqrt{10}\ x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}\ x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\left(\frac{5}{2}\right)^{1/4}x\right],\frac{1}{8}\left(4-\sqrt{10}\right)\right]}{2\times 10^{1/4}\ \sqrt{2+5x^2+5x^4}}$$

Result (type 4, 144 leaves):

$$-\frac{i\ \sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\ \sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-\frac{10}{-5-i\sqrt{15}}}\ x\right],\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right]}{\sqrt{10}\ \sqrt{-\frac{1}{-5-i\sqrt{15}}}\ \sqrt{2+5x^2+5x^4}}$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left(1+\sqrt{2}\, x^2\right) \sqrt{\frac{2+5 x^2+4 x^4}{\left(1+\sqrt{2}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{16}\left(8-5 \sqrt{2}\right)\right]}{2\times 2^{3/4} \sqrt{2+5 x^2+4 x^4}}$$

Result (type 4, 147 leaves):

$$-\frac{i \sqrt{1-\frac{8 x^2}{-5-i \sqrt{7}}}\ \sqrt{1-\frac{8 x^2}{-5+i \sqrt{7}}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{-\frac{2}{-5-i \sqrt{7}}}\ x\right], \frac{-5-i \sqrt{7}}{-5+i \sqrt{7}}\right]}{2 \sqrt{2} \sqrt{-\frac{1}{-5-i \sqrt{7}}}\ \sqrt{2+5 x^2+4 x^4}}$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5 x^2+3 x^4}}\, dx$$

Optimal (type 4, 52 leaves, 1 step):

$$\frac{\left(1+x^2\right) \sqrt{\frac{2+3 x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x], -\frac{1}{2}\right]}{\sqrt{2} \sqrt{2+5 x^2+3 x^4}}$$

Result (type 4, 58 leaves):

$$-\frac{i \sqrt{1+x^2}\ \sqrt{2+3 x^2}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right], \frac{2}{3}\right]}{\sqrt{6+15 x^2+9 x^4}}$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5 x^2+2 x^4}}\, dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{\sqrt{\frac{2+x^2}{1+2 x^2}}\ \left(1+2 x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{2}\, x\right], \frac{3}{4}\right]}{2 \sqrt{2+5 x^2+2 x^4}}$$

Result (type 4, 58 leaves):

$$-\frac{i \sqrt{2+x^2} \sqrt{1+2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} x\right], \frac{1}{4}\right]}{2 \sqrt{2+5 x^2+2 x^4}}$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5 x^2+x^4}} \, dx$$

Optimal (type 4, 108 leaves, 1 step):

$$\frac{\sqrt{\frac{4+\left(5-\sqrt{17}\right) x^2}{4+\left(5+\sqrt{17}\right) x^2}} \left(4+\left(5+\sqrt{17}\right) x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{1}{2} \sqrt{5+\sqrt{17}} x\right], \frac{1}{4} \left(-17+5 \sqrt{17}\right)\right]}{2 \sqrt{5+\sqrt{17}} \sqrt{2+5 x^2+x^4}}$$

Result (type 4, 103 leaves):

$$-\frac{i \sqrt{5-\sqrt{17}+2 x^2} \sqrt{5+\sqrt{17}+2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{17}}} x\right], \frac{21}{4}+\frac{5 \sqrt{17}}{4}\right]}{\sqrt{2 \left(5-\sqrt{17}\right)} \sqrt{2+5 x^2+x^4}}$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5 x^2-x^4}} \, dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{33}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{5+\sqrt{33}}} x\right], \frac{1}{4} \left(-29-5 \sqrt{33}\right)\right]$$

Result (type 4, 55 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{33}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{-5+\sqrt{33}}} x\right], -\frac{29}{4}+\frac{5 \sqrt{33}}{4}\right]$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{41}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{5+\sqrt{41}}}\right],\frac{1}{8}\left(-33-5\sqrt{41}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{41}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{2x}{\sqrt{-5+\sqrt{41}}}\right],-\frac{33}{8}+\frac{5\sqrt{41}}{8}\right]$$

Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-6\right]$$

Result (type 4, 65 leaves):

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\ \sqrt{1+3x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{3}\ x\right],-\frac{1}{6}\right]}{\sqrt{3}\ \sqrt{2+5x^2-3x^4}}$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{57}}}\ \text{EllipticF}\left[\text{ArcSin}\left[2\sqrt{\frac{2}{5+\sqrt{57}}}\ x\right],\frac{1}{16}\left(-41-5\sqrt{57}\right)\right]$$

Result (type 4, 56 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{57}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{\frac{2}{-5+\sqrt{57}}}\ x\right],\ \frac{1}{16}\left(-41+5\sqrt{57}\right)\right]$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{65}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{10}{5+\sqrt{65}}}\ x\right],\ \frac{1}{4}\left(-9-\sqrt{65}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{65}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{2}\sqrt{5+\sqrt{65}}\ x\right],\ \frac{1}{4}\left(-9+\sqrt{65}\right)\right]$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{73}}}\ \text{EllipticF}\left[\text{ArcSin}\left[2\sqrt{\frac{3}{5+\sqrt{73}}}\ x\right],\ \frac{1}{24}\left(-49-5\sqrt{73}\right)\right]$$

Result (type 4, 56 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{73}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[2\sqrt{\frac{3}{-5+\sqrt{73}}}\ x\right],\ \frac{1}{24}\left(-49+5\sqrt{73}\right)\right]$$

Problem 131: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{2+5x^2-7x^4}}\,dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x],-\frac{7}{2}\right]}{\sqrt{2}}$$

Result (type 4, 65 leaves):

$$-\frac{i\sqrt{1-x^2}\sqrt{2+7x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{7}{2}}\,x\right],-\frac{2}{7}\right]}{\sqrt{7}\sqrt{2+5x^2-7x^4}}$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{89}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{4x}{\sqrt{5+\sqrt{89}}}\right],\frac{1}{32}\left(-57-5\sqrt{89}\right)\right]$$

Result (type 4, 52 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{89}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{4x}{\sqrt{-5+\sqrt{89}}}\right],\frac{1}{32}\left(-57+5\sqrt{89}\right)\right]$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{97}}}\,\text{EllipticF}\left[\text{ArcSin}\left[3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right],\frac{1}{36}\left(-61-5\sqrt{97}\right)\right]$$

Result (type 4, 56 leaves):

$$-i\sqrt{\frac{2}{5+\sqrt{97}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[3\sqrt{\frac{2}{-5+\sqrt{97}}}\,x\right],\frac{1}{36}\left(-61+5\sqrt{97}\right)\right]$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(b x^2+c x^4\right)^3}{x^{15}}\,dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{\left(b+c x^2\right)^4}{8 b x^8}$$

Result (type 1, 43 leaves):

$$-\frac{b^3}{8 x^8}-\frac{b^2 c}{2 x^6}-\frac{3 b c^2}{4 x^4}-\frac{c^3}{2 x^2}$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2} \sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 323 leaves, 8 steps):

$$\frac{28 b^3 x^{3/2} \left(b+c x^2\right)}{195 c^{5/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{28 b^2 \sqrt{x} \sqrt{b x^2+c x^4}}{585 c^2}+\frac{4 b x^{5/2} \sqrt{b x^2+c x^4}}{117 c}+\frac{2}{13} x^{9/2} \sqrt{b x^2+c x^4}-$$

$$\frac{28 b^{13/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{11/4} \sqrt{b x^2+c x^4}}+\frac{14 b^{13/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{11/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 201 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(-14 b^3-4 b^2 c x^2+55 b c^2 x^4+45 c^3 x^6\right)+42 b^{7/2} \sqrt{1+\frac{c x^2}{b}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]-\right.\right.$$

$$\left.\left.42 b^{7/2} \sqrt{1+\frac{c x^2}{b}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(585 c^{5/2} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} \sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 176 leaves, 6 steps):

$$-\frac{20 b^2 \sqrt{b x^2+c x^4}}{231 c^2 \sqrt{x}}+\frac{4 b x^{3/2} \sqrt{b x^2+c x^4}}{77 c}+\frac{2}{11} x^{7/2} \sqrt{b x^2+c x^4}+\frac{10 b^{11/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 c^{9/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 133 leaves):

$$\frac{1}{231} \sqrt{x^2 \left(b+c x^2\right)} \left(\frac{2 \left(-10 b^2+6 b c x^2+21 c^2 x^4\right)}{c^2 \sqrt{x}}+\frac{20 i b^3 \sqrt{1+\frac{b}{c x^2}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^2 \left(b+c x^2\right)}\right)$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} \sqrt{b x^2+c x^4}\, dx$$

Optimal (type 4, 293 leaves, 7 steps):

$$-\frac{4 b^2 x^{3/2} \left(b+c x^2\right)}{15 c^{3/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{4 b \sqrt{x} \sqrt{b x^2+c x^4}}{45 c}+\frac{2}{9} x^{5/2} \sqrt{b x^2+c x^4}+$$
$$\frac{4 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{7/4} \sqrt{b x^2+c x^4}}-\frac{2 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{7/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(2 b^2+7 b c x^2+5 c^2 x^4\right)-6 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]+\right.\right.$$
$$\left.\left.6 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(45 c^{3/2} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 355: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x}\,\sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$\frac{4 b \sqrt{b x^2+c x^4}}{21 c \sqrt{x}}+\frac{2}{7} x^{3/2} \sqrt{b x^2+c x^4}-\frac{2 b^{7/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{21 c^{5/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 120 leaves):

$$\frac{1}{21} \sqrt{x^2 \left(b+c x^2\right)} \left(\frac{4 b}{c \sqrt{x}}+6 x^{3/2}-\frac{4 i b^2 \sqrt{1+\frac{b}{c x^2}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c \left(b+c x^2\right)}\right)$$

Problem 356: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2+c x^4}}{\sqrt{x}}\,dx$$

Optimal (type 4, 263 leaves, 6 steps):

$$\frac{4 b x^{3/2} \left(b+c x^2\right)}{5 \sqrt{c} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{2}{5} \sqrt{x} \sqrt{b x^2+c x^4}-\frac{4 b^{5/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5 c^{3/4} \sqrt{b x^2+c x^4}}+$$

$$\frac{2 b^{5/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5 c^{3/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 176 leaves):

$$\left(2\,x^{3/2}\left(\sqrt{c}\,x\,\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\,\left(b+c\,x^2\right)+2\,b^{3/2}\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-\right.\right.$$

$$\left.\left.2\,b^{3/2}\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\right)\Bigg/\left(5\,\sqrt{c}\,\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

Problem 357: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{b\,x^2+c\,x^4}}{x^{3/2}}\,dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{2\,\sqrt{b\,x^2+c\,x^4}}{3\,\sqrt{x}}+\frac{2\,b^{3/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,c^{1/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 102 leaves):

$$\frac{2}{3}\,\sqrt{x^2\,\left(b+c\,x^2\right)}\,\left(\frac{1}{\sqrt{x}}+\frac{2\,\mathrm{i}\,b\,\sqrt{1+\frac{b}{c\,x^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{c}}}\,\left(b+c\,x^2\right)}\right)$$

Problem 358: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{b\,x^2+c\,x^4}}{x^{5/2}}\,dx$$

Optimal (type 4, 254 leaves, 6 steps):

$$\frac{4\sqrt{c}\,x^{3/2}\left(b+c\,x^2\right)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}-\frac{2\sqrt{b\,x^2+c\,x^4}}{x^{3/2}}-\frac{4\,b^{1/4}\,c^{1/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{b\,x^2+c\,x^4}}+$$
$$\frac{2\,b^{1/4}\,c^{1/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 175 leaves) :

$$-\frac{1}{\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\left(b+c\,x^2\right)}}2\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(b+c\,x^2\right)-\right.$$
$$\left.2\sqrt{b}\,\sqrt{c}\,x\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+2\sqrt{b}\,\sqrt{c}\,x\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)$$

Problem 359: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{b\,x^2+c\,x^4}}{x^{7/2}}\,dx$$

Optimal (type 4, 118 leaves, 4 steps) :

$$-\frac{2\sqrt{b\,x^2+c\,x^4}}{3\,x^{5/2}}+\frac{2\,c^{3/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,b^{1/4}\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 104 leaves) :

$$\frac{2}{3}\sqrt{x^2\left(b+c\,x^2\right)}\left(-\frac{1}{x^{5/2}}+\frac{2\,i\,c\sqrt{1+\frac{b}{c\,x^2}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c\,x^2\right)}\right)$$

Problem 360: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2 + c x^4}}{x^{9/2}} \, dx$$

Optimal (type 4, 293 leaves, 7 steps):

$$\frac{4 c^{3/2} x^{3/2} \left(b + c x^2\right)}{5 b \left(\sqrt{b} + \sqrt{c} x\right) \sqrt{b x^2 + c x^4}} - \frac{2 \sqrt{b x^2 + c x^4}}{5 x^{7/2}} - \frac{4 c \sqrt{b x^2 + c x^4}}{5 b x^{3/2}} - \frac{4 c^{5/4} x \left(\sqrt{b} + \sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^2 + c x^4}} + \frac{2 c^{5/4} x \left(\sqrt{b} + \sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 196 leaves):

$$-\left(\left(2 \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \left(b^2 + 3 b c x^2 + 2 c^2 x^4\right) - 2 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + 2 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(5 b x^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 \left(b + c x^2\right)}\right)\right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2 + c x^4}}{x^{11/2}} \, dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{2 \sqrt{b x^2 + c x^4}}{7 x^{9/2}} - \frac{4 c \sqrt{b x^2 + c x^4}}{21 b x^{5/2}} - \frac{2 c^{7/4} x \left(\sqrt{b} + \sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21 b^{5/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 122 leaves):

$$\frac{1}{21}\sqrt{x^2\left(b+c x^2\right)}\left(-\frac{2\left(3 b+2 c x^2\right)}{b x^{9/2}}-\frac{4 i c^2 \sqrt{1+\frac{b}{c x^2}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)}\right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2+c x^4}}{x^{13/2}}\, dx$$

Optimal (type 4, 323 leaves, 8 steps):

$$-\frac{4 c^{5/2} x^{3/2}\left(b+c x^2\right)}{15 b^2\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{2\sqrt{b x^2+c x^4}}{9 x^{11/2}}-\frac{4 c \sqrt{b x^2+c x^4}}{45 b x^{7/2}}+\frac{4 c^2 \sqrt{b x^2+c x^4}}{15 b^2 x^{3/2}}+$$
$$\frac{4 c^{9/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{15 b^{7/4}\sqrt{b x^2+c x^4}}-\frac{2 c^{9/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{15 b^{7/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 209 leaves):

$$\left(2\left(\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\left(-5 b^3-7 b^2 c x^2+4 b c^2 x^4+6 c^3 x^6\right)-6\sqrt{b}\, c^{5/2} x^5\sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]+\right.\right.$$
$$\left.\left.6\sqrt{b}\, c^{5/2} x^5\sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(45 b^2 x^{7/2}\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}\right)$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2+c x^4}}{x^{15/2}}\, dx$$

Optimal (type 4, 176 leaves, 6 steps):

$$-\frac{2\sqrt{b x^2+c x^4}}{11 x^{13/2}}-\frac{4 c\sqrt{b x^2+c x^4}}{77 b x^{9/2}}+\frac{20 c^2\sqrt{b x^2+c x^4}}{231 b^2 x^{5/2}}+\frac{10 c^{11/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{231 b^{9/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 133 leaves):

$$\frac{1}{231}\sqrt{x^2\left(b+c x^2\right)}\left(\frac{2\left(-21 b^2-6 b c x^2+10 c^2 x^4\right)}{b^2 x^{13/2}}+\frac{20\, i\, c^3\sqrt{1+\frac{b}{c x^2}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)}\right)$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2}\left(b x^2+c x^4\right)^{3/2}\, dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{56 b^4 x^{3/2}\left(b+c x^2\right)}{1105 c^{5/2}\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{56 b^3\sqrt{x}\sqrt{b x^2+c x^4}}{3315 c^2}+\frac{8 b^2 x^{5/2}\sqrt{b x^2+c x^4}}{663 c}+\frac{12}{221} b x^{9/2}\sqrt{b x^2+c x^4}+\frac{2}{17} x^{5/2}\left(b x^2+c x^4\right)^{3/2}-$$
$$\frac{56 b^{17/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{1105 c^{11/4}\sqrt{b x^2+c x^4}}+\frac{28 b^{17/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{1105 c^{11/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 212 leaves):

$$\left(2 x^{3/2}\left(\sqrt{c}\, x\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\left(-28 b^4-8 b^3 c x^2+305 b^2 c^2 x^4+480 b c^3 x^6+195 c^4 x^8\right)+84 b^{9/2}\sqrt{1+\frac{c x^2}{b}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]-\right.\right.$$
$$\left.\left.84 b^{9/2}\sqrt{1+\frac{c x^2}{b}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(3315 c^{5/2}\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}\right)$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{x}\left(b x^2+c x^4\right)^{3/2}\, dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$-\frac{8 b^3 \sqrt{b x^2+c x^4}}{231 c^2 \sqrt{x}}+\frac{8 b^2 x^{3/2} \sqrt{b x^2+c x^4}}{385 c}+\frac{4}{55} b x^{7/2} \sqrt{b x^2+c x^4}+$$
$$\frac{2}{15} x^{3/2} \left(b x^2+c x^4\right)^{3/2}+\frac{4 b^{15/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 c^{9/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 164 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(-20 b^4-8 b^3 c x^2+131 b^2 c^2 x^4+196 b c^3 x^6+77 c^4 x^8\right)+40\, i\, b^4 \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/$$
$$\left(1155 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^2 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 366: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{\sqrt{x}}\, dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$-\frac{8 b^3 x^{3/2} \left(b+c x^2\right)}{65 c^{3/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{8 b^2 \sqrt{x} \sqrt{b x^2+c x^4}}{195 c}+\frac{4}{39} b x^{5/2} \sqrt{b x^2+c x^4}+\frac{2}{13} \sqrt{x} \left(b x^2+c x^4\right)^{3/2}+$$
$$\frac{8 b^{13/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{65 c^{7/4} \sqrt{b x^2+c x^4}}-\frac{4 b^{13/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{65 c^{7/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 201 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(4 b^3+29 b^2 c x^2+40 b c^2 x^4+15 c^3 x^6\right)-12 b^{7/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]+\right.\right.$$
$$\left.\left.12 b^{7/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(195 c^{3/2} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 367: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{3/2}}\,dx$$

Optimal (type 4, 173 leaves, 6 steps):

$$\frac{8 b^2 \sqrt{b x^2+c x^4}}{77 c \sqrt{x}}+\frac{12}{77} b x^{3/2} \sqrt{b x^2+c x^4}+\frac{2 \left(b x^2+c x^4\right)^{3/2}}{11 \sqrt{x}}-\frac{4 b^{11/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 c^{5/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 153 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(4 b^3+17 b^2 c x^2+20 b c^2 x^4+7 c^3 x^6\right)-8 i b^3 \sqrt{1+\frac{b}{c x^2}}\, x^2 \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Bigg/ \left(77 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 368: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{5/2}}\,dx$$

Optimal (type 4, 290 leaves, 7 steps):

$$\frac{8 b^2 x^{3/2} \left(b+c x^2\right)}{15 \sqrt{c} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{4}{15} b \sqrt{x} \sqrt{b x^2+c x^4}+\frac{2 \left(b x^2+c x^4\right)^{3/2}}{9 x^{3/2}}-$$

$$\frac{8 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{b x^2+c x^4}}+\frac{4 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(11 b^2+16 b c x^2+5 c^2 x^4\right)+12 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]-\right.\right.$$

$$\left.\left.12 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Bigg/ \left(45 \sqrt{c} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 369: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{7/2}} \, dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$\frac{4 b \sqrt{b x^2+c x^4}}{7 \sqrt{x}}+\frac{2 \left(b x^2+c x^4\right)^{3/2}}{7 x^{5/2}}+\frac{4 b^{7/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{7 c^{1/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 119 leaves):

$$\frac{2 x^{3/2} \left(3 b^2+4 b c x^2+c^2 x^4+\frac{4 i b^2 \sqrt{1+\frac{b}{c x^2}} \sqrt{x}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}\right)}{7 \sqrt{x^2 \left(b+c x^2\right)}}$$

Problem 370: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{9/2}} \, dx$$

Optimal (type 4, 286 leaves, 7 steps):

$$\frac{24 b \sqrt{c}\, x^{3/2} \left(b+c x^2\right)}{5 \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{12}{5} c \sqrt{x} \sqrt{b x^2+c x^4}-\frac{2 \left(b x^2+c x^4\right)^{3/2}}{x^{7/2}}-$$

$$\frac{24 b^{5/4} c^{1/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{b x^2+c x^4}}+$$

$$\frac{12 b^{5/4} c^{1/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{b x^2+c x^4}}$$

Result (type 4, 190 leaves):

$$\frac{1}{5\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\sqrt{x^2\left(b+c\,x^2\right)}}2\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(-5\,b^2-4\,b\,c\,x^2+c^2\,x^4\right)+\right.$$

$$\left.12\,b^{3/2}\sqrt{c}\,x\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-12\,b^{3/2}\sqrt{c}\,x\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)$$

Problem 371: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(b\,x^2+c\,x^4\right)^{3/2}}{x^{11/2}}\,dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$\frac{4\,c\sqrt{b\,x^2+c\,x^4}}{3\sqrt{x}}-\frac{2\left(b\,x^2+c\,x^4\right)^{3/2}}{3\,x^{9/2}}+\frac{4\,b^{3/4}\,c^{3/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 111 leaves):

$$\frac{2\left(-b^2+c^2\,x^4+\frac{4\,i\,b\,c\sqrt{1+\frac{b}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}\right)}{3\sqrt{x}\sqrt{x^2\left(b+c\,x^2\right)}}$$

Problem 372: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(b\,x^2+c\,x^4\right)^{3/2}}{x^{13/2}}\,dx$$

Optimal (type 4, 287 leaves, 7 steps):

$$\frac{24\,c^{3/2}\,x^{3/2}\,\left(b+c\,x^{2}\right)}{5\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{b\,x^{2}+c\,x^{4}}}-\frac{12\,c\,\sqrt{b\,x^{2}+c\,x^{4}}}{5\,x^{3/2}}-\frac{2\,\left(b\,x^{2}+c\,x^{4}\right)^{3/2}}{5\,x^{11/2}}-\frac{24\,b^{1/4}\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^{2}}{\left(\sqrt{b}+\sqrt{c}\,x\right)^{2}}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\,\frac{1}{2}\right]}{5\,\sqrt{b\,x^{2}+c\,x^{4}}}+$$

$$\frac{12\,b^{1/4}\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^{2}}{\left(\sqrt{b}+\sqrt{c}\,x\right)^{2}}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\,\frac{1}{2}\right]}{5\,\sqrt{b\,x^{2}+c\,x^{4}}}$$

Result (type 4, 193 leaves):

$$-\left(\left(2\left(\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,\left(b^{2}+8\,b\,c\,x^{2}+7\,c^{2}\,x^{4}\right)-12\,\sqrt{b}\,c^{3/2}\,x^{3}\,\sqrt{1+\frac{c\,x^{2}}{b}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],\,-1\right]+\right.\right.\right.$$

$$\left.\left.\left.12\,\sqrt{b}\,c^{3/2}\,x^{3}\,\sqrt{1+\frac{c\,x^{2}}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],\,-1\right]\right)\right)\Big/\left(5\,x^{3/2}\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^{2}\,\left(b+c\,x^{2}\right)}\right)\right)$$

Problem 373: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(b\,x^{2}+c\,x^{4}\right)^{3/2}}{x^{15/2}}\,dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$-\frac{4\,c\,\sqrt{b\,x^{2}+c\,x^{4}}}{7\,x^{5/2}}-\frac{2\,\left(b\,x^{2}+c\,x^{4}\right)^{3/2}}{7\,x^{13/2}}+\frac{4\,c^{7/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^{2}}{\left(\sqrt{b}+\sqrt{c}\,x\right)^{2}}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\,\frac{1}{2}\right]}{7\,b^{1/4}\,\sqrt{b\,x^{2}+c\,x^{4}}}$$

Result (type 4, 120 leaves):

$$\frac{2\left(-b^{2}-4\,b\,c\,x^{2}-3\,c^{2}\,x^{4}+\frac{4\,i\,c^{2}\,\sqrt{1+\frac{b}{c\,x^{2}}}\;x^{9/2}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}\right)}{7\,x^{5/2}\,\sqrt{x^{2}\,\left(b+c\,x^{2}\right)}}$$

Problem 374: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{17/2}}\, dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$\frac{8 c^{5/2} x^{3/2} \left(b+c x^2\right)}{15 b \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{4 c \sqrt{b x^2+c x^4}}{15 x^{7/2}}-\frac{8 c^2 \sqrt{b x^2+c x^4}}{15 b x^{3/2}}-\frac{2 \left(b x^2+c x^4\right)^{3/2}}{9 x^{15/2}}-$$
$$\frac{8 c^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{3/4} \sqrt{b x^2+c x^4}}+\frac{4 c^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{3/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 209 leaves):

$$-\left(\left(2 \left(\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(5 b^3+16 b^2 c x^2+23 b c^2 x^4+12 c^3 x^6\right)-12 \sqrt{b}\, c^{5/2} x^5 \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]+\right.\right.\right.$$
$$\left.\left.\left.12 \sqrt{b}\, c^{5/2} x^5 \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(45 b x^{7/2} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)\right)$$

Problem 375: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{19/2}}\, dx$$

Optimal (type 4, 173 leaves, 6 steps):

$$-\frac{12 c \sqrt{b x^2+c x^4}}{77 x^{9/2}}-\frac{8 c^2 \sqrt{b x^2+c x^4}}{77 b x^{5/2}}-\frac{2 \left(b x^2+c x^4\right)^{3/2}}{11 x^{17/2}}-\frac{4 c^{11/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 b^{5/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 154 leaves):

$$-\left(\left(2\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(7 b^3+20 b^2 c x^2+17 b c^2 x^4+4 c^3 x^6\right)+4 i c^3 \sqrt{1+\frac{b}{c x^2}}\, x^{13/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(77 b \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{9/2} \sqrt{x^2\left(b+c x^2\right)}\right)\right)$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{21/2}}\, dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$-\frac{8 c^{7/2} x^{3/2}\left(b+c x^2\right)}{65 b^2\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{4 c \sqrt{b x^2+c x^4}}{39 x^{11/2}}-\frac{8 c^2 \sqrt{b x^2+c x^4}}{195 b x^{7/2}}+\frac{8 c^3 \sqrt{b x^2+c x^4}}{65 b^2 x^{3/2}}-\frac{2\left(b x^2+c x^4\right)^{3/2}}{13 x^{19/2}}+$$
$$\frac{8 c^{13/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{65 b^{7/4}\sqrt{b x^2+c x^4}}-\frac{4 c^{13/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{65 b^{7/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 220 leaves):

$$\left(2\left(\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\left(-15 b^4-40 b^3 c x^2-29 b^2 c^2 x^4+8 b c^3 x^6+12 c^4 x^8\right)-12\sqrt{b}\, c^{7/2} x^7 \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]+12\sqrt{b}\, c^{7/2} x^7 \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(195 b^2 x^{11/2}\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}\right)$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(b x^2+c x^4\right)^{3/2}}{x^{23/2}}\, dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$-\frac{4 c \sqrt{b x^2+c x^4}}{55 x^{13/2}}-\frac{8 c^2 \sqrt{b x^2+c x^4}}{385 b x^{9/2}}+\frac{8 c^3 \sqrt{b x^2+c x^4}}{231 b^2 x^{5/2}}-\frac{2 \left(b x^2+c x^4\right)^{3/2}}{15 x^{21/2}}+\frac{4 c^{15/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 b^{9/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 165 leaves):

$$\left(2 \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(-77 b^4-196 b^3 c x^2-131 b^2 c^2 x^4+8 b c^3 x^6+20 c^4 x^8\right)+20 i c^4 \sqrt{1+\frac{b}{c x^2}}\, x^{17/2}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Bigg/ \left(1155 b^2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{13/2} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2}}{\sqrt{b x^2+c x^4}}\, dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$\frac{30 b^2 \sqrt{b x^2+c x^4}}{77 c^3 \sqrt{x}}-\frac{18 b x^{3/2} \sqrt{b x^2+c x^4}}{77 c^2}+\frac{2 x^{7/2} \sqrt{b x^2+c x^4}}{11 c}-\frac{15 b^{11/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 c^{13/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 153 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(15 b^3+6 b^2 c x^2-2 b c^2 x^4+7 c^3 x^6\right)-30 i b^3 \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Bigg/ \left(77 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^3 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2}}{\sqrt{b x^2+c x^4}}\, dx$$

Optimal (type 4, 296 leaves, 7 steps):

$$\frac{14 b^2 x^{3/2} \left(b+c x^2\right)}{15 c^{5/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{14 b \sqrt{x} \sqrt{b x^2+c x^4}}{45 c^2}+\frac{2 x^{5/2} \sqrt{b x^2+c x^4}}{9 c}-$$

$$\frac{14 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{11/4} \sqrt{b x^2+c x^4}}+\frac{7 b^{9/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{11/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(-7 b^2-2 b c x^2+5 c^2 x^4\right)+21 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\ \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]-\right.\right.$$

$$\left.\left.21 b^{5/2} \sqrt{1+\frac{c x^2}{b}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(45 c^{5/2} \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2}}{\sqrt{b x^2+c x^4}}\, dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$-\frac{10 b \sqrt{b x^2+c x^4}}{21 c^2 \sqrt{x}}+\frac{2 x^{3/2} \sqrt{b x^2+c x^4}}{7 c}+\frac{5 b^{7/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21 c^{9/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 144 leaves):

$$\frac{x \left(b+c x^2\right) \left(-\frac{10 b \sqrt{x}}{21 c^2}+\frac{2 x^{5/2}}{7 c}\right)}{\sqrt{x^2 \left(b+c x^2\right)}}+\frac{10\, i\, b^2 \sqrt{1+\frac{b}{c x^2}}\ x^2\ \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{21 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\ c^2 \sqrt{x^2 \left(b+c x^2\right)}}$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2}}{\sqrt{b x^2+c x^4}}\, dx$$

Optimal (type 4, 266 leaves, 6 steps):

$$-\frac{6\,b\,x^{3/2}\left(b+c\,x^2\right)}{5\,c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}+\frac{2\,\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}{5\,c}+$$

$$\frac{6\,b^{5/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{7/4}\,\sqrt{b\,x^2+c\,x^4}}-\frac{3\,b^{5/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{7/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 176 leaves):

$$\left(2\,x^{3/2}\left(\sqrt{c}\,x\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\left(b+c\,x^2\right)-3\,b^{3/2}\,\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+\right.\right.$$

$$\left.\left.3\,b^{3/2}\,\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(5\,c^{3/2}\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

Problem 382: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{5/2}}{\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 121 leaves, 4 steps):

$$\frac{2\,\sqrt{b\,x^2+c\,x^4}}{3\,c\,\sqrt{x}}-\frac{b^{3/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,c^{5/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 126 leaves):

$$\frac{2\,x^{3/2}\left(b+c\,x^2\right)}{3\,c\,\sqrt{x^2\left(b+c\,x^2\right)}}-\frac{2\,i\,b\,\sqrt{1+\frac{b}{c\,x^2}}\;x^2\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{3\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\;c\,\sqrt{x^2\left(b+c\,x^2\right)}}$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}}{\sqrt{b x^2 + c x^4}} \, dx$$

Optimal (type 4, 231 leaves, 5 steps):

$$\frac{2 x^{3/2} \left(b + c x^2\right)}{\sqrt{c} \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{b x^2 + c x^4}} - \frac{2 b^{1/4} x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b + c x^2}{\left(\sqrt{b} + \sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{b^{1/4} x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b + c x^2}{\left(\sqrt{b} + \sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 112 leaves):

$$\frac{2 i x^{5/2} \sqrt{1 + \frac{c x^2}{b}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)}{\left(\frac{i \sqrt{c}\, x}{\sqrt{b}}\right)^{3/2} \sqrt{x^2 \left(b + c x^2\right)}}$$

Problem 384: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\sqrt{b x^2 + c x^4}} \, dx$$

Optimal (type 4, 90 leaves, 3 steps):

$$\frac{x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b + c x^2}{\left(\sqrt{b} + \sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{1/4} c^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 85 leaves):

$$\frac{2 i \sqrt{1 + \frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x^2 \left(b + c x^2\right)}}$$

Problem 385: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{x}\,\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$\frac{2\sqrt{c}\,x^{3/2}\left(b+c x^2\right)}{b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b x^2+c x^4}}-\frac{2\sqrt{b x^2+c x^4}}{b\,x^{3/2}}-\frac{2\,c^{1/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{b x^2+c x^4}}+$$

$$\frac{c^{1/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 177 leaves):

$$-\frac{1}{b\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}}2\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(b+c x^2\right)-\right.$$

$$\left.\sqrt{b}\,\sqrt{c}\,x\,\sqrt{1+\frac{c x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+\sqrt{b}\,\sqrt{c}\,x\,\sqrt{1+\frac{c x^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{3/2}\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 121 leaves, 4 steps):

$$-\frac{2\sqrt{b x^2+c x^4}}{3\,b\,x^{5/2}}-\frac{c^{3/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,b^{5/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 110 leaves):

$$\frac{2\left(-b-c\,x^2-\frac{i\,c\,\sqrt{1+\frac{b}{c\,x^2}}\;x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}\right)}{3\,b\,\sqrt{x}\;\sqrt{x^2\,\left(b+c\,x^2\right)}}$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{5/2}\,\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 296 leaves, 7 steps):

$$-\frac{6\,c^{3/2}\,x^{3/2}\,\left(b+c\,x^2\right)}{5\,b^2\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{b\,x^2+c\,x^4}}-\frac{2\,\sqrt{b\,x^2+c\,x^4}}{5\,b\,x^{7/2}}+\frac{6\,c\,\sqrt{b\,x^2+c\,x^4}}{5\,b^2\,x^{3/2}}+$$
$$\frac{6\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,b^{7/4}\,\sqrt{b\,x^2+c\,x^4}}-\frac{3\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,b^{7/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 199 leaves):

$$\left(2\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,\left(-b^2+2\,b\,c\,x^2+3\,c^2\,x^4\right)-6\,\sqrt{b}\,c^{3/2}\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+\right.$$
$$\left.6\,\sqrt{b}\,c^{3/2}\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Big/\left(5\,b^2\,x^{3/2}\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\;\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

Problem 388: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{7/2}\,\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$-\frac{2\,\sqrt{b\,x^2+c\,x^4}}{7\,b\,x^{9/2}}+\frac{10\,c\,\sqrt{b\,x^2+c\,x^4}}{21\,b^2\,x^{5/2}}+\frac{5\,c^{7/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{21\,b^{9/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 144 leaves):

$$\frac{\left(-\frac{2}{7\,b\,x^{7/2}}+\frac{10\,c}{21\,b^2\,x^{3/2}}\right)x\left(b+c\,x^2\right)}{\sqrt{x^2\left(b+c\,x^2\right)}}+\frac{10\,i\,c^2\sqrt{1+\frac{b}{c\,x^2}}\;x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{21\,b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\;\sqrt{x^2\left(b+c\,x^2\right)}}$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^{9/2}\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 326 leaves, 8 steps):

$$\frac{14\,c^{5/2}\,x^{3/2}\left(b+c\,x^2\right)}{15\,b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}-\frac{2\sqrt{b\,x^2+c\,x^4}}{9\,b\,x^{11/2}}+\frac{14\,c\sqrt{b\,x^2+c\,x^4}}{45\,b^2\,x^{7/2}}-\frac{14\,c^2\sqrt{b\,x^2+c\,x^4}}{15\,b^3\,x^{3/2}}-$$

$$\frac{14\,c^{9/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{15\,b^{11/4}\sqrt{b\,x^2+c\,x^4}}+\frac{7\,c^{9/4}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{15\,b^{11/4}\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 210 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(5\,b^3-2\,b^2\,c\,x^2+14\,b\,c^2\,x^4+21\,c^3\,x^6\right)+42\sqrt{b}\,c^{5/2}\,x^5\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-\right.$$

$$\left.42\sqrt{b}\,c^{5/2}\,x^5\sqrt{1+\frac{c\,x^2}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Bigg/\left(45\,b^3\,x^{7/2}\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\;\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^{11/2}\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$-\frac{2\sqrt{b x^2+c x^4}}{11 b x^{13/2}}+\frac{18 c \sqrt{b x^2+c x^4}}{77 b^2 x^{9/2}}-\frac{30 c^2 \sqrt{b x^2+c x^4}}{77 b^3 x^{5/2}}-\frac{15 c^{11/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 b^{13/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 134 leaves):

$$\frac{2 \left(-7 b^3+2 b^2 c x^2-6 b c^2 x^4-15 c^3 x^6-\frac{15\, i\, c^3 \sqrt{1+\frac{b}{c x^2}}\, x^{13/2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}\right)}{77 b^3 x^{9/2} \sqrt{x^2 \left(b+c x^2\right)}}$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{17/2}}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 174 leaves, 6 steps):

$$-\frac{x^{11/2}}{c \sqrt{b x^2+c x^4}}-\frac{15 b \sqrt{b x^2+c x^4}}{7 c^3 \sqrt{x}}+\frac{9 x^{3/2} \sqrt{b x^2+c x^4}}{7 c^2}+\frac{15 b^{7/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{14 c^{13/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 141 leaves):

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(-15 b^2-6 b c x^2+2 c^2 x^4\right)+15\, i\, b^2 \sqrt{1+\frac{b}{c x^2}}\, x^2\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^3 \sqrt{x^2 \left(b+c x^2\right)}}$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{15/2}}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 291 leaves, 7 steps):

$$-\frac{x^{9/2}}{c\sqrt{b x^2+c x^4}}-\frac{21 b x^{3/2}\left(b+c x^2\right)}{5 c^{5/2}\left(\sqrt{b}+\sqrt{c} x\right)\sqrt{b x^2+c x^4}}+\frac{7\sqrt{x}\sqrt{b x^2+c x^4}}{5 c^2}+$$
$$\frac{21 b^{5/4} x\left(\sqrt{b}+\sqrt{c} x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5 c^{11/4}\sqrt{b x^2+c x^4}}-\frac{21 b^{5/4} x\left(\sqrt{b}+\sqrt{c} x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{10 c^{11/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 179 leaves):

$$\left(x^{3/2}\left(\sqrt{c}\,x\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(7 b+2 c x^2\right)-21 b^{3/2}\sqrt{1+\frac{c x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+\right.\right.$$
$$\left.\left.21 b^{3/2}\sqrt{1+\frac{c x^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(5 c^{5/2}\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}\right)$$

Problem 393: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{13/2}}{\left(b x^2+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{x^{7/2}}{c\sqrt{b x^2+c x^4}}+\frac{5\sqrt{b x^2+c x^4}}{3 c^2\sqrt{x}}-\frac{5 b^{3/4} x\left(\sqrt{b}+\sqrt{c} x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{6 c^{9/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 128 leaves):

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{3/2}\left(5 b+2 c x^2\right)-5\,i\,b\sqrt{1+\frac{b}{c x^2}}\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,c^2\sqrt{x^2\left(b+c x^2\right)}}$$

Problem 394: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{11/2}}{\left(b x^2+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$-\frac{x^{5/2}}{c\sqrt{b x^2+c x^4}}+\frac{3 x^{3/2}\left(b+c x^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{3 b^{1/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{c^{7/4}\sqrt{b x^2+c x^4}}+$$
$$\frac{3 b^{1/4} x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{2 c^{7/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 167 leaves):

$$-\left(\left(x^{3/2}\left(\sqrt{c}\, x\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}-3\sqrt{b}\sqrt{1+\frac{c x^2}{b}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]+\right.\right.\right.$$
$$\left.\left.\left.3\sqrt{b}\sqrt{1+\frac{c x^2}{b}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(c^{3/2}\sqrt{\frac{i\sqrt{c}\, x}{\sqrt{b}}}\sqrt{x^2\left(b+c x^2\right)}\right)\right)$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2}}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$-\frac{x^{3/2}}{c\sqrt{b x^2+c x^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{2 b^{1/4} c^{5/4}\sqrt{b x^2+c x^4}}$$

Result (type 4, 115 leaves):

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{3/2}+i\sqrt{1+\frac{b}{c x^2}}\, x^2\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, c\sqrt{x^2\left(b+c x^2\right)}}$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2}}{\left(b x^2 + c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 260 leaves, 6 steps):

$$\frac{x^{5/2}}{b \sqrt{b x^2 + c x^4}} - \frac{x^{3/2} \left(b + c x^2\right)}{b \sqrt{c} \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{b x^2 + c x^4}} + \frac{x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} c^{3/4} \sqrt{b x^2 + c x^4}} - \frac{x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{3/4} c^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 168 leaves):

$$\left(i\, x^{5/2} \left(\sqrt{c}\, x \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} - \sqrt{b} \sqrt{1 + \frac{c x^2}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right] + \sqrt{b} \sqrt{1 + \frac{c x^2}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right], -1\right]\right)\right) \Big/ \left(b^{3/2} \left(\frac{i \sqrt{c}\, x}{\sqrt{b}}\right)^{3/2} \sqrt{x^2 \left(b + c x^2\right)}\right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2}}{\left(b x^2 + c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{x^{3/2}}{b \sqrt{b x^2 + c x^4}} + \frac{x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{5/4} c^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 114 leaves):

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} + i \sqrt{1 + \frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x^2 \left(b + c x^2\right)}}$$

Problem 398: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}}{\left(b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 286 leaves, 7 steps):

$$\frac{\sqrt{x}}{b \sqrt{b x^2+c x^4}}+\frac{3 \sqrt{c}\, x^{3/2} \left(b+c x^2\right)}{b^2 \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{3 \sqrt{b x^2+c x^4}}{b^2 x^{3/2}}-\frac{3 c^{1/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{b^{7/4} \sqrt{b x^2+c x^4}}+\frac{3 c^{1/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{2 b^{7/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 181 leaves):

$$-\left(\left(\sqrt{x} \left(\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \left(2 b+3 c x^2\right)-3 \sqrt{b} \sqrt{c}\, x \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]+3 \sqrt{b} \sqrt{c}\, x \sqrt{1+\frac{c x^2}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}}\right],-1\right]\right)\right) \Big/ \left(b^2 \sqrt{\frac{i \sqrt{c}\, x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)\right)$$

Problem 399: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\left(b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$\frac{1}{b \sqrt{x} \sqrt{b x^2+c x^4}}-\frac{5 \sqrt{b x^2+c x^4}}{3 b^2 x^{5/2}}-\frac{5 c^{3/4} x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{6 b^{9/4} \sqrt{b x^2+c x^4}}$$

Result (type 4, 110 leaves):

$$\frac{-2\,b-5\,c\,x^2-\frac{5\,\mathrm{i}\,c\,\sqrt{1+\frac{b}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{c}}}}}{3\,b^2\,\sqrt{x}\,\sqrt{x^2\,\left(b+c\,x^2\right)}}$$

Problem 400: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{x}\,\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$\frac{1}{b\,x^{3/2}\,\sqrt{b\,x^2+c\,x^4}}-\frac{21\,c^{3/2}\,x^{3/2}\,\left(b+c\,x^2\right)}{5\,b^3\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{b\,x^2+c\,x^4}}-\frac{7\,\sqrt{b\,x^2+c\,x^4}}{5\,b^2\,x^{7/2}}+\frac{21\,c\,\sqrt{b\,x^2+c\,x^4}}{5\,b^3\,x^{3/2}}+$$
$$\frac{21\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,b^{11/4}\,\sqrt{b\,x^2+c\,x^4}}-\frac{21\,c^{5/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{10\,b^{11/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 198 leaves):

$$\left(\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\,\left(-2\,b^2+14\,b\,c\,x^2+21\,c^2\,x^4\right)-21\,\sqrt{b}\,c^{3/2}\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+\right.$$
$$\left.21\,\sqrt{b}\,c^{3/2}\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Big/\left(5\,b^3\,x^{3/2}\,\sqrt{\frac{\mathrm{i}\,\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

Problem 401: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{3/2}\,\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 173 leaves, 6 steps):

$$\frac{1}{b\,x^{5/2}\,\sqrt{b\,x^2+c\,x^4}}-\frac{9\,\sqrt{b\,x^2+c\,x^4}}{7\,b^2\,x^{9/2}}+\frac{15\,c\,\sqrt{b\,x^2+c\,x^4}}{7\,b^3\,x^{5/2}}+\frac{15\,c^{7/4}\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{14\,b^{13/4}\,\sqrt{b\,x^2+c\,x^4}}$$

Result (type 4, 143 leaves):

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-2b^2+6bcx^2+15c^2x^4\right)+15\,i\,c^2\sqrt{1+\frac{b}{cx^2}}\,x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{7b^3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Problem 402: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}+\frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}}+\frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}-$$
$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{15b^{15/4}\sqrt{bx^2+cx^4}}+\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]}{30b^{15/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 210 leaves):

$$\left(-\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(10b^3-22b^2cx^2+154bc^2x^4+231c^3x^6\right)+231\sqrt{b}\,c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-\right.$$
$$\left.231\sqrt{b}\,c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Big/\left(45b^4x^{7/2}\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}\right)$$

Problem 407: Result more than twice size of optimal antiderivative.

$$\int\frac{(cx)^m}{\left(bx^2+cx^4\right)^2}\,dx$$

Optimal (type 5, 45 leaves, 3 steps):

$$-\frac{(cx)^m\,\text{Hypergeometric2F1}\left[2,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right]}{b^2(3-m)x^3}$$

Result (type 5, 109 leaves):

$$\frac{(c\,x)^m\left(b\left(\frac{b}{-3+m}-\frac{2\,c\,x^2}{-1+m}\right)+\frac{2\,c^2\,x^4\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{c\,x^2}{b}\right]}{1+m}+\frac{c^2\,x^4\,\text{Hypergeometric2F1}\left[2,\frac{1+m}{2},\frac{3+m}{2},-\frac{c\,x^2}{b}\right]}{1+m}\right)}{b^4\,x^3}$$

Problem 408: Result more than twice size of optimal antiderivative.

$$\int\frac{(c\,x)^m}{\left(b\,x^2+c\,x^4\right)^3}\,dx$$

Optimal (type 5, 45 leaves, 3 steps):

$$-\frac{(c\,x)^m\,\text{Hypergeometric2F1}\left[3,\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),-\frac{c\,x^2}{b}\right]}{b^3\,(5-m)\,x^5}$$

Result (type 5, 164 leaves):

$$\frac{1}{b^6\,x^5}(c\,x)^m\left(\frac{b^3}{-5+m}-\frac{3\,b^2\,c\,x^2}{-3+m}+\frac{6\,b\,c^2\,x^4}{-1+m}-\frac{6\,c^3\,x^6\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{c\,x^2}{b}\right]}{1+m}-\frac{3\,c^3\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{1+m}{2},\frac{3+m}{2},-\frac{c\,x^2}{b}\right]}{1+m}-\frac{c^3\,x^6\,\text{Hypergeometric2F1}\left[3,\frac{1+m}{2},\frac{3+m}{2},-\frac{c\,x^2}{b}\right]}{1+m}\right)$$

Problem 438: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^2}{x^{11}}\,dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{\left(a+b\,x^2\right)^5}{10\,a\,x^{10}}$$

Result (type 1, 52 leaves):

$$-\frac{a^4}{10\,x^{10}}-\frac{a^3\,b}{2\,x^8}-\frac{a^2\,b^2}{x^6}-\frac{a\,b^3}{x^4}-\frac{b^4}{2\,x^2}$$

Problem 449: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^3\,dx$$

Optimal (type 1, 34 leaves, 4 steps):

$$-\frac{a\left(a+b\,x^2\right)^7}{14\,b^2}+\frac{\left(a+b\,x^2\right)^8}{16\,b^2}$$

Result (type 1, 77 leaves):

$$\frac{a^6\,x^4}{4}+a^5\,b\,x^6+\frac{15}{8}\,a^4\,b^2\,x^8+2\,a^3\,b^3\,x^{10}+\frac{5}{4}\,a^2\,b^4\,x^{12}+\frac{3}{7}\,a\,b^5\,x^{14}+\frac{b^6\,x^{16}}{16}$$

Problem 467: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^3}{x^{15}}\,dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{\left(a+b\,x^2\right)^7}{14\,a\,x^{14}}$$

Result (type 1, 82 leaves):

$$-\frac{a^6}{14\,x^{14}}-\frac{a^5\,b}{2\,x^{12}}-\frac{3\,a^4\,b^2}{2\,x^{10}}-\frac{5\,a^3\,b^3}{2\,x^8}-\frac{5\,a^2\,b^4}{2\,x^6}-\frac{3\,a\,b^5}{2\,x^4}-\frac{b^6}{2\,x^2}$$

Problem 515: Result more than twice size of optimal antiderivative.

$$\int \frac{x^9}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^3}\,dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$\frac{x^{10}}{10\,a\left(a+b\,x^2\right)^5}$$

Result (type 1, 57 leaves):

$$-\frac{a^4+5\,a^3\,b\,x^2+10\,a^2\,b^2\,x^4+10\,a\,b^3\,x^6+5\,b^4\,x^8}{10\,b^5\left(a+b\,x^2\right)^5}$$

Problem 659: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}} \, dx$$

Optimal (type 4, 298 leaves, 4 steps):

$$\frac{3 x \left(a+b x^2\right)}{5 b \left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}}+\left(3\times 3^{3/4} \sqrt{2-\sqrt{3}}\, a^2 \left(1+\frac{b x^2}{a}\right)^{2/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right], -7+4 \sqrt{3}\right]\right) \Bigg/ \left(5 b^2 x \left(a^2+2 a b x^2+b^2 x^4\right)^{1/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 64 leaves):

$$\frac{3 x \left(a+b x^2-a \left(1+\frac{b x^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{b x^2}{a}\right]\right)}{5 b \left(\left(a+b x^2\right)^2\right)^{1/3}}$$

Problem 660: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}} \, dx$$

Optimal (type 4, 256 leaves, 3 steps):

$$-\left(\left(3^{3/4} \sqrt{2-\sqrt{3}}\, a \left(1+\frac{b x^2}{a}\right)^{2/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right], -7+4 \sqrt{3}\right]\right) \Bigg/ \left(b x \left(a^2+2 a b x^2+b^2 x^4\right)^{1/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)\right)$$

Result (type 5, 49 leaves):

$$\frac{x\left(\frac{a+b x^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{b x^2}{a}\right]}{\left(\left(a+b x^2\right)^2\right)^{1/3}}$$

Problem 661: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}}\,dx$$

Optimal (type 4, 289 leaves, 4 steps):

$$-\frac{a+b x^2}{a x\left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}}+$$

$$\left(\sqrt{2-\sqrt{3}}\left(1+\frac{b x^2}{a}\right)^{2/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4} x\left(a^2+2 a b x^2+b^2 x^4\right)^{1/3}\sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 72 leaves):

$$\frac{-3\left(a+b x^2\right)-b x^2\left(1+\frac{b x^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{b x^2}{a}\right]}{3 a x\left(\left(a+b x^2\right)^2\right)^{1/3}}$$

Problem 662: Result unnecessarily involves higher level functions.

$$\int\frac{x^2}{\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}}\,dx$$

Optimal (type 4, 618 leaves, 6 steps):

$$-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)}+$$

$$\left(9\times3^{1/4}\sqrt{2+\sqrt{3}}\;a^2\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)\sqrt{\frac{1+\left(1+\frac{bx^2}{a}\right)^{1/3}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\left(1+\frac{bx^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}}\right)-$$

$$\left(3\times3^{3/4}a^2\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)\sqrt{\frac{1+\left(1+\frac{bx^2}{a}\right)^{1/3}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(\sqrt{2}\,b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\left(1+\frac{bx^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 64 leaves):

$$\frac{3x\left(a+bx^2\right)\left(-1+\left(1+\frac{bx^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right]\right)}{2b\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Problem 663: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$$

Optimal (type 4, 609 leaves, 6 steps):

$$\frac{3 x \left(a+b x^2\right)}{2 a \left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}}+\frac{3 x \left(1+\frac{b x^2}{a}\right)^{4/3}}{2 \left(a^2+2 a b x^2+b^2 x^4\right)^{2/3} \left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)}-$$

$$\left(3\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a \left(1+\frac{b x^2}{a}\right)^{4/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right],\ -7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(4 b x \left(a^2+2 a b x^2+b^2 x^4\right)^{2/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)+$$

$$\left(3^{3/4} a \left(1+\frac{b x^2}{a}\right)^{4/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right],\ -7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\, b x \left(a^2+2 a b x^2+b^2 x^4\right)^{2/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 64 leaves):

$$-\frac{x \left(a+b x^2\right) \left(-3+\left(1+\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{b x^2}{a}\right]\right)}{2 a \left(\left(a+b x^2\right)^2\right)^{2/3}}$$

Problem 664: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}}\, dx$$

Optimal (type 4, 649 leaves, 7 steps):

$$\frac{3\left(a+b x^2\right)}{2 a x\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}}-\frac{5\left(a+b x^2\right)^2}{2 a^2 x\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}}-\frac{5 b x\left(1+\frac{b x^2}{a}\right)^{4/3}}{2 a\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)}+$$

$$\left(5\times 3^{1/4}\sqrt{2+\sqrt{3}}\left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(4 x\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}\sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)-$$

$$\left(5\left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(\sqrt{2}\ 3^{1/4} x\left(a^2+2 a b x^2+b^2 x^4\right)^{2/3}\sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 79 leaves) :

$$-\frac{\left(a+b x^2\right)\left(6 a+15 b x^2-5 b x^2\left(1+\frac{b x^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{b x^2}{a}\right]\right)}{6 a^2 x\left(\left(a+b x^2\right)^2\right)^{2/3}}$$

Problem 909: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x\left(a+b+2 a x^2+a x^4\right)}\,dx$$

Optimal (type 3, 69 leaves, 7 steps) :

$$-\frac{\sqrt{a}\ \text{ArcTan}\left[\frac{\sqrt{a}\ \left(1+x^2\right)}{\sqrt{b}}\right]}{2\sqrt{b}\ (a+b)}+\frac{\text{Log}[x]}{a+b}-\frac{\text{Log}\left[a+b+2 a x^2+a x^4\right]}{4 (a+b)}$$

Result (type 3, 105 leaves) :

$$\frac{4\sqrt{b}\,\text{Log}[x]+i\left(\sqrt{a}+i\sqrt{b}\right)\text{Log}\left[-i\sqrt{b}+\sqrt{a}\,(1+x^2)\right]+\left(-i\sqrt{a}-\sqrt{b}\right)\text{Log}\left[i\sqrt{b}+\sqrt{a}\,(1+x^2)\right]}{4\sqrt{b}\,(a+b)}$$

Problem 910: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^3\left(a+b+2\,a\,x^2+a\,x^4\right)}\,dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{1}{2\,(a+b)\,x^2}+\frac{\sqrt{a}\,(a-b)\,\text{ArcTan}\left[\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right]}{2\sqrt{b}\,(a+b)^2}-\frac{2\,a\,\text{Log}[x]}{(a+b)^2}+\frac{a\,\text{Log}\left[a+b+2\,a\,x^2+a\,x^4\right]}{2\,(a+b)^2}$$

Result (type 3, 163 leaves):

$$-\frac{1}{2\,(a+b)\,x^2}-\frac{2\,a\,\text{Log}[x]}{(a+b)^2}+\frac{\left(-i\,a^2+2\,a^{3/2}\sqrt{b}+i\,a\,b\right)\text{Log}\left[\sqrt{a}-i\sqrt{b}+\sqrt{a}\,x^2\right]}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2}+\frac{\left(i\,a^2+2\,a^{3/2}\sqrt{b}-i\,a\,b\right)\text{Log}\left[\sqrt{a}+i\sqrt{b}+\sqrt{a}\,x^2\right]}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2}$$

Problem 911: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{a+b+2\,a\,x^2+a\,x^4}\,dx$$

Optimal (type 3, 432 leaves, 10 steps):

$$\frac{x}{a}+\frac{\left(a+b+2\sqrt{a}\,\sqrt{a+b}\right)\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\,\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\left(a+b+2\sqrt{a}\,\sqrt{a+b}\right)\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\,\sqrt{\sqrt{a}+\sqrt{a+b}}}+$$

$$\frac{\left(a+b-2\sqrt{a}\,\sqrt{a+b}\right)\text{Log}\left[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}}-$$

$$\frac{\left(a+b-2\sqrt{a}\,\sqrt{a+b}\right)\text{Log}\left[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Result (type 3, 164 leaves):

$$\frac{x}{a}-\frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right]}{2a\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right]}{2a\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Problem 912: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{a+b+2ax^2+ax^4}\,dx$$

Optimal (type 3, 331 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\text{Log}\left[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}-\frac{\text{Log}\left[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Result (type 3, 143 leaves):

$$\frac{\frac{\left(i\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right]}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(-i\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right]}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Problem 913: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{a+b+2ax^2+ax^4}\,dx$$

Optimal (type 3, 359 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{1/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{1/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\text{Log}\left[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{1/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}+\frac{\text{Log}\left[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right]}{4\sqrt{2}\,a^{1/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Result (type 3, 119 leaves):

$$-\frac{i\,\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}\right]}{2\sqrt{a-i\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{i\,\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}\right]}{2\sqrt{a+i\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}$$

Problem 914: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^2\left(a+b+2\,a\,x^2+a\,x^4\right)}\,dx$$

Optimal (type 3, 433 leaves, 10 steps):

$$-\frac{1}{(a+b)\,x}+\frac{a^{1/4}\left(2\sqrt{a}+\sqrt{a+b}\right)\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}}-$$

$$\frac{a^{1/4}\left(2\sqrt{a}+\sqrt{a+b}\right)\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{a^{1/4}\left(2\sqrt{a}-\sqrt{a+b}\right)\text{Log}\left[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\;x+\sqrt{a}\;x^2\right]}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}-$$

$$\frac{a^{1/4}\left(2\sqrt{a}-\sqrt{a+b}\right)\text{Log}\left[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\;x+\sqrt{a}\;x^2\right]}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Result (type 3, 174 leaves):

$$\frac{1}{(-a-b)\,x}+\frac{\left(i\,a-\sqrt{a}\,\sqrt{b}\right)\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}\right]}{2\sqrt{a-i\sqrt{a}\,\sqrt{b}}\,\sqrt{b}\,(a+b)}+\frac{\left(-i\,a-\sqrt{a}\,\sqrt{b}\right)\text{ArcTan}\left[\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}\right]}{2\sqrt{a+i\sqrt{a}\,\sqrt{b}}\,\sqrt{b}\,(a+b)}$$

Problem 918: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{1-x^2+x^4}\,dx$$

Optimal (type 3, 74 leaves, 9 steps):

$$-\frac{1}{2}\text{ArcTan}\left[\sqrt{3}-2\,x\right]+\frac{1}{2}\text{ArcTan}\left[\sqrt{3}+2\,x\right]+\frac{\text{Log}\left[1-\sqrt{3}\;x+x^2\right]}{4\sqrt{3}}-\frac{\text{Log}\left[1+\sqrt{3}\;x+x^2\right]}{4\sqrt{3}}$$

Result (type 3, 94 leaves):

$$\frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1}{2}\left(1-i\sqrt{3}\right)x\right]+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1}{2}\left(1+i\sqrt{3}\right)x\right]}{2\sqrt{6}}$$

Problem 919: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{2-2x^2+x^4}\,dx$$

Optimal (type 3, 188 leaves, 9 steps):

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+$$

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+\frac{\text{Log}\left[\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right]}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\text{Log}\left[\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right]}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Result (type 3, 39 leaves):

$$-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1-i}}\right]}{(-1-i)^{3/2}}-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1+i}}\right]}{(-1+i)^{3/2}}$$

Problem 930: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\sqrt{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 4, 395 leaves, 5 steps):

$$-\frac{2\left(2 b^2-5 a c\right) x \sqrt{a+b x^2+c x^4}}{105 c^2}+\frac{b\left(8 b^2-29 a c\right) x \sqrt{a+b x^2+c x^4}}{105 c^{5/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{x^3\left(b+5 c x^2\right) \sqrt{a+b x^2+c x^4}}{35 c}-$$

$$\frac{a^{1/4} b\left(8 b^2-29 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\, \sqrt{c}}\right)\right]}{105 c^{11/4} \sqrt{a+b x^2+c x^4}}+\frac{1}{210 c^{11/4} \sqrt{a+b x^2+c x^4}}$$

$$a^{1/4}\left(8 b^3-29 a b c+2 \sqrt{a}\, \sqrt{c}\left(2 b^2-5 a c\right)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\, \sqrt{c}}\right)\right]$$

Result (type 4, 538 leaves):

$$\frac{1}{420 c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, \sqrt{a+b x^2+c x^4}}$$

$$\left(4 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\left(10 a^2 c-4 b^3 x^2-b^2 c x^4+18 b c^2 x^6+15 c^3 x^8+a\left(-4 b^2+13 b c x^2+25 c^2 x^4\right)\right)+i b\left(8 b^2-29 a c\right)\left(-b+\sqrt{b^2-4 a c}\right)\right.$$

$$\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\ \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\ \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$

$$i\left(-8 b^4+37 a b^2 c-20 a^2 c^2+8 b^3 \sqrt{b^2-4 a c}-29 a b c \sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}$$

$$\left.\sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 931: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a+b x^2+c x^4}\, dx$$

Optimal (type 4, 342 leaves, 4 steps):

$$-\frac{2\left(b^2-3 a c\right) x \sqrt{a+b x^2+c x^4}}{15 c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{x\left(b+3 c x^2\right) \sqrt{a+b x^2+c x^4}}{15 c}+$$

$$\frac{2 a^{1/4}\left(b^2-3 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{15 c^{7/4} \sqrt{a+b x^2+c x^4}}-\frac{1}{30 c^{7/4} \sqrt{a+b x^2+c x^4}}$$

$$a^{1/4}\left(2 b^2+\sqrt{a}\, b \sqrt{c}-6 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]$$

Result (type 4, 479 leaves) :

$$\frac{1}{30 c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}}$$

$$\left(2 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\left(b+3 c x^2\right)\left(a+b x^2+c x^4\right)-i\left(b^2-3 a c\right)\left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+i\left(-b^3+4 a b c+b^2 \sqrt{b^2-4 a c}-3 a c \sqrt{b^2-4 a c}\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 932: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b x^2+c x^4}\, dx$$

Optimal (type 4, 309 leaves, 4 steps) :

$$\frac{1}{3}x\sqrt{a+b x^2+c x^4}+\frac{b x\sqrt{a+b x^2+c x^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4} b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{3 c^{3/4}\sqrt{a+b x^2+c x^4}}+$$
$$\frac{a^{1/4}\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{6 c^{3/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 445 leaves):

$$\Bigg(4 c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\left(a+b x^2+c x^4\right)+$$
$$i b\left(-b+\sqrt{b^2-4 a c}\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$
$$i\left(-b^2+4 a c+b\sqrt{b^2-4 a c}\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$
$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\Bigg)\Bigg/\left(12 c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)$$

Problem 933: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^2+c x^4}}{x^2}\,dx$$

Optimal (type 4, 303 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^2+c x^4}}{x}+\frac{2\sqrt{c}\,x\sqrt{a+b x^2+c x^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{2 a^{1/4} c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{\sqrt{a+b x^2+c x^4}}+$$
$$\frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2 a^{1/4} c^{1/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 435 leaves):

$$\left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(a+bx^2+cx^4\right)+\right.$$

$$\mathrm{i}\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-$$

$$\left.\mathrm{i}\sqrt{2}\sqrt{b^2-4ac}\,x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\sqrt{a+bx^2+cx^4}\right)$$

Problem 934: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

Optimal (type 4, 341 leaves, 5 steps):

$$-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}+\frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{b\,c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{3a^{3/4}\sqrt{a+bx^2+cx^4}}+$$

$$\frac{\left(b+2\sqrt{a}\sqrt{c}\right)c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{6a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Result (type 4, 459 leaves):

$$\frac{1}{12\,a\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x^3\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(-4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;\left(a+b\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)+\mathrm{i}\,b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,x^3\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\mathrm{i}\,\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\,x^3$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 935: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^2+c\,x^4}}{x^6}\,dx$$

Optimal (type 4, 397 leaves, 6 steps):

$$-\frac{\sqrt{a+b\,x^2+c\,x^4}}{5\,x^5}-\frac{b\,\sqrt{a+b\,x^2+c\,x^4}}{15\,a\,x^3}+\frac{2\,\left(b^2-3\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}{15\,a^2\,x}-\frac{2\,\sqrt{c}\,\left(b^2-3\,a\,c\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{15\,a^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\frac{2\,c^{1/4}\,\left(b^2-3\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{15\,a^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{1}{30\,a^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$c^{1/4}\,\left(2\,b^2+\sqrt{a}\,b\,\sqrt{c}-6\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 530 leaves):

$$\frac{1}{30\,a^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x^5\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\left(3\,a^3-2\,b^2\,x^6\left(b+c\,x^2\right)+a^2\left(4\,b\,x^2+9\,c\,x^4\right)+a\left(-b^2\,x^4+7\,b\,c\,x^6+6\,c^2\,x^8\right)\right)-i\left(b^2-3\,a\,c\right)\left(-b+\sqrt{b^2-4\,a\,c}\right)\right.$$

$$x^5\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$i\left(-b^3+4\,a\,b\,c+b^2\sqrt{b^2-4\,a\,c}-3\,a\,c\sqrt{b^2-4\,a\,c}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 947: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\left(a+b\,x^2+c\,x^4\right)^{3/2}dx$$

Optimal (type 4, 495 leaves, 6 steps):

$$\frac{\left(8\,b^4-51\,a\,b^2\,c+60\,a^2\,c^2\right)x\sqrt{a+b\,x^2+c\,x^4}}{1155\,c^3}-\frac{8\,b\left(2\,b^2-9\,a\,c\right)\left(b^2-3\,a\,c\right)x\sqrt{a+b\,x^2+c\,x^4}}{1155\,c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\frac{x^3\left(b\left(2\,b^2+a\,c\right)+10\,c\left(b^2-3\,a\,c\right)x^2\right)\sqrt{a+b\,x^2+c\,x^4}}{385\,c^2}+\frac{x^3\left(b+3\,c\,x^2\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}}{33\,c}+\frac{1}{1155\,c^{15/4}\sqrt{a+b\,x^2+c\,x^4}}$$

$$8\,a^{1/4}\,b\left(2\,b^2-9\,a\,c\right)\left(b^2-3\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]-$$

$$\frac{1}{2310\,c^{15/4}\sqrt{a+b\,x^2+c\,x^4}}a^{1/4}\left(8\,b\left(2\,b^2-9\,a\,c\right)\left(b^2-3\,a\,c\right)+\sqrt{a}\sqrt{c}\left(8\,b^4-51\,a\,b^2\,c+60\,a^2\,c^2\right)\right)$$

$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 657 leaves):

$$\frac{1}{2310\,c^4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,\sqrt{a+b x^2+c x^4}}\Bigg(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(60a^3c^2+a^2c\left(-51b^2+92bcx^2+255c^2x^4\right)+a\left(8b^4-57b^3cx^2-14b^2c^2x^4+367bc^3x^6+300c^4x^8\right)+x^2\left(8b^5+2b^4cx^2-b^3c^2x^4+145b^2c^3x^6+245bc^4x^8+105c^5x^{10}\right)\right)-4\,i\,b\left(2b^4-15ab^2c+27a^2c^2\right)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+i\left(-8b^6+68ab^4c-159a^2b^2c^2+60a^3c^3+8b^5\sqrt{b^2-4ac}-60ab^3c\sqrt{b^2-4ac}+108a^2bc^2\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\Bigg)$$

Problem 948: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\left(a+b x^2+c x^4\right)^{3/2}dx$$

Optimal (type 4, 443 leaves, 5 steps):

$$\frac{\left(8b^4-57ab^2c+84a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{315\,c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315\,c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63\,c}-\frac{1}{315\,c^{11/4}\sqrt{a+bx^2+cx^4}}a^{1/4}\left(8b^4-57ab^2c+84a^2c^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]+\frac{1}{630\,c^{11/4}\sqrt{a+bx^2+cx^4}}a^{1/4}\left(8b^4-57ab^2c+84a^2c^2+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 602 leaves) :

$$\frac{1}{1260\, c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, \sqrt{a+b x^2+c x^4}}\left(4 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x \left(-4 b^4 x^2-b^3 c x^4+53 b^2 c^2 x^6+85 b c^3 x^8+35 c^4 x^{10}+a^2 c \left(24 b+77 c x^2\right)+a \left(-4 b^3+27 b^2 c x^2+151 b c^2 x^4+112 c^3 x^6\right)\right)+\right.$$
$$i \left(8 b^4-57 a b^2 c+84 a^2 c^2\right) \left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}$$
$$\sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$
$$i \left(-8 b^5+65 a b^3 c-132 a^2 b c^2+8 b^4 \sqrt{b^2-4 a c}-57 a b^2 c \sqrt{b^2-4 a c}+84 a^2 c^2 \sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}$$
$$\left.\sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 949: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^2+c x^4\right)^{3/2} dx$$

Optimal (type 4, 381 leaves, 5 steps) :

$$-\frac{2 b \left(b^2-8 a c\right) x \sqrt{a+b x^2+c x^4}}{35 c^{3/2} \left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{x \left(b^2+10 a c+3 b c x^2\right) \sqrt{a+b x^2+c x^4}}{35 c}+\frac{1}{7} x \left(a+b x^2+c x^4\right)^{3/2}+$$
$$\frac{2 a^{1/4} b \left(b^2-8 a c\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{35 c^{7/4} \sqrt{a+b x^2+c x^4}}-\frac{1}{70 c^{7/4} \sqrt{a+b x^2+c x^4}}$$
$$a^{1/4} \left(\sqrt{a} \sqrt{c} \left(b^2-20 a c\right)+2 b \left(b^2-8 a c\right)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]$$

Result (type 4, 533 leaves):

$$\frac{1}{70\,c^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}\left(2\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\left(15\,a^2\,c+a\,\left(b^2+23\,b\,c\,x^2+20\,c^2\,x^4\right)+x^2\,\left(b^3+9\,b^2\,c\,x^2+13\,b\,c^2\,x^4+5\,c^3\,x^6\right)\right)-\mathrm{i}\,b\,\left(b^2-8\,a\,c\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\right.$$
$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$
$$\mathrm{i}\,\left(-b^4+9\,a\,b^2\,c-20\,a^2\,c^2+b^3\,\sqrt{b^2-4\,a\,c}-8\,a\,b\,c\,\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}$$
$$\left.\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 950: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 361 leaves, 5 steps):

$$\frac{\left(b^2+12\,a\,c\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{5\,\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{5}\,x\,\left(7\,b+6\,c\,x^2\right)\,\sqrt{a+b\,x^2+c\,x^4}-\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{x}-$$
$$\frac{a^{1/4}\,\left(b^2+12\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{5\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}+\frac{1}{10\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$
$$a^{1/4}\,\left(b^2+8\,\sqrt{a}\,b\,\sqrt{c}+12\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 505 leaves):

$$\frac{1}{20\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(-5\,a^2-3\,a\,b\,x^2+2\,b^2\,x^4-4\,a\,c\,x^4+3\,b\,c\,x^6+c^2\,x^8\right)+i\,\left(b^2+12\,a\,c\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,x\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$i\,\left(-b^3+4\,a\,b\,c+b^2\,\sqrt{b^2-4\,a\,c}+12\,a\,c\,\sqrt{b^2-4\,a\,c}\right)\,x\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 951: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 353 leaves, 5 steps):

$$\frac{8\,b\,\sqrt{c}\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(3\,b-2\,c\,x^2\right)\,\sqrt{a+b\,x^2+c\,x^4}}{3\,x}-\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{3\,x^3}-$$

$$\frac{8\,a^{1/4}\,b\,c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{3\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{\left(3\,b^2+8\,\sqrt{a}\,b\,\sqrt{c}+4\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{6\,a^{1/4}\,c^{1/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 473 leaves):

$$\frac{1}{6\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x^3\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\left(-a^2-5\,a\,b\,x^2-4\,b^2\,x^4-3\,b\,c\,x^6+c^2\,x^8\right)+4\,\mathrm{i}\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\mathrm{i}\left(-b^2+4\,a\,c+4\,b\sqrt{b^2-4\,a\,c}\right)$$

$$\left.x^3\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 952: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 400 leaves, 6 steps):

$$-\frac{\left(b^2+12\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}}{5\,a\,x}+\frac{\sqrt{c}\,\left(b^2+12\,a\,c\right)x\sqrt{a+b\,x^2+c\,x^4}}{5\,a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(b-6\,c\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}}{5\,x^3}-\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{5\,x^5}-$$

$$\frac{c^{1/4}\left(b^2+12\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{5\,a^{3/4}\sqrt{a+b\,x^2+c\,x^4}}+\frac{1}{10\,a^{3/4}\sqrt{a+b\,x^2+c\,x^4}}$$

$$c^{1/4}\left(b^2+8\sqrt{a}\,b\sqrt{c}+12\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 527 leaves):

$$\frac{1}{20\,a\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x^5\sqrt{a+b\,x^2+c\,x^4}}\left(-4\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\left(a^3+b^2\,x^6\left(b+c\,x^2\right)+a^2\left(3\,b\,x^2+8\,c\,x^4\right)+a\left(3\,b^2\,x^4+9\,b\,c\,x^6+7\,c^2\,x^8\right)\right)+\mathbb{i}\left(b^2+12\,a\,c\right)\left(-b+\sqrt{b^2-4\,a\,c}\right)x^5\right.$$
$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$
$$\mathbb{i}\left(-b^3+4\,a\,b\,c+b^2\sqrt{b^2-4\,a\,c}+12\,a\,c\sqrt{b^2-4\,a\,c}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$
$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 953: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{x^8}\,dx$$

Optimal (type 4, 447 leaves, 7 steps):

$$-\frac{\left(b^2-20\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}}{35\,a\,x^3}+\frac{2\,b\left(b^2-8\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}}{35\,a^2\,x}-\frac{2\,b\sqrt{c}\left(b^2-8\,a\,c\right)x\sqrt{a+b\,x^2+c\,x^4}}{35\,a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3\left(b+10\,c\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}}{35\,x^5}-$$
$$\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{7\,x^7}+\frac{2\,b\,c^{1/4}\left(b^2-8\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{35\,a^{7/4}\sqrt{a+b\,x^2+c\,x^4}}-\frac{1}{70\,a^{7/4}\sqrt{a+b\,x^2+c\,x^4}}$$
$$c^{1/4}\left(\sqrt{a}\sqrt{c}\left(b^2-20\,a\,c\right)+2\,b\left(b^2-8\,a\,c\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 572 leaves):

$$\frac{1}{70\,a^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x^7\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(5\,a^4-2\,b^3\,x^8\,\left(b+c\,x^2\right)+a^3\,\left(13\,b\,x^2+20\,c\,x^4\right)+a\,b\,x^6\,\left(-b^2+17\,b\,c\,x^2+16\,c^2\,x^4\right)+3\,a^2\,\left(3\,b^2\,x^4+13\,b\,c\,x^6+5\,c^2\,x^8\right)\right)-\right.$$

$$i\,b\,\left(b^2-8\,a\,c\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,x^7\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+i\,\left(-b^4+9\,a\,b^2\,c-20\,a^2\,c^2+b^3\sqrt{b^2-4\,a\,c}-8\,a\,b\,c\,\sqrt{b^2-4\,a\,c}\right)$$

$$\left.x^7\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 954: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{3-2\,x^2-x^4}\,dx$$

Optimal (type 4, 48 leaves, 5 steps):

$$\frac{1}{3}\,x\,\sqrt{3-2\,x^2-x^4}-\frac{2\,\text{EllipticE}\left[\text{ArcSin}[x],\,-\frac{1}{3}\right]}{\sqrt{3}}+\frac{4\,\text{EllipticF}\left[\text{ArcSin}[x],\,-\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 59 leaves):

$$\frac{1}{3}\left(x\,\sqrt{3-2\,x^2-x^4}-2\,i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],\,-3\right]-4\,i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],\,-3\right]\right)$$

Problem 963: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 313 leaves, 4 steps):

$$\frac{x\sqrt{a+b x^2+c x^4}}{3 c}-\frac{2 b x\sqrt{a+b x^2+c x^4}}{3 c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{2 a^{1/4} b\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{3 c^{7/4}\sqrt{a+b x^2+c x^4}}-$$

$$\frac{a^{1/4}\left(2 b+\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{6 c^{7/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 444 leaves):

$$\left(2 c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\left(a+b x^2+c x^4\right)-\right.$$

$$i\, b\left(-b+\sqrt{b^2-4 a c}\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+$$

$$i\left(-b^2+a c+b\sqrt{b^2-4 a c}\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\Big/\left(6 c^2\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)$$

Problem 964: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 267 leaves, 3 steps):

$$\frac{x\sqrt{a+b x^2+c x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{a^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{c^{3/4}\sqrt{a+b x^2+c x^4}}+$$

$$\frac{a^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2 c^{3/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 278 leaves):

$$\left(i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)$$

Problem 965: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 114 leaves, 1 step):

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2a^{1/4}c^{1/4}\sqrt{a+bx^2+cx^4}}$$

Result (type 4, 186 leaves):

$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Problem 966: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$-\frac{\sqrt{a+b x^2+c x^4}}{a x}+\frac{\sqrt{c}\, x \sqrt{a+b x^2+c x^4}}{a \left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{c^{1/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{a^{3/4} \sqrt{a+b x^2+c x^4}}+$$
$$\frac{c^{1/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{2 a^{3/4} \sqrt{a+b x^2+c x^4}}$$

Result (type 4, 298 leaves) :

$$\frac{1}{4 a \sqrt{a+b x^2+c x^4}}\left(-\frac{4 \left(a+b x^2+c x^4\right)}{x}+\frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}}\, i \sqrt{2} \left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\right.$$
$$\left.\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right)$$

Problem 967: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 345 leaves, 5 steps) :

$$-\frac{\sqrt{a+b x^2+c x^4}}{3 a x^3}+\frac{2 b \sqrt{a+b x^2+c x^4}}{3 a^2 x}-\frac{2 b \sqrt{c}\, x \sqrt{a+b x^2+c x^4}}{3 a^2 \left(\sqrt{a}+\sqrt{c}\, x^2\right)}+$$
$$\frac{2 b\, c^{1/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{3 a^{7/4} \sqrt{a+b x^2+c x^4}}-$$
$$\frac{\left(2 b+\sqrt{a} \sqrt{c}\right) c^{1/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{6 a^{7/4} \sqrt{a+b x^2+c x^4}}$$

Result (type 4, 459 leaves) :

$$\frac{1}{6\,a^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x^3\sqrt{a+b\,x^2+c\,x^4}}\left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;(a-2\,b\,x^2)\;(a+b\,x^2+c\,x^4)-i\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+i\left(-b^2+a\,c+b\sqrt{b^2-4\,a\,c}\right)x^3$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 968: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7}{\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$-\frac{x^4\sqrt{a+b\,x^2-c\,x^4}}{6\,c}-\frac{(15\,b^2+16\,a\,c+10\,b\,c\,x^2)\sqrt{a+b\,x^2-c\,x^4}}{48\,c^3}-\frac{b\,(5\,b^2+12\,a\,c)\,\text{ArcTan}\left[\frac{b-2\,c\,x^2}{2\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}}\right]}{32\,c^{7/2}}$$

Result (type 3, 112 leaves):

$$\frac{1}{96\,c^{7/2}}\left(-2\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}\;\left(15\,b^2+10\,b\,c\,x^2+8\,c\,(2\,a+c\,x^4)\right)+3\,i\,(5\,b^3+12\,a\,b\,c)\,\text{Log}\left[\frac{i\,(b-2\,c\,x^2)}{\sqrt{c}}+2\sqrt{a+b\,x^2-c\,x^4}\right]\right)$$

Problem 969: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^5}{\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$-\frac{3\,b\sqrt{a+b\,x^2-c\,x^4}}{8\,c^2}-\frac{x^2\sqrt{a+b\,x^2-c\,x^4}}{4\,c}-\frac{(3\,b^2+4\,a\,c)\,\text{ArcTan}\left[\frac{b-2\,c\,x^2}{2\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}}\right]}{16\,c^{5/2}}$$

Result (type 3, 94 leaves):

$$-\frac{\left(3 b+2 c x^2\right) \sqrt{a+b x^2-c x^4}}{8 c^2}+\frac{i \left(3 b^2+4 a c\right) \text{Log}\left[\frac{i \left(b-2 c x^2\right)}{\sqrt{c}}+2 \sqrt{a+b x^2-c x^4}\right]}{16 c^{5/2}}$$

Problem 970: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{a+b x^2-c x^4}} \, dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^2-c x^4}}{2 c}-\frac{b \text{ArcTan}\left[\frac{b-2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2-c x^4}}\right]}{4 c^{3/2}}$$

Result (type 3, 77 leaves):

$$-\frac{\sqrt{a+b x^2-c x^4}}{2 c}+\frac{i b \text{Log}\left[-\frac{i \left(-b+2 c x^2\right)}{\sqrt{c}}+2 \sqrt{a+b x^2-c x^4}\right]}{4 c^{3/2}}$$

Problem 971: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a+b x^2-c x^4}} \, dx$$

Optimal (type 3, 44 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{b-2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2-c x^4}}\right]}{2 \sqrt{c}}$$

Result (type 3, 51 leaves):

$$\frac{i \text{Log}\left[-\frac{i \left(-b+2 c x^2\right)}{\sqrt{c}}+2 \sqrt{a+b x^2-c x^4}\right]}{2 \sqrt{c}}$$

Problem 976: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a+b x^2-c x^4}} \, dx$$

Optimal (type 4, 409 leaves, 5 steps):

$$-\frac{x\sqrt{a+b x^2-c x^4}}{3 c}-\frac{1}{3\sqrt{2}\, c^{5/2}\sqrt{a+b x^2-c x^4}} b\left(b-\sqrt{b^2+4 a c}\right)\sqrt{b+\sqrt{b^2+4 a c}}\,\sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\,\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+\frac{1}{3\sqrt{2}\, c^{5/2}\sqrt{a+b x^2-c x^4}}\sqrt{b+\sqrt{b^2+4 a c}}\left(b^2+a c-b\sqrt{b^2+4 a c}\right)\sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\,\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]$$

Result (type 4, 459 leaves) :

$$\frac{1}{6 c^2\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\sqrt{a+b x^2-c x^4}}\left(2 c\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\left(-a-b x^2+c x^4\right)-\mathrm{i}\sqrt{2}\, b\left(-b+\sqrt{b^2+4 a c}\right)\sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}}\sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+\mathrm{i}\sqrt{2}\left(-b^2-a c+b\sqrt{b^2+4 a c}\right)\sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}}\sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

Problem 977: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\sqrt{a+b x^2-c x^4}}\,dx$$

Optimal (type 4, 377 leaves, 4 steps) :

$$-\frac{1}{2\sqrt{2}\,c^{3/2}\sqrt{a+b\,x^2-c\,x^4}}\left(b-\sqrt{b^2+4\,a\,c}\right)\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]+\frac{1}{2\sqrt{2}\,c^{3/2}\sqrt{a+b\,x^2-c\,x^4}}\left(b-\sqrt{b^2+4\,a\,c}\right)\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]$$

Result (type 4, 271 leaves):

$$-\left(\left(i\left(-b+\sqrt{b^2+4\,a\,c}\right)\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],-\frac{b+\sqrt{b^2+4\,a\,c}}{-b+\sqrt{b^2+4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],-\frac{b+\sqrt{b^2+4\,a\,c}}{-b+\sqrt{b^2+4\,a\,c}}\right]\right)\right)\Big/\left(2\sqrt{2}\,c\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{a+b\,x^2-c\,x^4}\right)\right)$$

Problem 978: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 169 leaves, 2 steps):

$$\frac{\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]}{\sqrt{2}\,\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}}$$

Result (type 4, 177 leaves):

$$-\frac{i\,\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],-\frac{b+\sqrt{b^2+4\,a\,c}}{-b+\sqrt{b^2+4\,a\,c}}\right]}{\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{a+b\,x^2-c\,x^4}}$$

Problem 979: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2\sqrt{a+b x^2-c x^4}}\,dx$$

Optimal (type 4, 408 leaves, 6 steps):

$$-\frac{\sqrt{a+b x^2-c x^4}}{a x}+\left(\left(b-\sqrt{b^2+4 a c}\right)\sqrt{b+\sqrt{b^2+4 a c}}\sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)\Big/\left(2\sqrt{2}\,a\sqrt{c}\sqrt{a+b x^2-c x^4}\right)-\left(\left(b-\sqrt{b^2+4 a c}\right)\sqrt{b+\sqrt{b^2+4 a c}}\sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)\Big/\left(2\sqrt{2}\,a\sqrt{c}\sqrt{a+b x^2-c x^4}\right)$$

Result (type 4, 283 leaves):

$$\frac{1}{4 a\sqrt{a+b x^2-c x^4}}\left(-\frac{4 a}{x}-4 b x+4 c x^3+\frac{1}{\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}}\,i\left(-b+\sqrt{b^2+4 a c}\right)\sqrt{2+\frac{4 c x^2}{-b+\sqrt{b^2+4 a c}}}\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\,x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\,x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)\right)$$

Problem 980: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4\sqrt{a+b x^2-c x^4}}\,dx$$

Optimal (type 4, 445 leaves, 6 steps):

$$-\frac{\sqrt{a+b\,x^2-c\,x^4}}{3\,a\,x^3}+\frac{2\,b\,\sqrt{a+b\,x^2-c\,x^4}}{3\,a^2\,x}-\left(b\left(b-\sqrt{b^2+4\,a\,c}\right)\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(3\,\sqrt{2}\,a^2\,\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}\right)+\left(\sqrt{b+\sqrt{b^2+4\,a\,c}}\left(b^2+a\,c-b\,\sqrt{b^2+4\,a\,c}\right)\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(3\,\sqrt{2}\,a^2\,\sqrt{c}\,\sqrt{a+b\,x^2-c\,x^4}\right)$$

Result (type 4, 472 leaves):

$$\frac{1}{6\,a^2\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x^3\,\sqrt{a+b\,x^2-c\,x^4}}\left(-2\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\left(a-2\,b\,x^2\right)\left(a+b\,x^2-c\,x^4\right)-i\,\sqrt{2}\,b\left(-b+\sqrt{b^2+4\,a\,c}\right)x^3\,\sqrt{\frac{b+\sqrt{b^2+4\,a\,c}-2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{\frac{-b+\sqrt{b^2+4\,a\,c}+2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]+i\,\sqrt{2}\left(-b^2-a\,c+b\,\sqrt{b^2+4\,a\,c}\right)x^3\,\sqrt{\frac{b+\sqrt{b^2+4\,a\,c}-2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{\frac{-b+\sqrt{b^2+4\,a\,c}+2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)$$

Problem 989: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 408 leaves, 5 steps):

$$\frac{x^3\left(2 a+b x^2\right)}{\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}}-\frac{b x\sqrt{a+b x^2+c x^4}}{c\left(b^2-4 a c\right)}+\frac{2\left(b^2-3 a c\right) x\sqrt{a+b x^2+c x^4}}{c^{3/2}\left(b^2-4 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-$$

$$\frac{2 a^{1/4}\left(b^2-3 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\ \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{c^{7/4}\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}}+$$

$$\left(a^{1/4}\left(2 b^2+\sqrt{a}\, b\sqrt{c}-6 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\ \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2 c^{7/4}\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 489 leaves) :

$$\frac{1}{2 c^2\left(-b^2+4 a c\right)\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}}$$

$$\left(2 c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\left(b^2 x^2+a\left(b-2 c x^2\right)\right)-i\left(b^2-3 a c\right)\left(-b+\sqrt{b^2-4 a c}\right)\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\ \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+i\left(-b^3+4 a b c+b^2\sqrt{b^2-4 a c}-3 a c\sqrt{b^2-4 a c}\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{2 b-2\sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\ \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 990: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{\left(a+b x^2+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 342 leaves, 4 steps) :

$$\frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{a^{1/4}b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-$$

$$\frac{a^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Result (type 4, 452 leaves):

$$\Bigg(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(2a+bx^2\right)-$$

$$i\,b\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+$$

$$i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/\left(4c\left(b^2-4ac\right)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)$$

Problem 991: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 341 leaves, 4 steps):

$$-\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2a^{1/4}c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+$$

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2a^{1/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{1/4}\sqrt{a+bx^2+cx^4}}$$

Result (type 4, 437 leaves):

$$\left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\left(b+2\,c\,x^2\right)+\right.$$
$$i\left(-b+\sqrt{b^2-4\,a\,c}\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$
$$\left.i\sqrt{2}\sqrt{b^2-4\,a\,c}\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(2\left(b^2-4\,a\,c\right)\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)$$

Problem 992: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 353 leaves, 4 steps):

$$\frac{x\left(b^2-2\,a\,c+b\,c\,x^2\right)}{a\left(b^2-4\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}}-\frac{b\,\sqrt{c}\;x\,\sqrt{a+b\,x^2+c\,x^4}}{a\left(b^2-4\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\;x^2\right)}+\frac{b\,c^{1/4}\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\;\sqrt{c}}\right)\right]}{a^{3/4}\left(b^2-4\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}}-$$
$$\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\;\sqrt{c}}\right)\right]}{2\,a^{3/4}\left(b-2\,\sqrt{a}\;\sqrt{c}\right)\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 456 leaves):

$$-\left(\left(-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\,\left(b^2-2ac+bcx^2\right)+i\,b\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\Big/\left(4a\left(b^2-4ac\right)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)\right)$$

Problem 993: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 428 leaves, 5 steps):

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x}+\frac{2\sqrt{c}\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2c^{1/4}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\left(c^{1/4}\left(2b^2+\sqrt{a}\,b\sqrt{c}-6ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\right)$$

Result (type 4, 515 leaves):

$$-\frac{1}{2 a^2 \left(b^2-4 a c\right) \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x \sqrt{a+b x^2+c x^4}}$$

$$\left(2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \left(-4 a^2 c+2 b^2 x^2 \left(b+c x^2\right)+a \left(b^2-7 b c x^2-6 c^2 x^4\right)\right)-\mathrm{i} \left(b^2-3 a c\right) \left(-b+\sqrt{b^2-4 a c}\right) x\right.$$

$$\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}\ \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+$$

$$\mathrm{i} \left(-b^3+4 a b c+b^2 \sqrt{b^2-4 a c}-3 a c \sqrt{b^2-4 a c}\right) x \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 1003: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a+\left(2+2 b-2 (1+b)\right) x^2+c x^4}}\, dx$$

Optimal (type 4, 108 leaves, 3 steps):

$$\frac{x \sqrt{a+c x^4}}{3 c}-\frac{a^{3/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 c^{5/4} \sqrt{a+c x^4}}$$

Result (type 4, 92 leaves):

$$\frac{x \left(a+c x^4\right)+\frac{\mathrm{i}\, a \sqrt{1+\frac{c x^4}{a}}\ \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{\mathrm{i} \sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]}{\sqrt{\frac{\mathrm{i} \sqrt{c}}{\sqrt{a}}}}}{3 c \sqrt{a+c x^4}}$$

Problem 1005: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a+\left(2+2\,b-2\,\left(1+b\right)\right)\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 210 leaves, 4 steps):

$$\frac{x\,\sqrt{a+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\,\sqrt{a+c\,x^4}}+$$
$$\frac{a^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,c^{3/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 104 leaves):

$$\frac{i\,\sqrt{1+\frac{c\,x^4}{a}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)}{\left(\frac{i\,\sqrt{c}}{\sqrt{a}}\right)^{3/2}\,\sqrt{a+c\,x^4}}$$

Problem 1007: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+\left(2+2\,b-2\,\left(1+b\right)\right)\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,c^{1/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 74 leaves):

$$-\frac{i\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+c\,x^4}}$$

Problem 1009: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} \, dx$$

Optimal (type 4, 232 leaves, 5 steps):

$$-\frac{\sqrt{a + c x^4}}{a x} + \frac{\sqrt{c} \, x \sqrt{a + c x^4}}{a \left(\sqrt{a} + \sqrt{c} \, x^2\right)} - \frac{c^{1/4} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticE}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} \sqrt{a + c x^4}} + \frac{c^{1/4} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} \sqrt{a + c x^4}}$$

Result (type 4, 121 leaves):

$$\frac{1}{\sqrt{a + c x^4}} \left(-\frac{a + c x^4}{a x} - i \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{1 + \frac{c x^4}{a}} \left(\text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \, x\right], -1\right] - \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \, x\right], -1\right]\right)\right)$$

Problem 1011: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} \, dx$$

Optimal (type 4, 110 leaves, 3 steps):

$$-\frac{\sqrt{a + c x^4}}{3 a x^3} - \frac{c^{3/4} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 a^{5/4} \sqrt{a + c x^4}}$$

Result (type 4, 95 leaves):

$$\frac{-\frac{a+c x^4}{x^3} + \frac{i c \sqrt{1+\frac{c x^4}{a}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \, x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 a \sqrt{a + c x^4}}$$

Problem 1039: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x],-\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 18 leaves):

$$-i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]$$

Problem 1040: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$$

Optimal (type 4, 39 leaves, 2 steps):

$$-\frac{\text{EllipticF}\left[\text{ArcCos}\left[\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right],\frac{1}{42}\left(21+5\sqrt{21}\right)\right]}{21^{1/4}}$$

Result (type 4, 87 leaves):

$$\frac{\sqrt{5-\sqrt{21}-2x^2}\,\sqrt{2+\left(-5+\sqrt{21}\right)x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right],\frac{23}{2}-\frac{5\sqrt{21}}{2}\right]}{2\sqrt{-1+5x^2-x^4}}$$

Problem 1062: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 389 leaves, 9 steps):

$$\frac{2\,x^{3/2}}{3\,c}-\frac{\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,c^{7/4}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-\frac{\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,c^{7/4}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}+$$
$$\frac{\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,c^{7/4}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+\frac{\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,c^{7/4}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 80 leaves):

$$\frac{4\,x^{3/2}-3\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{a\,\text{Log}\left[\sqrt{x}-\#1\right]+b\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{6\,c}$$

Problem 1063: Result is not expressed in closed-form.

$$\int\frac{x^{7/2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 385 leaves, 9 steps):

$$\frac{2\,\sqrt{x}}{c}+\frac{\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,c^{5/4}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,c^{5/4}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}+$$
$$\frac{\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,c^{5/4}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,c^{5/4}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}$$

Result (type 7, 80 leaves):

$$-\frac{-4\,\sqrt{x}+\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{a\,\text{Log}\left[\sqrt{x}-\#1\right]+b\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]}{2\,c}$$

Problem 1064: Result is not expressed in closed-form.

$$\int\frac{x^{5/2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Result (type 7, 48 leaves):

$$\frac{1}{2}\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{\text{Log}\left[\sqrt{x}-\#1\right]\#1^3}{b+2c\,\#1^4}\,\&\right]$$

Problem 1065: Result is not expressed in closed-form.

$$\int\frac{x^{3/2}}{a+bx^2+cx^4}\,dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$\frac{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{1/4}c^{1/4}\sqrt{b^2-4ac}}-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{1/4}c^{1/4}\sqrt{b^2-4ac}}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{1/4}c^{1/4}\sqrt{b^2-4ac}}-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2^{1/4}c^{1/4}\sqrt{b^2-4ac}}$$

Result (type 7, 46 leaves):

$$\frac{1}{2}\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{\text{Log}\left[\sqrt{x}-\#1\right]\#1}{b+2c\,\#1^4}\,\&\right]$$

Problem 1066: Result is not expressed in closed-form.

$$\int\frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$-\frac{2^{1/4} c^{1/4} \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{2^{1/4} c^{1/4} \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{2^{1/4} c^{1/4} \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{2^{1/4} c^{1/4} \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 47 leaves):

$$\frac{1}{2}\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{\text{Log}\left[\sqrt{x}-\#1\right]}{b\,\#1+2\,c\,\#1^5}\,\&\right]$$

Problem 1067: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x}\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$\frac{2^{3/4} c^{3/4} \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{2^{3/4} c^{3/4} \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}+\frac{2^{3/4} c^{3/4} \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{2^{3/4} c^{3/4} \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{\sqrt{b^2-4 a c}\,\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 49 leaves):

$$\frac{1}{2}\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{\text{Log}\left[\sqrt{x}-\#1\right]}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]$$

Problem 1068: Result is not expressed in closed-form.

$$\int \frac{1}{x^{3/2}\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$-\frac{2}{a\sqrt{x}}-\frac{c^{1/4}\left(1-\frac{b}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2^{3/4}\,a\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{c^{1/4}\left(1+\frac{b}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2^{3/4}\,a\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}+$$

$$\frac{c^{1/4}\left(1-\frac{b}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2^{3/4}\,a\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{c^{1/4}\left(1+\frac{b}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2^{3/4}\,a\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 78 leaves):

$$-\frac{\frac{4}{\sqrt{x}}+\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{b\,\text{Log}\left[\sqrt{x}-\#1\right]+c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{2\,a}$$

Problem 1069: Result is not expressed in closed-form.

$$\int \frac{1}{x^{5/2}\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$-\frac{2}{3\,a\,x^{3/2}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,a\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,a\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}+$$

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,a\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{1/4}\,a\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}$$

Result (type 7, 82 leaves):

$$-\frac{\frac{4}{x^{3/2}}+3\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{b\,\text{Log}\left[\sqrt{x}-\#1\right]+c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]}{6\,a}$$

Problem 1070: Result is not expressed in closed-form.

$$\int \frac{1}{x^{7/2}\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 412 leaves, 10 steps):

$$-\frac{2}{5\,a\,x^{5/2}}+\frac{2\,b}{a^2\sqrt{x}}+\frac{c^{1/4}\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,a^2\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+\frac{c^{1/4}\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,a^2\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{c^{1/4}\left(b-\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,a^2\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-\frac{c^{1/4}\left(b+\frac{b^2-2\,a\,c}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2^{3/4}\,a^2\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 107 leaves):

$$-\frac{\frac{4\,a}{x^{5/2}}-\frac{20\,b}{\sqrt{x}}-5\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{b^2\,\text{Log}\left[\sqrt{x}-\#1\right]-a\,c\,\text{Log}\left[\sqrt{x}-\#1\right]+b\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{10\,a^2}$$

Problem 1071: Result is not expressed in closed-form.

$$\int\frac{x^{13/2}}{\left(a+b\,x^2+c\,x^4\right)^2}\,dx$$

Optimal (type 3, 544 leaves, 10 steps):

$$-\frac{b\,x^{3/2}}{2\,c\left(b^2-4\,a\,c\right)}+\frac{x^{7/2}\left(2\,a+b\,x^2\right)}{2\left(b^2-4\,a\,c\right)\left(a+b\,x^2+c\,x^4\right)}+\frac{\left(3\,b^3-20\,a\,b\,c+\left(3\,b^2-14\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,c^{7/4}\left(b^2-4\,a\,c\right)^{3/2}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{\left(3\,b^3-20\,a\,b\,c-\left(3\,b^2-14\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,c^{7/4}\left(b^2-4\,a\,c\right)^{3/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{\left(3\,b^3-20\,a\,b\,c+\left(3\,b^2-14\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,c^{7/4}\left(b^2-4\,a\,c\right)^{3/2}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+\frac{\left(3\,b^3-20\,a\,b\,c-\left(3\,b^2-14\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,c^{7/4}\left(b^2-4\,a\,c\right)^{3/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 144 leaves):

$$\frac{-\frac{4\,x^{3/2}\left(b^2\,x^2+a\left(b-2\,c\,x^2\right)\right)}{a+b\,x^2+c\,x^4}+\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{3\,a\,b\,\text{Log}\left[\sqrt{x}-\#1\right]+3\,b^2\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4-14\,a\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{8\,c\left(b^2-4\,a\,c\right)}$$

Problem 1072: Result is not expressed in closed-form.

$$\int \frac{x^{11/2}}{\left(a+b x^2+c x^4\right)^2}\,dx$$

Optimal (type 3, 520 leaves, 10 steps):

$$-\frac{b \sqrt{x}}{2 c \left(b^2-4 a c\right)}+\frac{x^{5/2} \left(2 a+b x^2\right)}{2 \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)}-\frac{\left(b^2-10 a c+\frac{b \left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{1/4} c^{5/4} \left(b^2-4 a c\right) \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(b^2-10 a c-\frac{b \left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{1/4} c^{5/4} \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(b^2-10 a c+\frac{b \left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{1/4} c^{5/4} \left(b^2-4 a c\right) \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(b^2-10 a c-\frac{b \left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{1/4} c^{5/4} \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 144 leaves):

$$\frac{-\frac{4 \sqrt{x} \left(b^2 x^2+a \left(b-2 c x^2\right)\right)}{a+b x^2+c x^4}+\text{RootSum}\left[a+b \#1^4+c \#1^8\&,\frac{a b \,\text{Log}\left[\sqrt{x}-\#1\right]+b^2 \text{Log}\left[\sqrt{x}-\#1\right] \#1^4-10 a c \,\text{Log}\left[\sqrt{x}-\#1\right] \#1^4}{b \#1^3+2 c \#1^7}\&\right]}{8 c \left(b^2-4 a c\right)}$$

Problem 1073: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{\left(a+b x^2+c x^4\right)^2}\,dx$$

Optimal (type 3, 471 leaves, 9 steps):

$$\frac{x^{3/2} \left(2 a+b x^2\right)}{2 \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)}+\frac{\left(b^2+12 a c+b \sqrt{b^2-4 a c}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} c^{3/4} \left(b^2-4 a c\right)^{3/2} \left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{\left(b-\frac{b^2+12 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} c^{3/4} \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{\left(b^2+12 a c+b \sqrt{b^2-4 a c}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} c^{3/4} \left(b^2-4 a c\right)^{3/2} \left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{\left(b-\frac{b^2+12 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} c^{3/4} \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 124 leaves):

$$-\frac{-2\,a\,x^{3/2}-b\,x^{7/2}}{2\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)}+\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{-6\,a\,\text{Log}\left[\sqrt{x}-\#1\right]+b\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{8\,\left(b^2-4\,a\,c\right)}$$

Problem 1074: Result is not expressed in closed-form.

$$\int \frac{x^{7/2}}{\left(a+b\,x^2+c\,x^4\right)^2}\,dx$$

Optimal (type 3, 483 leaves, 9 steps):

$$\frac{\sqrt{x}\,\left(2\,a+b\,x^2\right)}{2\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)}-\frac{\left(3\,b^2+4\,a\,c+3\,b\,\sqrt{b^2-4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{1/4}\,c^{1/4}\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{\left(3\,b^2+4\,a\,c-3\,b\,\sqrt{b^2-4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{1/4}\,c^{1/4}\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}-\frac{\left(3\,b^2+4\,a\,c+3\,b\,\sqrt{b^2-4\,a\,c}\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{1/4}\,c^{1/4}\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}+\frac{\left(3\,b^2+4\,a\,c-3\,b\,\sqrt{b^2-4\,a\,c}\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{1/4}\,c^{1/4}\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}$$

Result (type 7, 127 leaves):

$$-\frac{-2\,a\,\sqrt{x}-b\,x^{5/2}}{2\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)}+\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{-2\,a\,\text{Log}\left[\sqrt{x}-\#1\right]+3\,b\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]}{8\,\left(b^2-4\,a\,c\right)}$$

Problem 1075: Result is not expressed in closed-form.

$$\int \frac{x^{5/2}}{\left(a+b\,x^2+c\,x^4\right)^2}\,dx$$

Optimal (type 3, 450 leaves, 9 steps):

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{c^{1/4}\left(4b+\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{3/4}\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}+\frac{c^{1/4}\left(4b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{3/4}\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}+$$

$$\frac{c^{1/4}\left(4b+\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{3/4}\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}-\frac{c^{1/4}\left(4b-\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{3/4}\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}$$

Result (type 7, 109 leaves):

$$-\frac{\frac{4x^{3/2}\left(b+2cx^2\right)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{-3b\,\text{Log}\left[\sqrt{x}-\#1\right]+2c\,\text{Log}\left[\sqrt{x}-\#1\right]\#1^4}{b\,\#1+2c\,\#1^5}\,\&\right]}{8\left(b^2-4ac\right)}$$

Problem 1076: Result is not expressed in closed-form.

$$\int\frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Optimal (type 3, 442 leaves, 9 steps):

$$-\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{1/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{1/4}\left(b^2-4ac\right)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+$$

$$\frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{1/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{2\times2^{1/4}\left(b^2-4ac\right)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Result (type 7, 111 leaves):

$$-\frac{\frac{4\sqrt{x}\left(b+2cx^2\right)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{-b\,\text{Log}\left[\sqrt{x}-\#1\right]+6c\,\text{Log}\left[\sqrt{x}-\#1\right]\#1^4}{b\,\#1^3+2c\,\#1^7}\,\&\right]}{8\left(b^2-4ac\right)}$$

Problem 1077: Result is not expressed in closed-form.

$$\int \frac{\sqrt{x}}{\left(a+b x^2+c x^4\right)^2} \, dx$$

Optimal (type 3, 489 leaves, 9 steps):

$$\frac{x^{3/2} \left(b^2-2 a c+b c x^2\right)}{2 a \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)}+\frac{c^{1/4} \left(b-\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} a \left(b^2-4 a c\right) \left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{c^{1/4} \left(b+\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} a \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{c^{1/4} \left(b-\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} a \left(b^2-4 a c\right) \left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{c^{1/4} \left(b+\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4\times 2^{3/4} a \left(b^2-4 a c\right) \left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 149 leaves):

$$-\left(\left(4 x^{3/2} \left(b^2-2 a c+b c x^2\right)+\left(a+b x^2+c x^4\right) \text{RootSum}\left[a+b \#1^4+c \#1^8 \&,\frac{b^2 \text{Log}\left[\sqrt{x}-\#1\right]-10 a c \text{Log}\left[\sqrt{x}-\#1\right]+b c \text{Log}\left[\sqrt{x}-\#1\right] \#1^4}{b \#1+2 c \#1^5}\&\right]\right)\Big/\left(8 a \left(-b^2+4 a c\right) \left(a+b x^2+c x^4\right)\right)\right)$$

Problem 1078: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x} \left(a+b x^2+c x^4\right)^2} \, dx$$

Optimal (type 3, 503 leaves, 9 steps):

$$\frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{4\times 2^{1/4}a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-$$

$$\frac{c^{3/4}\left(3b^2-28ac+3b\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{4\times 2^{1/4}a\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+$$

$$\frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{4\times 2^{1/4}a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{c^{3/4}\left(3b^2-28ac+3b\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{4\times 2^{1/4}a\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Result (type 7, 153 leaves):

$$-\left(\left(4\sqrt{x}\left(b^2-2ac+bcx^2\right)+\left(a+bx^2+cx^4\right)\text{RootSum}\left[a+b\,\sharp1^4+c\,\sharp1^8\,\&,\frac{3b^2\,\text{Log}\left[\sqrt{x}-\sharp1\right]-14ac\,\text{Log}\left[\sqrt{x}-\sharp1\right]+3bc\,\text{Log}\left[\sqrt{x}-\sharp1\right]\sharp1^4}{b\,\sharp1^3+2c\,\sharp1^7}\,\&\right]\right)\Big/\left(8a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)\right)\right)$$

Problem 1079: Result is not expressed in closed-form.

$$\int\frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx$$

Optimal (type 3, 573 leaves, 10 steps):

$$-\frac{5\,b^2-18\,a\,c}{2\,a^2\left(b^2-4\,a\,c\right)\sqrt{x}}+\frac{b^2-2\,a\,c+b\,c\,x^2}{2\,a\left(b^2-4\,a\,c\right)\sqrt{x}\left(a+b\,x^2+c\,x^4\right)}+\frac{c^{1/4}\left(5\,b^3-28\,a\,b\,c-\left(5\,b^2-18\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,a^2\left(b^2-4\,a\,c\right)^{3/2}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{c^{1/4}\left(5\,b^3-28\,a\,b\,c+\left(5\,b^2-18\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,a^2\left(b^2-4\,a\,c\right)^{3/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{c^{1/4}\left(5\,b^3-28\,a\,b\,c-\left(5\,b^2-18\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,a^2\left(b^2-4\,a\,c\right)^{3/2}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+$$

$$\frac{c^{1/4}\left(5\,b^3-28\,a\,b\,c+\left(5\,b^2-18\,a\,c\right)\sqrt{b^2-4\,a\,c}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{4\times 2^{3/4}\,a^2\left(b^2-4\,a\,c\right)^{3/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 190 leaves):

$$-\frac{1}{8\,a^2}$$

$$\left(\frac{16}{\sqrt{x}}+\frac{4\,x^{3/2}\left(b^3-3\,a\,b\,c+b^2\,c\,x^2-2\,a\,c^2\,x^2\right)}{\left(b^2-4\,a\,c\right)\left(a+b\,x^2+c\,x^4\right)}+\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{5\,b^3\,\text{Log}\left[\sqrt{x}-\#1\right]-23\,a\,b\,c\,\text{Log}\left[\sqrt{x}-\#1\right]+5\,b^2\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4-18\,a\,c^2\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]}{b^2-4\,a\,c}\right)$$

Problem 1080: Result is not expressed in closed-form.

$$\int\frac{x^{15/2}}{\left(a+b\,x^2+c\,x^4\right)^3}\,dx$$

Optimal (type 3, 621 leaves, 11 steps):

$$-\frac{3\left(b^2+12 a c\right)\sqrt{x}}{16 c\left(b^2-4 a c\right)^2}+\frac{x^{9/2}\left(2 a+b x^2\right)}{4\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^2}+\frac{3 x^{5/2}\left(8 a b+\left(b^2+12 a c\right) x^2\right)}{16\left(b^2-4 a c\right)^2\left(a+b x^2+c x^4\right)}-$$

$$\frac{3\left(b^3-28 a b c+\frac{b^4-30 a b^2 c-24 a^2 c^2}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{5/4}\left(b^2-4 a c\right)^2\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{3\left(b^3-28 a b c-\frac{b^4-30 a b^2 c-24 a^2 c^2}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{5/4}\left(b^2-4 a c\right)^2\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}-$$

$$\frac{3\left(b^3-28 a b c+\frac{b^4-30 a b^2 c-24 a^2 c^2}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{5/4}\left(b^2-4 a c\right)^2\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{3\left(b^3-28 a b c-\frac{b^4-30 a b^2 c-24 a^2 c^2}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{5/4}\left(b^2-4 a c\right)^2\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 254 leaves):

$$\Bigg(4\sqrt{x}\left(-4 b^4+21 a b^2 c-68 a^2 c^2+b^3 c x^2-28 a b c^2 x^2\right)\left(a+b x^2+c x^4\right)+$$
$$16\left(b^2-4 a c\right)\sqrt{x}\left(-2 a^2 c+b^3 x^2+a b\left(b-3 c x^2\right)\right)+3 c\left(a+b x^2+c x^4\right)^2\text{RootSum}\Big[a+b\,\sharp1^4+c\,\sharp1^8\,\&,$$
$$\frac{a b^2\text{Log}\left[\sqrt{x}-\sharp1\right]+12 a^2 c\,\text{Log}\left[\sqrt{x}-\sharp1\right]+b^3\text{Log}\left[\sqrt{x}-\sharp1\right]\sharp1^4-28 a b c\,\text{Log}\left[\sqrt{x}-\sharp1\right]\sharp1^4}{b\,\sharp1^3+2 c\,\sharp1^7}\,\&\Big]\Bigg)\Big/\left(64 c^2\left(b^2-4 a c\right)^2\left(a+b x^2+c x^4\right)^2\right)$$

Problem 1081: Result is not expressed in closed-form.

$$\int\frac{x^{13/2}}{\left(a+b x^2+c x^4\right)^3}\,dx$$

Optimal (type 3, 569 leaves, 10 steps):

$$\frac{x^{7/2}\left(2 a+b x^2\right)}{4\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^2}+\frac{x^{3/2}\left(24 a b+\left(5 b^2+28 a c\right) x^2\right)}{16\left(b^2-4 a c\right)^2\left(a+b x^2+c x^4\right)}+$$

$$\frac{\left(5 b^3+172 a b c+\sqrt{b^2-4 a c}\left(5 b^2+28 a c\right)\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{3/4} c^{3/4}\left(b^2-4 a c\right)^{5/2}\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{\left(5 b^2+28 a c-\frac{5 b^3+172 a b c}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{3/4} c^{3/4}\left(b^2-4 a c\right)^2\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}-$$

$$\frac{\left(5 b^3+172 a b c+\sqrt{b^2-4 a c}\left(5 b^2+28 a c\right)\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{3/4} c^{3/4}\left(b^2-4 a c\right)^{5/2}\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{\left(5 b^2+28 a c-\frac{5 b^3+172 a b c}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{3/4} c^{3/4}\left(b^2-4 a c\right)^2\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 216 leaves):

$$\left(4 x^{3/2} \left(4 b^3+8 a b c+5 b^2 c x^2+28 a c^2 x^2\right) \left(a+b x^2+c x^4\right)-16 \left(b^2-4 a c\right) x^{3/2} \left(b^2 x^2+a \left(b-2 c x^2\right)\right)+c \left(a+b x^2+c x^4\right)^2 \text{RootSum}\left[a+b \#1^4+c \#1^8\&,\frac{-72 a b \text{Log}\left[\sqrt{x}-\#1\right]+5 b^2 \text{Log}\left[\sqrt{x}-\#1\right] \#1^4+28 a c \text{Log}\left[\sqrt{x}-\#1\right] \#1^4}{b \#1+2 c \#1^5}\&\right]\right) \Big/ \left(64 c \left(b^2-4 a c\right)^2 \left(a+b x^2+c x^4\right)^2\right)$$

Problem 1082: Result is not expressed in closed-form.

$$\int \frac{x^{11/2}}{\left(a+b x^2+c x^4\right)^3} \, dx$$

Optimal (type 3, 569 leaves, 10 steps):

$$\frac{x^{5/2} \left(2 a+b x^2\right)}{4 \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)^2}+\frac{\sqrt{x} \left(24 a b+\left(7 b^2+20 a c\right) x^2\right)}{16 \left(b^2-4 a c\right)^2 \left(a+b x^2+c x^4\right)}-$$
$$\frac{3 \left(7 b^3+36 a b c+\sqrt{b^2-4 a c} \left(7 b^2+20 a c\right)\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{1/4} \left(b^2-4 a c\right)^{5/2} \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{3 \left(7 b^2+20 a c-\frac{7 b^3+36 a b c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{1/4} \left(b^2-4 a c\right)^2 \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}-$$
$$\frac{3 \left(7 b^3+36 a b c+\sqrt{b^2-4 a c} \left(7 b^2+20 a c\right)\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{1/4} \left(b^2-4 a c\right)^{5/2} \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{3 \left(7 b^2+20 a c-\frac{7 b^3+36 a b c}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{32\times 2^{1/4} c^{1/4} \left(b^2-4 a c\right)^2 \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 219 leaves):

$$\left(4 \sqrt{x} \left(4 b^3+8 a b c+7 b^2 c x^2+20 a c^2 x^2\right) \left(a+b x^2+c x^4\right)-16 \left(b^2-4 a c\right) \sqrt{x} \left(b^2 x^2+a \left(b-2 c x^2\right)\right)+3 c \left(a+b x^2+c x^4\right)^2 \text{RootSum}\left[a+b \#1^4+c \#1^8\&,\frac{-8 a b \text{Log}\left[\sqrt{x}-\#1\right]+7 b^2 \text{Log}\left[\sqrt{x}-\#1\right] \#1^4+20 a c \text{Log}\left[\sqrt{x}-\#1\right] \#1^4}{b \#1^3+2 c \#1^7}\&\right]\right) \Big/ \left(64 c \left(b^2-4 a c\right)^2 \left(a+b x^2+c x^4\right)^2\right)$$

Problem 1083: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{\left(a+b x^2+c x^4\right)^3} \, dx$$

Optimal (type 3, 533 leaves, 10 steps):

$$\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-$$

$$\frac{3c^{1/4}\left(11b^2+20ac+4b\sqrt{b^2-4ac}\right)\mathrm{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{3/4}\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}+\frac{3c^{1/4}\left(11b^2+20ac-4b\sqrt{b^2-4ac}\right)\mathrm{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{3/4}\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}+$$

$$\frac{3c^{1/4}\left(11b^2+20ac+4b\sqrt{b^2-4ac}\right)\mathrm{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{3/4}\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}-\frac{3c^{1/4}\left(11b^2+20ac-4b\sqrt{b^2-4ac}\right)\mathrm{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{3/4}\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}$$

Result (type 7, 176 leaves):

$$\frac{1}{64\left(b^2-4ac\right)^2}\left(\frac{16\left(b^2-4ac\right)x^{3/2}\left(2a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2}-\frac{12x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{a+bx^2+cx^4}-\right.$$
$$\left.3\,\mathrm{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\frac{-7b^2\,\mathrm{Log}\left[\sqrt{x}-\#1\right]-20ac\,\mathrm{Log}\left[\sqrt{x}-\#1\right]+8bc\,\mathrm{Log}\left[\sqrt{x}-\#1\right]\#1^4}{b\,\#1+2c\,\#1^5}\,\&\right]\right)$$

Problem 1084: Result is not expressed in closed-form.

$$\int\frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Optimal (type 3, 533 leaves, 10 steps):

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+$$

$$\frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2-4ac}\right)\mathrm{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{1/4}\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{c^{3/4}\left(41b^2+28ac-36b\sqrt{b^2-4ac}\right)\mathrm{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{1/4}\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+$$

$$\frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2-4ac}\right)\mathrm{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{1/4}\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{c^{3/4}\left(41b^2+28ac-36b\sqrt{b^2-4ac}\right)\mathrm{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{16\times2^{1/4}\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Result (type 7, 177 leaves):

$$-\frac{1}{64\,\left(b^2-4\,a\,c\right)^2}\left(\frac{4\,\sqrt{x}\,\left(28\,a^2\,c+a\,\left(5\,b^2+36\,b\,c\,x^2-4\,c^2\,x^4\right)+b\,x^2\,\left(9\,b^2+37\,b\,c\,x^2+24\,c^2\,x^4\right)\right)}{\left(a+b\,x^2+c\,x^4\right)^2}+\right.$$
$$\left.\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{-5\,b^2\,\text{Log}\left[\sqrt{x}-\#1\right]-28\,a\,c\,\text{Log}\left[\sqrt{x}-\#1\right]+72\,b\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]\right)$$

Problem 1085: Result is not expressed in closed-form.

$$\int\frac{x^{5/2}}{\left(a+b\,x^2+c\,x^4\right)^3}\,dx$$

Optimal (type 3, 594 leaves, 10 steps):

$$-\frac{x^{3/2}\,\left(b+2\,c\,x^2\right)}{4\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)^2}+\frac{3\,x^{3/2}\,\left(b\,\left(b^2+4\,a\,c\right)+c\,\left(b^2+12\,a\,c\right)\,x^2\right)}{16\,a\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x^2+c\,x^4\right)}+$$
$$\frac{3\,c^{1/4}\,\left(b^2+12\,a\,c-\frac{b^3}{\sqrt{b^2-4\,a\,c}}+\frac{68\,a\,b\,c}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{3/4}\,a\,\left(b^2-4\,a\,c\right)^2\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+\frac{3\,c^{1/4}\,\left(b^3-68\,a\,b\,c+\sqrt{b^2-4\,a\,c}\,\left(b^2+12\,a\,c\right)\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{3/4}\,a\,\left(b^2-4\,a\,c\right)^{5/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$
$$\frac{3\,c^{1/4}\,\left(b^2+12\,a\,c-\frac{b^3}{\sqrt{b^2-4\,a\,c}}+\frac{68\,a\,b\,c}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{3/4}\,a\,\left(b^2-4\,a\,c\right)^2\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-\frac{3\,c^{1/4}\,\left(b^3-68\,a\,b\,c+\sqrt{b^2-4\,a\,c}\,\left(b^2+12\,a\,c\right)\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{3/4}\,a\,\left(b^2-4\,a\,c\right)^{5/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 222 leaves):

$$\left(-16\,a\,\left(b^2-4\,a\,c\right)\,x^{3/2}\,\left(b+2\,c\,x^2\right)+12\,x^{3/2}\,\left(b^3+4\,a\,b\,c+b^2\,c\,x^2+12\,a\,c^2\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)+\right.$$
$$\left.3\,\left(a+b\,x^2+c\,x^4\right)^2\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{b^3\,\text{Log}\left[\sqrt{x}-\#1\right]-28\,a\,b\,c\,\text{Log}\left[\sqrt{x}-\#1\right]+b^2\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4+12\,a\,c^2\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]\right)\Big/$$
$$\left(64\,a\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x^2+c\,x^4\right)^2\right)$$

Problem 1086: Result is not expressed in closed-form.

$$\int\frac{x^{3/2}}{\left(a+b\,x^2+c\,x^4\right)^3}\,dx$$

Optimal (type 3, 594 leaves, 10 steps):

$$-\frac{\sqrt{x}\,\left(b+2\,c\,x^2\right)}{4\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)^2}+\frac{\sqrt{x}\,\left(b\,\left(b^2+20\,a\,c\right)+c\,\left(b^2+44\,a\,c\right)\,x^2\right)}{16\,a\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x^2+c\,x^4\right)}-$$

$$\frac{3\,c^{3/4}\,\left(b^2+44\,a\,c-\frac{b^3}{\sqrt{b^2-4\,a\,c}}+\frac{68\,a\,b\,c}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{1/4}\,a\,\left(b^2-4\,a\,c\right)^2\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}-\frac{3\,c^{3/4}\,\left(b^3-68\,a\,b\,c+\sqrt{b^2-4\,a\,c}\,\left(b^2+44\,a\,c\right)\right)\,\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{1/4}\,a\,\left(b^2-4\,a\,c\right)^{5/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}-$$

$$\frac{3\,c^{3/4}\,\left(b^2+44\,a\,c-\frac{b^3}{\sqrt{b^2-4\,a\,c}}+\frac{68\,a\,b\,c}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{1/4}\,a\,\left(b^2-4\,a\,c\right)^2\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}-\frac{3\,c^{3/4}\,\left(b^3-68\,a\,b\,c+\sqrt{b^2-4\,a\,c}\,\left(b^2+44\,a\,c\right)\right)\,\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,\sqrt{x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{32\times 2^{1/4}\,a\,\left(b^2-4\,a\,c\right)^{5/2}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}$$

Result (type 7, 224 leaves) :

$$\left(-16\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{x}\,\left(b+2\,c\,x^2\right)+4\,\sqrt{x}\,\left(b^3+20\,a\,b\,c+b^2\,c\,x^2+44\,a\,c^2\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)+\right.$$
$$\left.3\,\left(a+b\,x^2+c\,x^4\right)^2\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{b^3\,\text{Log}\left[\sqrt{x}-\#1\right]-12\,a\,b\,c\,\text{Log}\left[\sqrt{x}-\#1\right]+b^2\,c\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4+44\,a\,c^2\,\text{Log}\left[\sqrt{x}-\#1\right]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]\right)\Big/$$
$$\left(64\,a\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x^2+c\,x^4\right)^2\right)$$

Problem 1087: Result is not expressed in closed-form.

$$\int\frac{\sqrt{x}}{\left(a+b\,x^2+c\,x^4\right)^3}\,dx$$

Optimal (type 3, 658 leaves, 10 steps) :

$$\frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-$$

$$\frac{c^{1/4}\left(5b^4-54ab^2c+520a^2c^2-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}\right)\operatorname{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}+$$

$$\frac{c^{1/4}\left(5b^4-54ab^2c+520a^2c^2+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}\right)\operatorname{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}+$$

$$\frac{c^{1/4}\left(5b^4-54ab^2c+520a^2c^2-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}\right)\operatorname{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}-$$

$$\frac{c^{1/4}\left(5b^4-54ab^2c+520a^2c^2+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}\right)\operatorname{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}$$

Result (type 7, 254 leaves):

$$\frac{1}{64a^2\left(b^2-4ac\right)^2}\left(-\frac{16a\left(-b^2+4ac\right)x^{3/2}\left(b^2-2ac+bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}+\frac{4x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+5b^3cx^2-44abc^2x^2\right)}{a+bx^2+cx^4}+\operatorname{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\right.\right.$$
$$\left.\left.\frac{1}{b\,\#1+2c\,\#1^5}\left(5b^4\operatorname{Log}\left[\sqrt{x}-\#1\right]-49ab^2c\operatorname{Log}\left[\sqrt{x}-\#1\right]+260a^2c^2\operatorname{Log}\left[\sqrt{x}-\#1\right]+5b^3c\operatorname{Log}\left[\sqrt{x}-\#1\right]\#1^4-44abc^2\operatorname{Log}\left[\sqrt{x}-\#1\right]\#1^4\right)\&\right]\right)$$

Problem 1088: Result is not expressed in closed-form.

$$\int\frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx$$

Optimal (type 3, 658 leaves, 10 steps):

$$\frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+$$

$$\frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times 2^{1/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-$$

$$\frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times 2^{1/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+$$

$$\frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times 2^{1/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-$$

$$\frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{b^2-4ac}\right)^{1/4}}\right]}{32\times 2^{1/4}a^2\left(b^2-4ac\right)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Result (type 7, 258 leaves):

$$\frac{1}{64a^2\left(b^2-4ac\right)^2}\left(-\frac{16a\left(-b^2+4ac\right)\sqrt{x}\left(b^2-2ac+bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}+\frac{4\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+7b^3cx^2-52abc^2x^2\right)}{a+bx^2+cx^4}+3\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\right.\right.$$

$$\left.\left.\frac{1}{b\,\#1^3+2c\,\#1^7}\left(7b^4\,\text{Log}\left[\sqrt{x}-\#1\right]-59ab^2c\,\text{Log}\left[\sqrt{x}-\#1\right]+140a^2c^2\,\text{Log}\left[\sqrt{x}-\#1\right]+7b^3c\,\text{Log}\left[\sqrt{x}-\#1\right]\#1^4-52abc^2\,\text{Log}\left[\sqrt{x}-\#1\right]\#1^4\right)\&\right]\right)$$

Problem 1089: Result more than twice size of optimal antiderivative.

$$\int (d\,x)^{3/2}\sqrt{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\frac{2\,(d\,x)^{5/2}\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Result (type 6, 1048 leaves):

$$\frac{1}{225\, c^2 \left(a+b x^2+c x^4\right)^{3/2}}\, d \sqrt{d x} \Bigg(10 c \left(2 b+5 c x^2\right) \left(a+b x^2+c x^4\right)^2 -$$

$$\Big(25 a^2 b \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(5 a\, \text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-x^2 \Big(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)+$$

$$\Big(90 a^2 c x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(9 a\, \text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-x^2 \Big(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)+$$

$$\Big(27 a b^2 x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(-9 a\, \text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+x^2 \Big(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Bigg)$$

Problem 1090: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d x}\, \sqrt{a+b x^2+c x^4}\, dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\frac{2 (d x)^{3/2} \sqrt{a+b x^2+c x^4}\, \text{AppellF1}\left[\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{3 d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}}$$

Result (type 6, 706 leaves):

$$\frac{1}{147\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,2\,x\,\sqrt{d\,x}\,\left(21\,\left(a+b\,x^2+c\,x^4\right)^2+\left(49\,a^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+\left(33\,a\,b\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(22\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,c\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1091: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{d\,x}}\,dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$\frac{2\,\sqrt{d\,x}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 709 leaves):

$$
\begin{aligned}
&\frac{1}{25\sqrt{d\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\\
&2\,x\left(5\,\left(a+b\,x^2+c\,x^4\right)^2+\left(25\,a^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.\\
&\quad\left(c\left(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.\\
&\qquad\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+\\
&\quad\left(9\,a\,b\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\\
&\quad\left(2\,c\left(9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\\
&\qquad\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)
\end{aligned}
$$

Problem 1092: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{a+b\,x^2+c\,x^4}}{(d\,x)^{3/2}}\,dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$-\frac{2\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\sqrt{d\,x}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 707 leaves):

$$\frac{1}{21\,(d\,x)^{3/2}\,(a+b\,x^2+c\,x^4)^{3/2}}\,2\,x\,\Bigg(-21\,(a+b\,x^2+c\,x^4)^2+\Big(49\,a\,b\,x^2\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big(2\,c\,\Big(7\,a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+\Big(33\,a\,x^4\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big(11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Bigg)$$

Problem 1093: Result more than twice size of optimal antiderivative.

$$\int (d\,x)^{3/2}\,(a+b\,x^2+c\,x^4)^{3/2}\,dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$\frac{2\,a\,(d\,x)^{5/2}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\Big[\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{5\,d\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1751 leaves):

$$\frac{1}{16\,575\,c^3\,(a+b\,x^2+c\,x^4)^{3/2}}\,2\,d\,\sqrt{d\,x}\,\Big(5\,c\,(a+b\,x^2+c\,x^4)^2\,\big(-28\,b^3+20\,b^2\,c\,x^2+65\,c^2\,x^2\,(7\,a+3\,c\,x^4)+b\,c\,(176\,a+285\,c\,x^4)\big)+\Big(175\,a^2\,b^3\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big(5\,a\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)-$$

$$\Big(1100\, a^3 b\, c \Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(5\, a\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)+$$

$$\Big(189\, a\, b^4 x^2 \Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(9\, a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)+$$

$$\Big(2340\, a^3 c^2 x^2 \Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(9\, a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-$$

$$x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+$$

$$\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)+$$

$$\Big(1413\, a^2 b^2 c\, x^2 \Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(-9\, a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+$$

$$x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+$$

$$\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)$$

Problem 1094: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\,dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$\frac{2\,a\,(d\,x)^{3/2}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,d\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1395 leaves):

$$
\begin{aligned}
&\frac{1}{8085\, c^2 \left(a+b x^2+c x^4\right)^{3/2}} 2 x \sqrt{d x} \left(7 c \left(a+b x^2+c x^4\right)^2 \left(12 b^2+119 b c x^2+11 c \left(19 a+7 c x^4\right)\right) -\right.\\
&\quad \left(147 a^2 b^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \\
&\quad \left(7 a\, \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.\\
&\qquad\quad \left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+\\
&\quad \left(2156 a^3 c \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \\
&\quad \left(7 a\, \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.\\
&\qquad\quad \left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+\\
&\quad \left(1188 a^2 b c x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \\
&\quad \left(11 a\, \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-\right.\\
&\qquad x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\\
&\qquad\quad \left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+\\
&\quad \left(165 a b^3 x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \\
&\quad \left(-11 a\, \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\\
&\qquad x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\\
&\qquad\quad \left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)
\end{aligned}
$$

Problem 1095: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b x^2 + c x^4\right)^{3/2}}{\sqrt{d x}} \, dx$$

Optimal (type 6, 146 leaves, 2 steps):

$$\frac{2 a \sqrt{d x} \sqrt{a + b x^2 + c x^4} \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{5}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]}{d \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}}}$$

Result (type 6, 1395 leaves):

$$\frac{1}{975\,c^2\sqrt{d\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,2\,x\,\Bigg(5\,c\,\left(a+b\,x^2+c\,x^4\right)^2\,\left(4\,b^2+25\,b\,c\,x^2+3\,c\,\left(17\,a+5\,c\,x^4\right)\right)-$$

$$\left(25\,a^2\,b^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(900\,a^3\,c\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(252\,a^2\,b\,c\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(27\,a\,b^3\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Bigg)$$

Problem 1096: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x^2+c\,x^4\right)^{3/2}}{\left(d\,x\right)^{3/2}}\,dx$$

Optimal (type 6, 146 leaves, 2 steps):

$$-\frac{2\,a\,\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\,\sqrt{d\,x}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1059 leaves):

$$\frac{1}{539\,(d\,x)^{3/2}\,(a+b\,x^2+c\,x^4)^{3/2}}\,2\,x\,\Bigg(7\,(a+b\,x^2+c\,x^4)^2\,(-77\,a+13\,b\,x^2+7\,c\,x^4)+$$
$$\left(784\,a^2\,b\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(c\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$
$$\left(924\,a^2\,x^4\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$
$$x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$
$$\left(33\,a\,b^2\,x^4\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(c\left(11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Bigg)$$

Problem 1097: Result more than twice size of optimal antiderivative.

$$\int\frac{(d\,x)^{3/2}}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\frac{2\,(d\,x)^{5/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,d\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 386 leaves):

$$-\left(\left(18\,a^2\,x\,(d\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$
$$\left(5\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)$$

Problem 1098: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{d\,x}}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\frac{2\,(d\,x)^{3/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,d\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 386 leaves):

$$-\left(\left(14\,a^2\,x\,\sqrt{d\,x}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$
$$\left(3\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)$$

Problem 1099: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{d x}\,\sqrt{a+b x^2+c x^4}}\,dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$\frac{2\sqrt{d x}\,\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\,\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{d\sqrt{a+b x^2+c x^4}}$$

Result (type 6, 384 leaves):

$$-\left(\left(10 a^2 x\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\right.$$
$$\left(\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\sqrt{d x}\left(a+b x^2+c x^4\right)^{3/2}\right.$$
$$\left(-5 a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2\left(\left(b+\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right)$$

Problem 1100: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(d x)^{3/2}\sqrt{a+b x^2+c x^4}}\,dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$-\frac{2\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\,\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\,\text{AppellF1}\left[-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{d\sqrt{d x}\,\sqrt{a+b x^2+c x^4}}$$

Result (type 6, 710 leaves):

$$\frac{1}{21\,a\,(d\,x)^{3/2}\,(a+b\,x^2+c\,x^4)^{3/2}}\,2\,x\,\Bigg(-21\,(a+b\,x^2+c\,x^4)^2+\Big(49\,a\,b\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big/\Big(4\,c\,\Big(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\Big)+\Big(99\,a\,x^4\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big/\Big(44\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-4\,x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\Bigg)$$

Problem 1101: Result more than twice size of optimal antiderivative.

$$\int \frac{(d\,x)^{3/2}}{(a+b\,x^2+c\,x^4)^{3/2}}\,dx$$

Optimal (type 6, 150 leaves, 2 steps):

$$\frac{2\,(d\,x)^{5/2}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,a\,d\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 720 leaves):

$$\frac{1}{5\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} d\sqrt{dx}\left(-5\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)+\right.$$
$$\left(25ab\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(4c\left(5a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)+$$
$$\left(9ax^2\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(18a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-\right.$$
$$2x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)$$

Problem 1102: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 6, 150 leaves, 2 steps):

$$\frac{2\,(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left[\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]}{3ad\sqrt{a+bx^2+cx^4}}$$

Result (type 6, 1058 leaves):

$$\frac{1}{84\,a\,\left(-b^2+4\,a\,c\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,x\,\sqrt{d\,x}\,\left(-84\,\left(b^2-2\,a\,c+b\,c\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)+\right.$$

$$\left(196\,a^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(14\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(49\,a\,b^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$

$$\left(99\,a\,b\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(-11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1103: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{d\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$\frac{2\,\sqrt{d\,x}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{a\,d\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 1058 leaves):

$$\frac{1}{20\,a\,\left(-b^2+4\,a\,c\right)\,\sqrt{d\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,x\,\Bigg(-20\,\left(b^2-2\,a\,c+b\,c\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)+$$

$$\left(300\,a^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Bigg(10\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)-$$

$$\left(25\,a\,b^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Bigg(c\,\Bigg(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.$$

$$\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg)-$$

$$\left(9\,a\,b\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Bigg(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 1104: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(d\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$-\frac{2\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{a\,d\,\sqrt{d\,x}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 1600 leaves):

$$\frac{1}{7\ (d\,x)^{3/2}\ (a+b\,x^2+c\,x^4)^{3/2}}$$

$$\times\Bigg(\frac{7\,x^2\ (b^3-3\,a\,b\,c+b^2\,c\,x^2-2\,a\,c^2\,x^2)\ (a+b\,x^2+c\,x^4)}{a^2\ (-b^2+4\,a\,c)}-\frac{14\ (a+b\,x^2+c\,x^4)^2}{a^2}+\Big(49\,b^3\,x^2\ \big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)$$

$$\mathrm{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big((b^2-4\,a\,c)\ \big(-b+\sqrt{b^2-4\,a\,c}\big)\ \big(b+\sqrt{b^2-4\,a\,c}\big)$$

$$\Big(-7\,a\,\mathrm{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\ \Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(147\,a\,b\,c\,x^2\ \big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \mathrm{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big((b^2-4\,a\,c)\ \big(-b+\sqrt{b^2-4\,a\,c}\big)\ \big(b+\sqrt{b^2-4\,a\,c}\big)\ \Big(-7\,a\,\mathrm{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\ \Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\big(b-\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$

$$\Big(99\,b^2\,c\,x^4\ \big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big((b^2-4\,a\,c)\ \big(-b+\sqrt{b^2-4\,a\,c}\big)\ \big(b+\sqrt{b^2-4\,a\,c}\big)\ \Big(-11\,a\,\mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\ \Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\big(b-\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(330\,a\,c^2\,x^4\ \big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\ \mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big((b^2-4\,a\,c)\ \big(-b+\sqrt{b^2-4\,a\,c}\big)\ \big(b+\sqrt{b^2-4\,a\,c}\big)\ \Big(-11\,a\,\mathrm{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\ \Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\ \mathrm{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 1108: Result is not expressed in closed-form.

$$\int\frac{(dx)^m}{a+bx^2+cx^4}\,dx$$

Optimal (type 5, 173 leaves, 3 steps):

$$\frac{2c\,(dx)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right]}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)d\,(1+m)}-\frac{2c\,(dx)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}\right)d\,(1+m)}$$

Result (type 7, 82 leaves):

$$\frac{(dx)^m\,\text{RootSum}\left[a+b\,\#1^2+c\,\#1^4\,\&,\frac{\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right]\left(\frac{x}{x-\#1}\right)^{-m}}{b\,\#1+2c\,\#1^3}\,\&\right]}{2m}$$

Problem 1109: Result unnecessarily involves higher level functions.

$$\int\frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Optimal (type 5, 315 leaves, 4 steps):

$$\frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)}+$$
$$\left(c\left(b^2(1-m)+b\sqrt{b^2-4ac}\,(1-m)-4ac\,(3-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d\,(1+m)\right)-$$
$$\left(c\left(b^2(1-m)-b\sqrt{b^2-4ac}\,(1-m)-4ac\,(3-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d\,(1+m)\right)$$

Result (type 6, 376 leaves):

$$\left(a\,(3+m)\,x\,(d\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(4\,c\,(1+m)\left(a+b\,x^2+c\,x^4\right)^3\left(a\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.\right.$$
$$2\,x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},2,3,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1110: Result more than twice size of optimal antiderivative.

$$\int (d\,x)^m\left(a+b\,x^2+c\,x^4\right)^{3/2}dx$$

Optimal (type 6, 158 leaves, 2 steps):

$$\frac{a\,(d\,x)^{1+m}\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{1+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\,(1+m)\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1080 leaves):

$$\frac{1}{8 c^2 \sqrt{a+b x^2+c x^4}} \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) x (d x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right)$$

$$\left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(\left(a (3+m) \text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right/$$

$$\left((1+m) \left(2 a (3+m) \text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)+$$

$$\left(b (5+m) x^2 \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$

$$\left((3+m) \left(2 a (5+m) \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2},-\frac{1}{2},\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2},\frac{1}{2},-\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)+$$

$$\left(c (7+m) x^4 \text{AppellF1}\left[\frac{5+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$

$$\left((5+m) \left(2 a (7+m) \text{AppellF1}\left[\frac{5+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2},-\frac{1}{2},\frac{1}{2},\frac{9+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$

$$\left.\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2},\frac{1}{2},-\frac{1}{2},\frac{9+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right)$$

Problem 1111: Result more than twice size of optimal antiderivative.

$$\int (d x)^m \sqrt{a+b x^2+c x^4}\, dx$$

Optimal (type 6, 157 leaves, 2 steps):

$$\frac{(d\,x)^{1+m}\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\,(1+m)\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 423 leaves) :

$$\left(\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)(3+m)\,x\,(d\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(8\,c^2\,(1+m)\sqrt{a+b\,x^2+c\,x^4}\left(2\,a\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$
$$x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1112: Result more than twice size of optimal antiderivative.

$$\int\frac{(d\,x)^m}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 157 leaves, 2 steps) :

$$\frac{(d\,x)^{1+m}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\text{AppellF1}\left[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{d\,(1+m)\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 425 leaves) :

$$-\left(\left(2 a^2 (3+m) x (d x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(\left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) (1+m) \left(a+b x^2+c x^4\right)^{3/2} \left(-2 a (3+m) \text{AppellF1}\left[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{1}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{3}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right)$$

Problem 1113: Result more than twice size of optimal antiderivative.

$$\int \frac{(d x)^m}{\left(a+b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 6, 160 leaves, 2 steps):

$$\frac{(d x)^{1+m} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{a d (1+m) \sqrt{a+b x^2+c x^4}}$$

Result (type 6, 426 leaves):

$$\left(2 a^2 (3+m) x (d x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(\left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) (1+m) \left(a+b x^2+c x^4\right)^{5/2} \left(2 a (3+m) \text{AppellF1}\left[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-3 x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{3}{2},\frac{5}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{5}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 1114: Result more than twice size of optimal antiderivative.

$$\int (d x)^m \left(a+b x^2+c x^4\right)^p \, dx$$

Optimal (type 6, 155 leaves, 2 steps):

$$\frac{1}{d\,(1+m)}(d\,x)^{1+m}\left(1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(a+b\,x^2+c\,x^4\right)^p \text{AppellF1}\left[\frac{1+m}{2},-p,-p,\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]$$

Result (type 6, 499 leaves) :

$$-\left(\left(2^{-2-p}\,c\left(b+\sqrt{b^2-4\,a\,c}\right)(3+m)\,x\,(d\,x)^m\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^2\right)^{-p}\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{c}\right)^{1+p}\right.\right.$$
$$\left.\left(-2\,a+\left(-b+\sqrt{b^2-4\,a\,c}\right)x^2\right)^2\left(a+b\,x^2+c\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{1+m}{2},-p,-p,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)(1+m)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(a\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-p,-p,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$
$$p\,x^2\left(\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},1-p,-p,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{3+m}{2},-p,1-p,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)$$

Problem 1115: Result unnecessarily involves higher level functions.

$$\int x^7\left(a+b\,x^2+c\,x^4\right)^p\,dx$$

Optimal (type 5, 257 leaves, 4 steps) :

$$\frac{x^4\left(a+b\,x^2+c\,x^4\right)^{1+p}}{4\,c\,(2+p)}+\frac{\left(b^2\,(2+p)\,(3+p)-2\,a\,c\,(3+2\,p)-2\,b\,c\,(1+p)\,(3+p)\,x^2\right)\left(a+b\,x^2+c\,x^4\right)^{1+p}}{8\,c^3\,(1+p)\,(2+p)\,(3+2\,p)}-$$
$$\left(2^{-2+p}\,b\left(6\,a\,c-b^2\,(3+p)\right)\left(-\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{\sqrt{b^2-4\,a\,c}}\right)^{-1-p}\left(a+b\,x^2+c\,x^4\right)^{1+p}\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{2\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(c^3\sqrt{b^2-4\,a\,c}\,(1+p)\,(3+2\,p)\right)$$

Result (type 6, 440 leaves) :

$$\left(5\times 2^{-4-p} c \left(b+\sqrt{b^2-4 a c}\right) x^8 \left(\frac{b-\sqrt{b^2-4 a c}}{2 c}+x^2\right)^{-p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c}\right)^{1+p} \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x^2\right)^2 \left(a+x^2 \left(b+c x^2\right)\right)^{-1+p} \text{AppellF1}\left[4,-p,-p,5,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\left(-b+\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(-10 a \text{AppellF1}\left[4,-p,-p,5,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+p x^2 \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5,1-p,-p,6,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5,-p,1-p,6,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 1116: Result unnecessarily involves higher level functions.

$$\int x^5 \left(a+b x^2+c x^4\right)^p dx$$

Optimal (type 5, 223 leaves, 4 steps):

$$-\frac{b (2+p) \left(a+b x^2+c x^4\right)^{1+p}}{4 c^2 (1+p) (3+2 p)}+\frac{x^2 \left(a+b x^2+c x^4\right)^{1+p}}{2 c (3+2 p)}+\left(2^{-1+p} \left(2 a c-b^2 (2+p)\right) \left(-\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{\sqrt{b^2-4 a c}}\right)^{-1-p} \left(a+b x^2+c x^4\right)^{1+p} \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{2 \sqrt{b^2-4 a c}}\right]\right) \Big/ \left(c^2 \sqrt{b^2-4 a c} (1+p) (3+2 p)\right)$$

Result (type 6, 395 leaves):

$$\left(\left(b+\sqrt{b^2-4 a c}\right) x^6 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x^2\right)^2 \left(a+x^2 \left(b+c x^2\right)\right)^{-1+p} \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(3 \left(-b+\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(-8 a \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+p x^2 \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,1-p,-p,5,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,-p,1-p,5,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 1117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b x^2+c x^4\right)^p \, dx$$

Optimal (type 5, 160 leaves, 3 steps):

$$\frac{\left(a+b x^2+c x^4\right)^{1+p}}{4 c (1+p)}+\frac{2^{-1+p} b \left(-\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{\sqrt{b^2-4 a c}}\right)^{-1-p} \left(a+b x^2+c x^4\right)^{1+p} \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{2 \sqrt{b^2-4 a c}}\right]}{c \sqrt{b^2-4 a c} (1+p)}$$

Result (type 6, 440 leaves):

$$\left(3\times 2^{-3-p} c \left(b+\sqrt{b^2-4 a c}\right) x^4 \left(\frac{b-\sqrt{b^2-4 a c}}{2 c}+x^2\right)^{-p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c}\right)^{1+p} \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x^2\right)^2 \left(a+x^2 \left(b+c x^2\right)\right)^{-1+p} \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/ \left(\left(-b+\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(-6 a \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+p x^2 \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,1-p,-p,4,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,-p,1-p,4,-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 1119: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^2+c x^4\right)^p}{x} \, dx$$

Optimal (type 6, 152 leaves, 3 steps):

$$\frac{1}{p}4^{-1+p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c x^2}\right)^{-p} \left(\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{c x^2}\right)^{-p} \left(a+b x^2+c x^4\right)^p \text{AppellF1}\left[-2 p,-p,-p,1-2 p,-\frac{b-\sqrt{b^2-4 a c}}{2 c x^2},-\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}\right]$$

Result (type 6, 497 leaves):

$$\left(2^{-3-2p}\, c\, (-1+2p) \left(1+\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right)^{-p} x^2 \left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p} \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{c}\right)^{1+p} \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{p} \left(b+\sqrt{b^2-4ac}+2cx^2\right) \left(a+bx^2+cx^4\right)^{-1+p} \text{AppellF1}\left[-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]\right) \Big/ \left(p\left(-\left(b+\sqrt{b^2-4ac}\right) p\,\text{AppellF1}\left[1-2p,1-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]+\left(-b+\sqrt{b^2-4ac}\right) p\,\text{AppellF1}\left[1-2p,-p,1-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]+2c(-1+2p)\,x^2\,\text{AppellF1}\left[-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]\right)\right)$$

Problem 1120: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^3}\,dx$$

Optimal (type 6, 166 leaves, 3 steps):

$$-\frac{1}{(1-2p)\,x^2} 2^{-1+2p} \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{AppellF1}\left[1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right]$$

Result (type 6, 516 leaves):

$$\left(2^{-1-2p} (-1+p) \left(1+\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right)^{-p} \left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p} \left(-b+\sqrt{b^2-4ac}-2cx^2\right) \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{c}\right)^{p} \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{p} \left(b+\sqrt{b^2-4ac}+2cx^2\right) \left(a+bx^2+cx^4\right)^{-1+p} \text{AppellF1}\left[1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]\right) \Big/ \left((-1+2p)\left(-4c(-1+p)\,x^2\,\text{AppellF1}\left[1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]+\left(b+\sqrt{b^2-4ac}\right) p\,\text{AppellF1}\left[2-2p,1-p,-p,3-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]+\left(b-\sqrt{b^2-4ac}\right) p\,\text{AppellF1}\left[2-2p,-p,1-p,3-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right]\right)\right)$$

Problem 1121: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^2+c x^4\right)^p}{x^5} \, dx$$

Optimal (type 6, 164 leaves, 3 steps):

$$-\frac{1}{(1-p)\, x^4} 4^{-1+p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c x^2}\right)^{-p} \left(\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{c x^2}\right)^{-p} \left(a+b x^2+c x^4\right)^p \text{AppellF1}\left[2 (1-p), -p, -p, 3-2 p, -\frac{b-\sqrt{b^2-4 a c}}{2 c x^2}, -\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}\right]$$

Result (type 6, 504 leaves):

$$\left(2^{-3-2 p} c\, (-3+2 p) \left(1+\frac{b-\sqrt{b^2-4 a c}}{2 c x^2}\right)^{-p} \left(\frac{b-\sqrt{b^2-4 a c}}{2 c}+x^2\right)^{-p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c}\right)^{1+p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{c x^2}\right)^{p} \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(a+b x^2+c x^4\right)^{-1+p} \text{AppellF1}\left[2-2 p, -p, -p, 3-2 p, -\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}, \frac{-b+\sqrt{b^2-4 a c}}{2 c x^2}\right]\right) \Big/ \left((-1+p)\, x^2 \left(2 c\, (-3+2 p)\, x^2 \,\text{AppellF1}\left[2-2 p, -p, -p, 3-2 p, -\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}, \frac{-b+\sqrt{b^2-4 a c}}{2 c x^2}\right] - p \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3-2 p, 1-p, -p, 4-2 p, -\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}, \frac{-b+\sqrt{b^2-4 a c}}{2 c x^2}\right] + \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3-2 p, -p, 1-p, 4-2 p, -\frac{b+\sqrt{b^2-4 a c}}{2 c x^2}, \frac{-b+\sqrt{b^2-4 a c}}{2 c x^2}\right]\right)\right)\right)$$

Problem 1122: Result more than twice size of optimal antiderivative.

$$\int x^4 \left(a+b x^2+c x^4\right)^p \, dx$$

Optimal (type 6, 138 leaves, 2 steps):

$$\frac{1}{5} x^5 \left(1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}\right)^{-p} \left(1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right)^{-p} \left(a+b x^2+c x^4\right)^p \text{AppellF1}\left[\frac{5}{2}, -p, -p, \frac{7}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]$$

Result (type 6, 457 leaves):

$$\left(7\times 2^{-2-p}\,c\left(b+\sqrt{b^2-4\,a\,c}\right)x^5\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^2\right)^{-p}\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{c}\right)^{1+p}\right.$$
$$\left.\left(-2\,a+\left(-b+\sqrt{b^2-4\,a\,c}\right)x^2\right)^2\left(a+b\,x^2+c\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\bigg/$$
$$\left(5\left(-b+\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(-7\,a\,\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$
$$p\,x^2\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{2},1-p,-p,\frac{9}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{2},-p,1-p,\frac{9}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1123: Result more than twice size of optimal antiderivative.

$$\int x^2\left(a+b\,x^2+c\,x^4\right)^p\,dx$$

Optimal (type 6, 138 leaves, 2 steps):

$$\frac{1}{3}x^3\left(1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(a+b\,x^2+c\,x^4\right)^p\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]$$

Result (type 6, 457 leaves):

$$\left(5\times 2^{-2-p}\,c\left(b+\sqrt{b^2-4\,a\,c}\right)x^3\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^2\right)^{-p}\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{c}\right)^{1+p}\right.$$
$$\left.\left(-2\,a+\left(-b+\sqrt{b^2-4\,a\,c}\right)x^2\right)^2\left(a+b\,x^2+c\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\bigg/$$
$$\left(3\left(-b+\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(-5\,a\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$
$$p\,x^2\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{2},-p,1-p,\frac{7}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 1124: Result more than twice size of optimal antiderivative.

$$\int \left(a + b x^2 + c x^4\right)^p \, dx$$

Optimal (type 6, 133 leaves, 2 steps):

$$x \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}\right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right)^{-p} \left(a + b x^2 + c x^4\right)^p \text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 487 leaves):

$$\left(3 \times 4^{-1-p} \left(b + \sqrt{b^2 - 4 a c}\right) x \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2\right)^{-p} \left(\frac{b + \sqrt{b^2 - 4 a c}}{2 c} + x^2\right)^{-p} \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c}\right)^{1+p} \left(\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{c}\right)^{-1+p} \left(-2 a + \left(-b + \sqrt{b^2 - 4 a c}\right) x^2\right)^2 \left(a + b x^2 + c x^4\right)^{-1+p} \text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right]\right) \Big/ \left(\left(-b + \sqrt{b^2 - 4 a c}\right) \left(-3 a \,\text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + p x^2 \left(\left(-b + \sqrt{b^2 - 4 a c}\right) \text{AppellF1}\left[\frac{3}{2}, 1 - p, -p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \left(b + \sqrt{b^2 - 4 a c}\right) \text{AppellF1}\left[\frac{3}{2}, -p, 1 - p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right]\right)\right)\right)$$

Problem 1125: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b x^2 + c x^4\right)^p}{x^2} \, dx$$

Optimal (type 6, 136 leaves, 2 steps):

$$-\frac{1}{x} \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}\right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right)^{-p} \left(a + b x^2 + c x^4\right)^p \text{AppellF1}\left[-\frac{1}{2}, -p, -p, \frac{1}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 472 leaves):

$$-\left(\left(2^{-2-p}\left(b+\sqrt{b^2-4ac}\right)\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{c}\right)^{p}\right.\right.$$
$$\left.\left(-2a+\left(-b+\sqrt{b^2-4ac}\right)x^2\right)^2\left(a+bx^2+cx^4\right)^{-1+p}\text{AppellF1}\left[-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(\left(-b+\sqrt{b^2-4ac}\right)x\left(b+\sqrt{b^2-4ac}+2cx^2\right)\left(a\,\text{AppellF1}\left[-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\right.\right.$$
$$p\,x^2\left(\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1}{2},1-p,-p,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\right.$$
$$\left.\left.\left.\left.\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1}{2},-p,1-p,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)\right)$$

Problem 1126: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+bx^2+cx^4\right)^p}{x^4}\,dx$$

Optimal (type 6, 138 leaves, 2 steps):

$$-\frac{1}{3x^3}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\,\text{AppellF1}\left[-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right]$$

Result (type 6, 456 leaves):

$$\left(2^{-2-p}c\left(b+\sqrt{b^2-4ac}\right)\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{c}\right)^{1+p}\right.$$
$$\left.\left(-2a+\left(-b+\sqrt{b^2-4ac}\right)x^2\right)^2\left(a+bx^2+cx^4\right)^{-1+p}\text{AppellF1}\left[-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$
$$\left(3\left(-b+\sqrt{b^2-4ac}\right)x^3\left(b+\sqrt{b^2-4ac}+2cx^2\right)\left(a\,\text{AppellF1}\left[-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\right.\right.$$
$$p\,x^2\left(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[-\frac{1}{2},1-p,-p,\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[-\frac{1}{2},-p,1-p,\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)$$

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b\,x^2}{\sqrt{1-b^2\,x^4}}\,dx$$

Optimal (type 4, 16 leaves, 2 steps):

$$\frac{\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\,x\right],-1\right]}{\sqrt{b}}$$

Result (type 4, 27 leaves):

$$-\frac{i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-b}\,x\right],-1\right]}{\sqrt{-b}}$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b\,x^2}{\sqrt{1-b^2\,x^4}}\,dx$$

Optimal (type 4, 35 leaves, 5 steps):

$$-\frac{\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\,x\right],-1\right]}{\sqrt{b}}+\frac{2\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{b}\,x\right],-1\right]}{\sqrt{b}}$$

Result (type 4, 46 leaves):

$$\frac{i\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-b}\,x\right],-1\right]-2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-b}\,x\right],-1\right]\right)}{\sqrt{-b}}$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b\,x^2}{\sqrt{-1+b^2\,x^4}}\,dx$$

Optimal (type 4, 43 leaves, 3 steps):

$$\frac{\sqrt{1-b^2\,x^4}\;\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\,x\right],-1\right]}{\sqrt{b}\,\sqrt{-1+b^2\,x^4}}$$

Result (type 4, 54 leaves):

$$-\frac{i\sqrt{1-b^2 x^4}\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]}{\sqrt{-b}\ \sqrt{-1+b^2 x^4}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b x^2}{\sqrt{-1+b^2 x^4}}\, dx$$

Optimal (type 4, 89 leaves, 6 steps):

$$-\frac{\sqrt{1-b^2 x^4}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\ x\right],-1\right]}{\sqrt{b}\ \sqrt{-1+b^2 x^4}}+\frac{2\sqrt{1-b^2 x^4}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{b}\ x\right],-1\right]}{\sqrt{b}\ \sqrt{-1+b^2 x^4}}$$

Result (type 4, 73 leaves):

$$\frac{i\sqrt{1-b^2 x^4}\ \left(\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]-2\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]\right)}{\sqrt{-b}\ \sqrt{-1+b^2 x^4}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b x^2}{\sqrt{1+b^2 x^4}}\, dx$$

Optimal (type 4, 89 leaves, 1 step):

$$-\frac{x\sqrt{1+b^2 x^4}}{1+b x^2}+\frac{\left(1+b x^2\right)\sqrt{\frac{1+b^2 x^4}{\left(1+b x^2\right)^2}}\ \text{EllipticE}\left[2\ \text{ArcTan}\left[\sqrt{b}\ x\right],\frac{1}{2}\right]}{\sqrt{b}\ \sqrt{1+b^2 x^4}}$$

Result (type 4, 52 leaves):

$$-\frac{\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{i b}\ x\right],-1\right]-(1-i)\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{i b}\ x\right],-1\right]}{\sqrt{i b}}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b x^2}{\sqrt{1+b^2 x^4}}\, dx$$

Optimal (type 4, 152 leaves, 3 steps):

$$\frac{x\sqrt{1+b^2x^4}}{1+b\,x^2}-\frac{\left(1+b\,x^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+b\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{1+b^2x^4}}+\frac{\left(1+b\,x^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+b\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

Result (type 4, 51 leaves):

$$\frac{\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]-(1+i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]}{\sqrt{i\,b}}$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-b\,x^2}{\sqrt{-1-b^2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{x\sqrt{-1-b^2x^4}}{1+b\,x^2}+\frac{\left(1+b\,x^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+b\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Result (type 4, 79 leaves):

$$-\frac{\sqrt{1+b^2x^4}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]-(1-i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]\right)}{\sqrt{i\,b}\,\sqrt{-1-b^2x^4}}$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+b\,x^2}{\sqrt{-1-b^2x^4}}\,dx$$

Optimal (type 4, 156 leaves, 3 steps):

$$-\frac{x\sqrt{-1-b^2x^4}}{1+b\,x^2}-\frac{\left(1+b\,x^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+b\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}+\frac{\left(1+b\,x^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+b\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Result (type 4, 78 leaves):

$$\frac{\sqrt{1+b^2x^4}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]-(1+i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]\right)}{\sqrt{i\,b}\,\sqrt{-1-b^2x^4}}$$

Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+c^2 x^2}{\sqrt{1-c^4 x^4}} \, dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\frac{\text{EllipticE}[\text{ArcSin}[c\,x],-1]}{c}$$

Result (type 4, 31 leaves):

$$-\frac{i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\,x\right],-1\right]}{\sqrt{-c^2}}$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1-c^2 x^2}{\sqrt{1-c^4 x^4}} \, dx$$

Optimal (type 4, 23 leaves, 5 steps):

$$-\frac{\text{EllipticE}[\text{ArcSin}[c\,x],-1]}{c}+\frac{2\,\text{EllipticF}[\text{ArcSin}[c\,x],-1]}{c}$$

Result (type 4, 52 leaves):

$$\frac{i\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\,x\right],-1\right]-2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\,x\right],-1\right]\right)}{\sqrt{-c^2}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{d^2+b x^2+e^2 x^4} \, dx$$

Optimal (type 3, 82 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-b+2 d e}-2 e x}{\sqrt{b+2 d e}}\right]}{\sqrt{b+2 d e}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-b+2 d e}+2 e x}{\sqrt{b+2 d e}}\right]}{\sqrt{b+2 d e}}$$

Result (type 3, 181 leaves):

$$\frac{\frac{\left(-b+2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}+\frac{\left(b-2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int\frac{d+e\,x^2}{d^2+f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 82 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e-f}-2\,e\,x}{\sqrt{2\,d\,e+f}}\right]}{\sqrt{2\,d\,e+f}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e-f}+2\,e\,x}{\sqrt{2\,d\,e+f}}\right]}{\sqrt{2\,d\,e+f}}$$

Result (type 3, 181 leaves):

$$\frac{\frac{\left(2\,d\,e-f+\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}+\frac{\left(-2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int\frac{d+e\,x^2}{d^2-b\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b+2\,d\,e}-2\,e\,x}{\sqrt{b-2\,d\,e}}\right]}{\sqrt{b-2\,d\,e}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b+2\,d\,e}+2\,e\,x}{\sqrt{b-2\,d\,e}}\right]}{\sqrt{b-2\,d\,e}}$$

Result (type 3, 189 leaves):

$$\frac{\frac{\left(b+2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}+\frac{\left(-b-2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e\,x^2}{d^2-f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e+f}-2\,e\,x}{\sqrt{2\,d\,e-f}}\right]}{\sqrt{2\,d\,e-f}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e+f}+2\,e\,x}{\sqrt{2\,d\,e-f}}\right]}{\sqrt{2\,d\,e-f}}$$

Result (type 3, 189 leaves):

$$\frac{\frac{\left(2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}+\frac{\left(-2\,d\,e-f+\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{d-e\,x^2}{d^2+b\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{-b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{-b+2\,d\,e}}+\frac{\text{Log}\left[d+\sqrt{-b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{-b+2\,d\,e}}$$

Result (type 3, 182 leaves):

$$\frac{\frac{\left(b+2\,d\,e-\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}-\frac{\left(b+2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{d-e\,x^2}{d^2+f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{2\,d\,e-f}\;x+e\,x^2\right]}{2\,\sqrt{2\,d\,e-f}}+\frac{\text{Log}\left[d+\sqrt{2\,d\,e-f}\;x+e\,x^2\right]}{2\,\sqrt{2\,d\,e-f}}$$

Result (type 3, 182 leaves) :

$$\frac{\frac{\left(2\,d\,e+f-\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}-\frac{\left(2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{d-e\,x^2}{d^2-b\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 70 leaves, 3 steps) :

$$-\frac{\text{Log}\left[d-\sqrt{b+2\,d\,e}\;x+e\,x^2\right]}{2\,\sqrt{b+2\,d\,e}}+\frac{\text{Log}\left[d+\sqrt{b+2\,d\,e}\;x+e\,x^2\right]}{2\,\sqrt{b+2\,d\,e}}$$

Result (type 3, 190 leaves) :

$$\frac{-\frac{\left(b-2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}+\frac{\left(b-2\,d\,e-\sqrt{b^2-4\,d^2\,e^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int\frac{d-e\,x^2}{d^2-f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 70 leaves, 3 steps) :

$$-\frac{\text{Log}\left[d-\sqrt{2\,d\,e+f}\;x+e\,x^2\right]}{2\,\sqrt{2\,d\,e+f}}+\frac{\text{Log}\left[d+\sqrt{2\,d\,e+f}\;x+e\,x^2\right]}{2\,\sqrt{2\,d\,e+f}}$$

Result (type 3, 190 leaves) :

$$\frac{-\frac{\left(-2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}+\frac{\left(-2\,d\,e+f-\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\,x^2}{a^2+\left(-1+2\,a\,b\right)\,x^2+b^2\,x^4}\,dx$$

Optimal (type 3, 60 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{1-2\,b\,x}{\sqrt{1-4\,a\,b}}\right]}{\sqrt{1-4\,a\,b}}-\frac{\text{ArcTanh}\left[\frac{1+2\,b\,x}{\sqrt{1-4\,a\,b}}\right]}{\sqrt{1-4\,a\,b}}$$

Result (type 3, 138 leaves):

$$\frac{\frac{\left(1+\sqrt{1-4\,a\,b}\right)\,\text{ArcTan}\left[\frac{b\,x}{\sqrt{-\frac{1}{2}+a\,b-\frac{1}{2}\sqrt{1-4\,a\,b}}}\right]}{\sqrt{-1+2\,a\,b-\sqrt{1-4\,a\,b}}}+\frac{\left(-1+\sqrt{1-4\,a\,b}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,b\,x}{\sqrt{-1+2\,a\,b+\sqrt{1-4\,a\,b}}}\right]}{\sqrt{-1+2\,a\,b+\sqrt{1-4\,a\,b}}}}{\sqrt{2-8\,a\,b}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int\frac{1+2\,x^2}{1+b\,x^2+4\,x^4}\,dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{4-b}-4\,x}{\sqrt{4+b}}\right]}{\sqrt{4+b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{4-b}+4\,x}{\sqrt{4+b}}\right]}{\sqrt{4+b}}$$

Result (type 3, 126 leaves):

$$\frac{\frac{\left(4-b+\sqrt{-16+b^2}\right)\,\text{ArcTan}\left[\frac{2\,\sqrt{2}\,x}{\sqrt{b-\sqrt{-16+b^2}}}\right]}{\sqrt{b-\sqrt{-16+b^2}}}+\frac{\left(-4+b+\sqrt{-16+b^2}\right)\,\text{ArcTan}\left[\frac{2\,\sqrt{2}\,x}{\sqrt{b+\sqrt{-16+b^2}}}\right]}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\,\sqrt{-16+b^2}}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{1+2x^2}{1-b x^2+4x^4}\,dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right]}{\sqrt{4-b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right]}{\sqrt{4-b}}$$

Result (type 3, 134 leaves):

$$\frac{\frac{\left(4+b+\sqrt{-16+b^2}\right)\text{ArcTan}\left[\frac{2\sqrt{2}\,x}{\sqrt{-b-\sqrt{-16+b^2}}}\right]}{\sqrt{-b-\sqrt{-16+b^2}}}+\frac{\left(-4-b+\sqrt{-16+b^2}\right)\text{ArcTan}\left[\frac{2\sqrt{2}\,x}{\sqrt{-b+\sqrt{-16+b^2}}}\right]}{\sqrt{-b+\sqrt{-16+b^2}}}}{\sqrt{2}\,\sqrt{-16+b^2}}$$

Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-4x}{\sqrt{7}}\right]}{\sqrt{7}}+\frac{\text{ArcTan}\left[\frac{1+4x}{\sqrt{7}}\right]}{\sqrt{7}}$$

Result (type 3, 97 leaves):

$$\frac{\left(-i+\sqrt{7}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right]}{\sqrt{42-14\,i\sqrt{7}}}+\frac{\left(i+\sqrt{7}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right]}{\sqrt{42+14\,i\sqrt{7}}}$$

Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2\sqrt{2}\,x}{\sqrt{3}}\right]}{\sqrt{6}}+\frac{\text{ArcTan}\left[\frac{1+2\sqrt{2}\,x}{\sqrt{3}}\right]}{\sqrt{6}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{1-i\sqrt{3}}}\right]}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{1+i\sqrt{3}}}\right]}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+2x^2}{1+x^2+4x^4}\,dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}-4x}{\sqrt{5}}\right]}{\sqrt{5}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}+4x}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 97 leaves):

$$\frac{\left(-3i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right]}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(3i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right]}{\sqrt{30+30i\sqrt{15}}}$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+2x^2}{1-x^2+4x^4}\,dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{5}-4x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{5}+4x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 101 leaves):

$$\frac{\left(-5\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2\,x}{\sqrt{\frac{1}{2}\left(-1-i\,\sqrt{15}\right)}}\right]}{\sqrt{30\left(-1-i\,\sqrt{15}\right)}}+\frac{\left(5\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2\,x}{\sqrt{\frac{1}{2}\left(-1+i\,\sqrt{15}\right)}}\right]}{\sqrt{30\left(-1+i\,\sqrt{15}\right)}}$$

Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+2\,x^2}{1-2\,x^2+4\,x^4}\,dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-2\,\sqrt{2}\,x\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\sqrt{3}+2\,\sqrt{2}\,x\right]}{\sqrt{2}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2\,x}{\sqrt{-1-i\,\sqrt{3}}}\right]}{2\,\sqrt{3\left(-1-i\,\sqrt{3}\right)}}+\frac{\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2\,x}{\sqrt{-1+i\,\sqrt{3}}}\right]}{2\,\sqrt{3\left(-1+i\,\sqrt{3}\right)}}$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int\frac{1-2\,x^2}{1-4\,x^2+4\,x^4}\,dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2}\,x\right]}{\sqrt{2}}$$

Result (type 3, 32 leaves):

$$\frac{-\text{Log}\left[\sqrt{2}-2\,x\right]+\text{Log}\left[\sqrt{2}+2\,x\right]}{2\,\sqrt{2}}$$

Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+x^2}{1+x^2+x^4}\,dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-\mathrm{i}+\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-\mathrm{i}\sqrt{3}\right)}}\right]}{\sqrt{6\left(1-\mathrm{i}\sqrt{3}\right)}}+\frac{\left(\mathrm{i}+\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+\mathrm{i}\sqrt{3}\right)}}\right]}{\sqrt{6\left(1+\mathrm{i}\sqrt{3}\right)}}$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int\frac{1-x^2}{1+b\,x^2+x^4}\,dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{\text{Log}\left[1-\sqrt{2-b}\;x+x^2\right]}{2\sqrt{2-b}}+\frac{\text{Log}\left[1+\sqrt{2-b}\;x+x^2\right]}{2\sqrt{2-b}}$$

Result (type 3, 125 leaves):

$$\frac{\frac{\left(2+b-\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right]}{\sqrt{b-\sqrt{-4+b^2}}}-\frac{\left(2+b+\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\,\sqrt{-4+b^2}}$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int\frac{1-x^2}{1-2\,x^2+x^4}\,dx$$

Optimal (type 3, 2 leaves, 3 steps):

$$\text{ArcTanh}[x]$$

Result (type 3, 19 leaves):

$$-\frac{1}{2}\text{Log}[1-x]+\frac{1}{2}\text{Log}[1+x]$$

Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1-3x}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+3x}{\sqrt{2}}\right]}{2\sqrt{2}}$$

Result (type 3, 99 leaves):

$$-\frac{\left(-i+\sqrt{2}\right)\text{ArcTan}\left[\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right]}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\text{ArcTan}\left[\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right]}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Problem 93: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1-3x}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+3x}{\sqrt{2}}\right]}{2\sqrt{2}}$$

Result (type 3, 99 leaves):

$$-\frac{\left(-i+\sqrt{2}\right)\text{ArcTan}\left[\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right]}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\text{ArcTan}\left[\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right]}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{1+x^2+x^4}\,dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$-\frac{(a+b)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(a+b)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}(a-b)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(a-b)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 97 leaves):

$$\frac{\left(2\,i\,a+\left(-i+\sqrt{3}\right)b\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{6+6\,i\,\sqrt{3}}}+\frac{\left(-2\,i\,a+\left(i+\sqrt{3}\right)b\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{6-6\,i\,\sqrt{3}}}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{\left(1+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 119 leaves, 10 steps):

$$\frac{x\left(a+b-(a-2b)\,x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{(4a+b)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{(4a+b)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}-\frac{1}{8}(2a-b)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{8}(2a-b)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 147 leaves):

$$\frac{x\left(a+b-a\,x^2+2\,b\,x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{\left(\left(-11\,i+\sqrt{3}\right)a-2\left(-2\,i+\sqrt{3}\right)b\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{6\sqrt{6+6\,i\,\sqrt{3}}}-\frac{\left(\left(11\,i+\sqrt{3}\right)a-2\left(2\,i+\sqrt{3}\right)b\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{6\sqrt{6-6\,i\,\sqrt{3}}}$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{2+x^2+x^4}\,dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}\,b\right)\text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right]+\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}\,b\right)\text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right]-$$
$$\frac{\left(a-\sqrt{2}\,b\right)\text{Log}\left[\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right]}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\left(a-\sqrt{2}\,b\right)\text{Log}\left[\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2\right]}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

Result (type 3, 111 leaves):

$$\frac{\left(-2\,i\,a+\left(i+\sqrt{7}\right)b\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right]}{\sqrt{14-14\,i\sqrt{7}}}+\frac{\left(2\,i\,a+\left(-i+\sqrt{7}\right)b\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right]}{\sqrt{14+14\,i\sqrt{7}}}$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{\left(2+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 316 leaves, 10 steps):

$$\frac{x\left(3\,a+2\,b-(a-4\,b)\,x^2\right)}{28\left(2+x^2+x^4\right)}-\frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}-2\,x}{\sqrt{1+2\sqrt{2}}}\right]+$$
$$\frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}+2\,x}{\sqrt{1+2\sqrt{2}}}\right]-$$
$$\frac{\left(11\,a+\sqrt{2}\,(a-4\,b)-2\,b\right)\text{Log}\left[\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right]}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}\,b\right)\right)\text{Log}\left[\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2\right]}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

Result (type 3, 165 leaves):

$$\frac{-a\,x\left(-3+x^2\right)+2\,b\left(x+2\,x^3\right)}{28\left(2+x^2+x^4\right)}-\frac{\left(\left(23\,i+\sqrt{7}\right)a-4\left(2\,i+\sqrt{7}\right)b\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right]}{28\sqrt{14-14\,i\sqrt{7}}}-\frac{\left(\left(-23\,i+\sqrt{7}\right)a-4\left(-2\,i+\sqrt{7}\right)b\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right]}{28\sqrt{14+14\,i\sqrt{7}}}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2\,x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2+\sqrt{2}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2\,x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\,\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,x+x^2\right]+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\,\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,x+x^2\right]$$

Result (type 3, 53 leaves):

$$\frac{\sqrt{-1-\mathrm{i}}\ \mathrm{ArcTan}\left[\frac{2^{1/4}\,x}{\sqrt{-1-\mathrm{i}}}\right]+\sqrt{-1+\mathrm{i}}\ \mathrm{ArcTan}\left[\frac{2^{1/4}\,x}{\sqrt{-1+\mathrm{i}}}\right]}{2^{3/4}}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx$$

Optimal (type 3, 172 leaves, 9 steps):

$$-\frac{\mathrm{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2-\sqrt{2}}}+\frac{\mathrm{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2-\sqrt{2}}}-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\ \mathrm{Log}\left[1-\sqrt{2-\sqrt{2}}\ x+x^2\right]+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\ \mathrm{Log}\left[1+\sqrt{2-\sqrt{2}}\ x+x^2\right]$$

Result (type 3, 53 leaves):

$$\frac{\sqrt{1-\mathrm{i}}\ \mathrm{ArcTan}\left[\frac{2^{1/4}\,x}{\sqrt{1-\mathrm{i}}}\right]+\sqrt{1+\mathrm{i}}\ \mathrm{ArcTan}\left[\frac{2^{1/4}\,x}{\sqrt{1+\mathrm{i}}}\right]}{2^{3/4}}$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2a-x^2}{a^2-a\,x^2+x^4}\,dx$$

Optimal (type 3, 114 leaves, 9 steps):

$$-\frac{\mathrm{ArcTan}\left[\sqrt{3}-\frac{2x}{\sqrt{a}}\right]}{2\sqrt{a}}+\frac{\mathrm{ArcTan}\left[\sqrt{3}+\frac{2x}{\sqrt{a}}\right]}{2\sqrt{a}}-\frac{\sqrt{3}\ \mathrm{Log}\left[a-\sqrt{3}\ \sqrt{a}\ x+x^2\right]}{4\sqrt{a}}+\frac{\sqrt{3}\ \mathrm{Log}\left[a+\sqrt{3}\ \sqrt{a}\ x+x^2\right]}{4\sqrt{a}}$$

Result (type 3, 115 leaves):

$$\frac{(-1)^{1/4}\left(-\sqrt{\mathrm{i}+\sqrt{3}}\ \left(3\,\mathrm{i}+\sqrt{3}\right)\mathrm{ArcTan}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{-\mathrm{i}+\sqrt{3}}\ \sqrt{a}}\right]+\sqrt{-\mathrm{i}+\sqrt{3}}\ \left(-3\,\mathrm{i}+\sqrt{3}\right)\mathrm{ArcTanh}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{\mathrm{i}+\sqrt{3}}\ \sqrt{a}}\right]\right)}{2\sqrt{6}\ \sqrt{a}}$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}\,x^2+x^4}\,dx$$

Optimal (type 3, 122 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-\frac{2x}{a^{1/4}}\right]}{2a^{1/4}}+\frac{\text{ArcTan}\left[\sqrt{3}+\frac{2x}{a^{1/4}}\right]}{2a^{1/4}}-\frac{\sqrt{3}\,\text{Log}\left[\sqrt{a}-\sqrt{3}\,a^{1/4}x+x^2\right]}{4a^{1/4}}+\frac{\sqrt{3}\,\text{Log}\left[\sqrt{a}+\sqrt{3}\,a^{1/4}x+x^2\right]}{4a^{1/4}}$$

Result (type 3, 115 leaves):

$$\frac{(-1)^{1/4}\left(-\sqrt{i+\sqrt{3}}\,\left(3i+\sqrt{3}\right)\text{ArcTan}\left[\frac{(1+i)\,x}{\sqrt{-i+\sqrt{3}}\,a^{1/4}}\right]+\sqrt{-i+\sqrt{3}}\,\left(-3i+\sqrt{3}\right)\text{ArcTanh}\left[\frac{(1+i)\,x}{\sqrt{i+\sqrt{3}}\,a^{1/4}}\right]\right)}{2\sqrt{6}\,a^{1/4}}$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$$

Optimal (type 3, 124 leaves, 9 steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{b^{1/3}-2x}{\sqrt{3}\,b^{1/3}}\right]}{2b^{1/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{b^{1/3}+2x}{\sqrt{3}\,b^{1/3}}\right]}{2b^{1/3}}-\frac{\text{Log}\left[b^{2/3}-b^{1/3}x+x^2\right]}{4b^{1/3}}+\frac{\text{Log}\left[b^{2/3}+b^{1/3}x+x^2\right]}{4b^{1/3}}$$

Result (type 3, 115 leaves):

$$\frac{(-1)^{1/4}\left(\sqrt{-i+\sqrt{3}}\,\left(-3i+\sqrt{3}\right)\text{ArcTan}\left[\frac{(1+i)\,x}{\sqrt{i+\sqrt{3}}\,b^{1/3}}\right]-\sqrt{i+\sqrt{3}}\,\left(3i+\sqrt{3}\right)\text{ArcTanh}\left[\frac{(1+i)\,x}{\sqrt{-i+\sqrt{3}}\,b^{1/3}}\right]\right)}{2\sqrt{6}\,b^{1/3}}$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+Bx^2}{a^2-a x^2+x^4}\,dx$$

Optimal (type 3, 136 leaves, 9 steps):

$$-\frac{(A+aB)\,\text{ArcTan}\left[\sqrt{3}-\frac{2x}{\sqrt{a}}\right]}{2a^{3/2}}+\frac{(A+aB)\,\text{ArcTan}\left[\sqrt{3}+\frac{2x}{\sqrt{a}}\right]}{2a^{3/2}}-\frac{(A-aB)\,\text{Log}\left[a-\sqrt{3}\,\sqrt{a}\,x+x^2\right]}{4\sqrt{3}\,a^{3/2}}+\frac{(A-aB)\,\text{Log}\left[a+\sqrt{3}\,\sqrt{a}\,x+x^2\right]}{4\sqrt{3}\,a^{3/2}}$$

Result (type 3, 130 leaves):

$$\frac{(-1)^{1/4}\left(\frac{\left(-2\,\mathrm{i}\,A+\left(-\mathrm{i}+\sqrt{3}\right)a\,B\right)\mathrm{ArcTan}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{-\mathrm{i}+\sqrt{3}}\,\sqrt{a}}\right]}{\sqrt{-\mathrm{i}+\sqrt{3}}}-\frac{\left(2\,\mathrm{i}\,A+\left(\mathrm{i}+\sqrt{3}\right)a\,B\right)\mathrm{ArcTanh}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{\mathrm{i}+\sqrt{3}}\,\sqrt{a}}\right]}{\sqrt{\mathrm{i}+\sqrt{3}}}\right)}{\sqrt{6}\,a^{3/2}}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{a-\sqrt{a}\,x^2+x^4}\,dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$-\frac{\left(A+\sqrt{a}\,B\right)\mathrm{ArcTan}\left[\sqrt{3}-\frac{2\,x}{a^{1/4}}\right]}{2\,a^{3/4}}+\frac{\left(A+\sqrt{a}\,B\right)\mathrm{ArcTan}\left[\sqrt{3}+\frac{2\,x}{a^{1/4}}\right]}{2\,a^{3/4}}-\frac{\left(A-\sqrt{a}\,B\right)\mathrm{Log}\left[\sqrt{a}-\sqrt{3}\,a^{1/4}\,x+x^2\right]}{4\,\sqrt{3}\,a^{3/4}}+\frac{\left(A-\sqrt{a}\,B\right)\mathrm{Log}\left[\sqrt{a}+\sqrt{3}\,a^{1/4}\,x+x^2\right]}{4\,\sqrt{3}\,a^{3/4}}$$

Result (type 3, 138 leaves):

$$\frac{(-1)^{1/4}\left(\frac{\left(-2\,\mathrm{i}\,A+\left(-\mathrm{i}+\sqrt{3}\right)\sqrt{a}\,B\right)\mathrm{ArcTan}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{-\mathrm{i}+\sqrt{3}}\,a^{1/4}}\right]}{\sqrt{-\mathrm{i}+\sqrt{3}}}-\frac{\left(2\,\mathrm{i}\,A+\left(\mathrm{i}+\sqrt{3}\right)\sqrt{a}\,B\right)\mathrm{ArcTanh}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{\mathrm{i}+\sqrt{3}}\,a^{1/4}}\right]}{\sqrt{\mathrm{i}+\sqrt{3}}}\right)}{\sqrt{6}\,a^{3/4}}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{a-\sqrt{a\,c}\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 414 leaves, 9 steps):

$$-\frac{\left(\sqrt{a}\,B+A\,\sqrt{c}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}-2\,\sqrt{c}\,x}{\sqrt{2\,\sqrt{a}\,\sqrt{c}-\sqrt{a\,c}}}\right]}{2\,\sqrt{a}\,\sqrt{c}\,\sqrt{2\,\sqrt{a}\,\sqrt{c}-\sqrt{a\,c}}}+\frac{\left(\sqrt{a}\,B+A\,\sqrt{c}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}+2\,\sqrt{c}\,x}{\sqrt{2\,\sqrt{a}\,\sqrt{c}-\sqrt{a\,c}}}\right]}{2\,\sqrt{a}\,\sqrt{c}\,\sqrt{2\,\sqrt{a}\,\sqrt{c}-\sqrt{a\,c}}}-$$

$$\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\mathrm{Log}\left[\sqrt{a}-\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}\,x+\sqrt{c}\,x^2\right]}{4\,\sqrt{a}\,\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}}+\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\mathrm{Log}\left[\sqrt{a}+\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}\,x+\sqrt{c}\,x^2\right]}{4\,\sqrt{a}\,\sqrt{2\,\sqrt{a}\,\sqrt{c}+\sqrt{a\,c}}}$$

Result (type 3, 247 leaves):

$$\frac{\frac{\left(\sqrt{3}\ \sqrt{a}\ B\sqrt{c}-i\left(2Ac+B\sqrt{ac}\right)\right)\text{ArcTan}\left[\frac{\sqrt{2}\ \sqrt{c}\ x}{\sqrt{-i\sqrt{3}\ \sqrt{a}\ \sqrt{c}-\sqrt{ac}}}\right]}{\sqrt{-i\sqrt{3}\ \sqrt{a}\ \sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{3}\ \sqrt{a}\ B\sqrt{c}+i\left(2Ac+B\sqrt{ac}\right)\right)\text{ArcTan}\left[\frac{\sqrt{2}\ \sqrt{c}\ x}{\sqrt{i\sqrt{3}\ \sqrt{a}\ \sqrt{c}-\sqrt{ac}}}\right]}{\sqrt{i\sqrt{3}\ \sqrt{a}\ \sqrt{c}-\sqrt{ac}}}}{\sqrt{6}\ \sqrt{a}\ c}$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{a-\sqrt{a}\ \sqrt{c}\ x^2+c x^4}\,dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$-\frac{\left(\sqrt{a}\ B+A\sqrt{c}\right)\text{ArcTan}\left[\sqrt{3}-\frac{2c^{1/4}x}{a^{1/4}}\right]}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\ B+A\sqrt{c}\right)\text{ArcTan}\left[\sqrt{3}+\frac{2c^{1/4}x}{a^{1/4}}\right]}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\ B}{\sqrt{c}}\right)\text{Log}\left[\sqrt{a}-\sqrt{3}\ a^{1/4}c^{1/4}x+\sqrt{c}\ x^2\right]}{4\sqrt{3}\ a^{3/4}c^{1/4}}+\frac{\left(A-\frac{\sqrt{a}\ B}{\sqrt{c}}\right)\text{Log}\left[\sqrt{a}+\sqrt{3}\ a^{1/4}c^{1/4}x+\sqrt{c}\ x^2\right]}{4\sqrt{3}\ a^{3/4}c^{1/4}}$$

Result (type 3, 163 leaves):

$$\frac{(-1)^{1/4}\left(\frac{\left(\left(-i+\sqrt{3}\right)\sqrt{a}\ B-2iA\sqrt{c}\right)\text{ArcTan}\left[\frac{(1+i)c^{1/4}x}{\sqrt{-i+\sqrt{3}}\ a^{1/4}}\right]}{\sqrt{-i+\sqrt{3}}}-\frac{\left(\left(i+\sqrt{3}\right)\sqrt{a}\ B+2iA\sqrt{c}\right)\text{ArcTanh}\left[\frac{(1+i)c^{1/4}x}{\sqrt{i+\sqrt{3}}\ a^{1/4}}\right]}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}\ a^{3/4}c^{3/4}}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\ x\right],\frac{1}{6}\left(-7-\sqrt{13}\right)\right]+\sqrt{7+2\sqrt{13}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\ x\right],\frac{1}{6}\left(-7-\sqrt{13}\right)\right]$$

Result (type 4, 103 leaves):

$$-\frac{1}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

$$i\left(\left(1+\sqrt{13}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right],\frac{1}{6}\left(-7+\sqrt{13}\right)\right]-\left(-5+\sqrt{13}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right],\frac{1}{6}\left(-7+\sqrt{13}\right)\right]\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx$$

Optimal (type 4, 25 leaves, 5 steps):

$$-\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{3}}\right],-3\right]+4\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{3}}\right],-3\right]$$

Result (type 4, 19 leaves):

$$-i\sqrt{3}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{3}\right]$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}\,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right],\frac{1}{2}\left(-5-\sqrt{21}\right)\right]+\sqrt{9+2\sqrt{21}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right],\frac{1}{2}\left(-5-\sqrt{21}\right)\right]$$

Result (type 4, 103 leaves):

$$-\frac{1}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

$$i\left(\left(3+\sqrt{21}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right],\frac{1}{2}\left(-5+\sqrt{21}\right)\right]-\left(-3+\sqrt{21}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right],\frac{1}{2}\left(-5+\sqrt{21}\right)\right]\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\ x\right],\ \frac{1}{6}\left(-7+\sqrt{13}\right)\right]+\sqrt{5+2\sqrt{13}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\ x\right],\ \frac{1}{6}\left(-7+\sqrt{13}\right)\right]$$

Result (type 4, 107 leaves):

$$-\frac{1}{\sqrt{2\left(-1+\sqrt{13}\right)}}\,\mathrm{i}\left(\left(-1+\sqrt{13}\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\ x\right],\ -\frac{7}{6}-\frac{\sqrt{13}}{6}\right]-\left(-7+\sqrt{13}\right)\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\ x\right],\ -\frac{7}{6}-\frac{\sqrt{13}}{6}\right]\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$$

Optimal (type 4, 27 leaves, 4 steps):

$$-\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}[x],\ -\frac{1}{3}\right]+2\sqrt{3}\ \text{EllipticF}\left[\text{ArcSin}[x],\ -\frac{1}{3}\right]$$

Result (type 4, 35 leaves):

$$-\mathrm{i}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],\ -3\right]+2\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],\ -3\right]\right)$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\ x\right],\ \frac{1}{2}\left(-5+\sqrt{21}\right)\right]+\sqrt{3+2\sqrt{21}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\ x\right],\ \frac{1}{2}\left(-5+\sqrt{21}\right)\right]$$

Result (type 4, 107 leaves):

$$-\frac{1}{\sqrt{2\left(-3+\sqrt{21}\right)}}$$

$$i\left(\left(-3+\sqrt{21}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\ x\right],\ -\frac{5}{2}-\frac{\sqrt{21}}{2}\right]-\left(-9+\sqrt{21}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\ x\right],\ -\frac{5}{2}-\frac{\sqrt{21}}{2}\right]\right)$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 296 leaves, 3 steps):

$$\frac{2\sqrt{c}\ x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\ x^2}-\frac{2a^{1/4}c^{1/4}\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\ \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\ \sqrt{c}}\right)\right]}{\sqrt{a+bx^2+cx^4}}+\frac{1}{2a^{1/4}c^{1/4}\sqrt{a+bx^2+cx^4}}$$

$$\left(b+2\sqrt{a}\ \sqrt{c}-\sqrt{b^2-4ac}\right)\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\ \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\ \sqrt{c}}\right)\right]$$

Result (type 4, 187 leaves):

$$-\frac{1}{\sqrt{a+bx^2+cx^4}}$$

$$2i\sqrt{2}\ a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\ \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\ \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\ x\right],\ \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]$$

Problem 150: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+ex^2\right)^4}{\sqrt{a+cx^4}}\,dx$$

Optimal (type 4, 388 leaves, 6 steps):

$$\frac{e^2\left(42 c d^2-5 a e^2\right) x \sqrt{a+c x^4}}{21 c^2}+\frac{4 d e^3 x^3 \sqrt{a+c x^4}}{5 c}+\frac{e^4 x^5 \sqrt{a+c x^4}}{7 c}+\frac{4 d e\left(5 c d^2-3 a e^2\right) x \sqrt{a+c x^4}}{5 c^{3/2}\left(\sqrt{a}+\sqrt{c} x^2\right)}-$$

$$\frac{4 a^{1/4} d e\left(5 c d^2-3 a e^2\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{7/4} \sqrt{a+c x^4}}+\frac{1}{210 a^{1/4} c^{9/4} \sqrt{a+c x^4}}$$

$$\left(105 c^2 d^4+420 \sqrt{a} c^{3/2} d^3 e-210 a c d^2 e^2-252 a^{3/2} \sqrt{c} d e^3+25 a^2 e^4\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 298 leaves):

$$\frac{1}{105 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^2 \sqrt{a+c x^4}}\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} e^2 x\left(-25 a^2 e^2+2 a c\left(105 d^2+42 d e x^2-5 e^2 x^4\right)+3 c^2 x^4\left(70 d^2+28 d e x^2+5 e^2 x^4\right)\right)-\right.$$

$$84 \sqrt{a} \sqrt{c} d e\left(-5 c d^2+3 a e^2\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+$$

$$\left.\left(-105 i c^2 d^4-420 \sqrt{a} c^{3/2} d^3 e+210 i a c d^2 e^2+252 a^{3/2} \sqrt{c} d e^3-25 i a^2 e^4\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^3}{\sqrt{a+c x^4}} \, dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\frac{d\,e^2\,x\,\sqrt{a+c\,x^4}}{c}+\frac{e^3\,x^3\,\sqrt{a+c\,x^4}}{5\,c}+\frac{3\,e\,\left(5\,c\,d^2-a\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{5\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$
$$\frac{3\,a^{1/4}\,e\,\left(5\,c\,d^2-a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{7/4}\,\sqrt{a+c\,x^4}}+$$
$$\frac{a^{1/4}\,\left(15\,c\,d^2\,e-3\,a\,e^3+\frac{5\,\sqrt{c}\,d\,\left(c\,d^2-a\,e^2\right)}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{10\,c^{7/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 235 leaves) :

$$\frac{1}{5\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,c^{3/2}\,\sqrt{a+c\,x^4}}\left(\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}\,e^2\,x\,\left(5\,d+e\,x^2\right)\,\left(a+c\,x^4\right)-3\,\sqrt{a}\,e\,\left(-5\,c\,d^2+a\,e^2\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$
$$\left.\left(-5\,i\,c^{3/2}\,d^3-15\,\sqrt{a}\,c\,d^2\,e+5\,i\,a\,\sqrt{c}\,d\,e^2+3\,a^{3/2}\,e^3\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^2}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 264 leaves, 4 steps) :

$$\frac{e^2\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{2\,d\,e\,x\,\sqrt{a+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\,a^{1/4}\,d\,e\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\,\sqrt{a+c\,x^4}}+$$
$$\frac{\left(3\,c\,d^2+6\,\sqrt{a}\,\sqrt{c}\,d\,e-a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{6\,a^{1/4}\,c^{5/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 195 leaves) :

$$\frac{1}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c\sqrt{a+c x^4}}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,e^2 x\left(a+c x^4\right)+6\sqrt{a}\,\sqrt{c}\,d\,e\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$
$$\left.i\left(-3 c d^2+6 i\sqrt{a}\,\sqrt{c}\,d\,e+a e^2\right)\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e x^2}{\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 226 leaves, 3 steps):

$$\frac{e x\sqrt{a+c x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4} e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\sqrt{a+c x^4}}+$$
$$\frac{a^{1/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 c^{3/4}\sqrt{a+c x^4}}$$

Result (type 4, 131 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}\,\sqrt{a+c x^4}}\sqrt{1+\frac{c x^4}{a}}\left(\sqrt{a}\,e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\left(-i\sqrt{c}\,d-\sqrt{a}\,e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e x^2\right)\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 334 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{2\sqrt{d}\,\sqrt{c\,d^2+a\,e^2}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+c\,x^4}}-$$

$$\frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,c^{1/4}\,d\left(c\,d^2-a\,e^2\right)\sqrt{a+c\,x^4}}$$

Result (type 4, 95 leaves) :

$$-\frac{i\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{a+c\,x^4}}$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)^2\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 581 leaves, 6 steps) :

$$-\frac{\sqrt{c}\,e\,x\sqrt{a+c\,x^4}}{2\,d\left(c\,d^2+a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2\,x\sqrt{a+c\,x^4}}{2\,d\left(c\,d^2+a\,e^2\right)\left(d+e\,x^2\right)}+\frac{\sqrt{e}\left(3\,c\,d^2+a\,e^2\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{4\,d^{3/2}\left(c\,d^2+a\,e^2\right)^{3/2}}+$$

$$\frac{a^{1/4}\,c^{1/4}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,d\left(c\,d^2+a\,e^2\right)\sqrt{a+c\,x^4}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,d\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+c\,x^4}}-$$

$$\left(\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(3\,c\,d^2+a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,c^{1/4}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c\,d^2+a\,e^2\right)\sqrt{a+c\,x^4}\right)$$

Result (type 4, 522 leaves) :

$$\frac{1}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d^2 \left(c d^2+a e^2\right) \left(d+e x^2\right) \sqrt{a+c x^4}} \left(a \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d e^2 x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, c d e^2 x^5-\sqrt{a}\,\sqrt{c}\, d e \left(d+e x^2\right) \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\right.$$

$$\sqrt{c}\, d \left(i\sqrt{c}\, d+\sqrt{a}\, e\right) \left(d+e x^2\right) \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-3\, i\, c\, d^3 \sqrt{1+\frac{c x^4}{a}}$$

$$\text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-i\, a\, d\, e^2 \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-3\, i\, c\, d^2 e\, x^2$$

$$\left.\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-i\, a\, e^3 x^2 \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 156: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^3 \sqrt{a+c x^4}}\, dx$$

Optimal (type 4, 729 leaves, 7 steps):

$$-\frac{3\sqrt{c}\,e\left(3 c d^2+a e^2\right) x \sqrt{a+c x^4}}{8 d^2\left(c d^2+a e^2\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2 x \sqrt{a+c x^4}}{4 d\left(c d^2+a e^2\right)\left(d+e x^2\right)^2}+$$

$$\frac{3 e^2\left(3 c d^2+a e^2\right) x \sqrt{a+c x^4}}{8 d^2\left(c d^2+a e^2\right)^2\left(d+e x^2\right)}+\frac{3\sqrt{e}\left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right) \text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{16 d^{5/2}\left(c d^2+a e^2\right)^{5/2}}+$$

$$\frac{3 a^{1/4} c^{1/4} e\left(3 c d^2+a e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{8 d^2\left(c d^2+a e^2\right)^2\sqrt{a+c x^4}}+$$

$$\frac{c^{1/4}\left(4 c d^2-\sqrt{a}\,\sqrt{c}\,d e+3 a e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{8 a^{1/4} d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c d^2+a e^2\right)\sqrt{a+c x^4}}-$$

$$\left(3\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(32 a^{1/4} c^{1/4} d^3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c d^2+a e^2\right)^2\sqrt{a+c x^4}\right)$$

Result (type 4, 332 leaves) :

$$\left(\frac{d\,e^2\,x\,\left(a+c\,x^4\right)\,\left(a\,e^2\,\left(5\,d+3\,e\,x^2\right)+c\,d^2\,\left(11\,d+9\,e\,x^2\right)\right)}{\left(d+e\,x^2\right)^2}+\right.$$

$$\frac{1}{\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}}\sqrt{1+\frac{c\,x^4}{a}}\,\left(-3\,\sqrt{a}\,\sqrt{c}\,d\,e\,\left(3\,c\,d^2+a\,e^2\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\mathrm{i}\,\left(\sqrt{c}\,d\,\left(7\,c^{3/2}\,d^3-9\,\mathrm{i}\,\sqrt{a}\,c\,d^2\,e+a\,\sqrt{c}\,d\,e^2-3\,\mathrm{i}\,a^{3/2}\,e^3\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\right.$$

$$\left.\left.\left.3\,\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\,\text{EllipticPi}\left[-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\right)\Big/\left(8\,d^3\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}\right)$$

Problem 157: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^3}{\sqrt{a-c\,x^4}}\,dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{d\,e^2\,x\,\sqrt{a-c\,x^4}}{c}-\frac{e^3\,x^3\,\sqrt{a-c\,x^4}}{5\,c}+\frac{3\,a^{3/4}\,e\,\left(5\,c\,d^2+a\,e^2\right)\,\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{5\,c^{7/4}\,\sqrt{a-c\,x^4}}+$$

$$\frac{a^{3/4}\,\left(\frac{5\,\sqrt{c}\,d\,\left(c\,d^2+a\,e^2\right)}{\sqrt{a}}-3\,e\,\left(5\,c\,d^2+a\,e^2\right)\right)\,\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{5\,c^{7/4}\,\sqrt{a-c\,x^4}}$$

Result (type 4, 232 leaves):

$$\frac{1}{5\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;c^{3/2}\sqrt{a-c\,x^4}}\left(\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} e^2\,x\,\left(5\,d+e\,x^2\right)\,\left(a-c\,x^4\right)-3\,\mathrm{i}\,\sqrt{a}\;e\,\left(5\,c\,d^2+a\,e^2\right)\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]+\right.$$
$$\left.\mathrm{i}\,\left(-5\,c^{3/2}\,d^3+15\,\sqrt{a}\;c\,d^2\,e-5\,a\,\sqrt{c}\;d\,e^2+3\,a^{3/2}\,e^3\right)\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^2}{\sqrt{a-c\,x^4}}\,dx$$

Optimal (type 4, 162 leaves, 7 steps):

$$-\frac{e^2\,x\,\sqrt{a-c\,x^4}}{3\,c}+\frac{2\,a^{3/4}\,d\,e\,\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{3/4}\,\sqrt{a-c\,x^4}}+\frac{a^{1/4}\,\left(3\,c\,d^2-6\,\sqrt{a}\;\sqrt{c}\;d\,e+a\,e^2\right)\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{3\,c^{5/4}\,\sqrt{a-c\,x^4}}$$

Result (type 4, 192 leaves):

$$\frac{1}{3\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;c\,\sqrt{a-c\,x^4}}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;e^2\,x\,\left(-a+c\,x^4\right)-6\,\mathrm{i}\,\sqrt{a}\;\sqrt{c}\;d\,e\,\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]-\right.$$
$$\left.\mathrm{i}\,\left(3\,c\,d^2-6\,\sqrt{a}\;\sqrt{c}\;d\,e+a\,e^2\right)\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

Problem 159: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x^2}{\sqrt{a-c\,x^4}}\,dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{a^{3/4}\,e\,\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{3/4}\,\sqrt{a-c\,x^4}}+\frac{a^{3/4}\,\left(\frac{\sqrt{c}\;d}{\sqrt{a}}-e\right)\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{3/4}\,\sqrt{a-c\,x^4}}$$

Result (type 4, 127 leaves):

$$\frac{1}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a-c x^4}}\,i\,\sqrt{1-\frac{c x^4}{a}}\left(\sqrt{a}\,e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e x^2\right)\sqrt{a-c x^4}}\,dx$$

Optimal (type 4, 72 leaves, 2 steps):

$$\frac{a^{1/4}\sqrt{1-\frac{c x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\text{ArcSin}\left[\frac{c^{1/4}x}{a^{1/4}}\right],-1\right]}{c^{1/4}\,d\,\sqrt{a-c x^4}}$$

Result (type 4, 91 leaves):

$$-\frac{i\sqrt{1-\frac{c x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\,\sqrt{a-c x^4}}$$

Problem 161: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e x^2\right)^2\sqrt{a-c x^4}}\,dx$$

Optimal (type 4, 299 leaves, 10 steps):

$$-\frac{e^2 x\sqrt{a-c x^4}}{2 d\left(c d^2-a e^2\right)\left(d+e x^2\right)}-\frac{a^{3/4}c^{1/4}e\sqrt{1-\frac{c x^4}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}x}{a^{1/4}}\right],-1\right]}{2 d\left(c d^2-a e^2\right)\sqrt{a-c x^4}}-$$

$$\frac{a^{1/4}c^{1/4}\sqrt{1-\frac{c x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}x}{a^{1/4}}\right],-1\right]}{2 d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{a-c x^4}}+\frac{a^{1/4}\left(3 c d^2-a e^2\right)\sqrt{1-\frac{c x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\text{ArcSin}\left[\frac{c^{1/4}x}{a^{1/4}}\right],-1\right]}{2 c^{1/4}d^2\left(c d^2-a e^2\right)\sqrt{a-c x^4}}$$

Result (type 4, 508 leaves):

$$\frac{1}{2\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d^2\left(c\,d^2-a\,e^2\right)\left(d+e\,x^2\right)\sqrt{a-c\,x^4}}$$

$$\left(-a\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\,e^2\,x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,c\,d\,e^2\,x^5+i\sqrt{a}\,\sqrt{c}\,d\,e\left(d+e\,x^2\right)\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\right.$$

$$i\sqrt{c}\,d\left(-\sqrt{c}\,d+\sqrt{a}\,e\right)\left(d+e\,x^2\right)\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$3\,i\,c\,d^3\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+i\,a\,d\,e^2\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-3\,i$$

$$\left.c\,d^2\,e\,x^2\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+i\,a\,e^3\,x^2\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)^3\sqrt{a-c\,x^4}}\,dx$$

Optimal (type 4, 425 leaves, 11 steps):

$$-\frac{e^2\,x\sqrt{a-c\,x^4}}{4\,d\left(c\,d^2-a\,e^2\right)\left(d+e\,x^2\right)^2}-\frac{3\,e^2\left(3\,c\,d^2-a\,e^2\right)x\sqrt{a-c\,x^4}}{8\,d^2\left(c\,d^2-a\,e^2\right)^2\left(d+e\,x^2\right)}-\frac{3\,a^{3/4}\,c^{1/4}\,e\left(3\,c\,d^2-a\,e^2\right)\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,d^2\left(c\,d^2-a\,e^2\right)^2\sqrt{a-c\,x^4}}-$$

$$\frac{a^{1/4}\,c^{1/4}\left(7\,c\,d^2-2\sqrt{a}\,\sqrt{c}\,d\,e-3\,a\,e^2\right)\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,d^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(c\,d^2-a\,e^2\right)\sqrt{a-c\,x^4}}+$$

$$\frac{3\,a^{1/4}\left(5\,c^2\,d^4-2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\sqrt{1-\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,c^{1/4}\,d^3\left(c\,d^2-a\,e^2\right)^2\sqrt{a-c\,x^4}}$$

Result (type 4, 321 leaves):

$$\left(\frac{d\,e^2\,x\,\left(a-c\,x^4\right)\,\left(a\,e^2\,\left(5\,d+3\,e\,x^2\right)-c\,d^2\,\left(11\,d+9\,e\,x^2\right)\right)}{\left(d+e\,x^2\right)^2}-\right.$$

$$\frac{1}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\,\mathrm{i}\,\sqrt{1-\frac{c\,x^4}{a}}\,\left(3\,\sqrt{a}\,\sqrt{c}\,d\,e\,\left(-3\,c\,d^2+a\,e^2\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\left(-7\,c^2\,d^4+9\,\sqrt{a}\,c^{3/2}\,d^3\,e+a\,c\,d^2\,e^2-3\,a^{3/2}\,\sqrt{c}\,d\,e^3\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left.\left.3\,\left(5\,c^2\,d^4-2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\Big/\left(8\,d^3\,\left(c\,d^2-a\,e^2\right)^2\,\sqrt{a-c\,x^4}\right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)^4\,\sqrt{a-c\,x^4}}\,dx$$

Optimal (type 4, 563 leaves, 12 steps):

$$-\frac{e^2 x \sqrt{a-c x^4}}{6 d \left(c d^2-a e^2\right) \left(d+e x^2\right)^3}-\frac{5 e^2 \left(3 c d^2-a e^2\right) x \sqrt{a-c x^4}}{24 d^2 \left(c d^2-a e^2\right)^2 \left(d+e x^2\right)^2}-\frac{e^2 \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) x \sqrt{a-c x^4}}{16 d^3 \left(c d^2-a e^2\right)^3 \left(d+e x^2\right)}-$$

$$\frac{a^{3/4} c^{1/4} e \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{16 d^3 \left(c d^2-a e^2\right)^3 \sqrt{a-c x^4}}-$$

$$\left(a^{1/4} c^{1/4} \left(57 c^2 d^4-30 \sqrt{a}\, c^{3/2} d^3 e-32 a c d^2 e^2+10 a^{3/2} \sqrt{c}\, d e^3+15 a^2 e^4\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/$$

$$\left(48 d^3 \left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2 \left(\sqrt{c}\, d+\sqrt{a}\, e\right)^3 \sqrt{a-c x^4}\right)+$$

$$\frac{a^{1/4} \left(35 c^3 d^6-7 a c^2 d^4 e^2+17 a^2 c d^2 e^4-5 a^3 e^6\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{16 c^{1/4} d^4 \left(c d^2-a e^2\right)^3 \sqrt{a-c x^4}}$$

Result (type 4, 458 leaves):

$$\frac{1}{48 d^4 \sqrt{a-c x^4}}\left(-\frac{1}{\left(c d^2-a e^2\right)^3 \left(d+e x^2\right)^3}\right.$$

$$d e^2 x \left(a-c x^4\right) \left(8 \left(c d^3-a d e^2\right)^2+10 d \left(c d^2-a e^2\right) \left(3 c d^2-a e^2\right) \left(d+e x^2\right)+3 \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) \left(d+e x^2\right)^2\right)-$$

$$\frac{1}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} \left(-c d^2+a e^2\right)^3} i \sqrt{1-\frac{c x^4}{a}} \left(3 \sqrt{a} \sqrt{c}\, d e \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\right.$$

$$\sqrt{c}\, d \left(57 c^{5/2} d^5-87 \sqrt{a}\, c^2 d^4 e-2 a c^{3/2} d^3 e^2+42 a^{3/2} c d^2 e^3+5 a^2 \sqrt{c}\, d e^4-15 a^{5/2} e^5\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left.\left.3 \left(-35 c^3 d^6+7 a c^2 d^4 e^2-17 a^2 c d^2 e^4+5 a^3 e^6\right) \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{-a+c x^4}} \, dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$\frac{a^{3/4} e \sqrt{1-\frac{c x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{3/4} \sqrt{-a+c x^4}}+\frac{a^{3/4} \left(\frac{\sqrt{c} d}{\sqrt{a}}-e\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{3/4} \sqrt{-a+c x^4}}$$

Result (type 4, 128 leaves):

$$\frac{1}{\sqrt{a} \left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{-a+c x^4}} i \sqrt{1-\frac{c x^4}{a}} \left(\sqrt{a} e \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} x\right],-1\right]+\left(\sqrt{c} d-\sqrt{a} e\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right) \sqrt{-a+c x^4}} \, dx$$

Optimal (type 4, 73 leaves, 2 steps):

$$\frac{a^{1/4} \sqrt{1-\frac{c x^4}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} e}{\sqrt{c} d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{1/4} d \sqrt{-a+c x^4}}$$

Result (type 4, 92 leaves):

$$-\frac{i \sqrt{1-\frac{c x^4}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} e}{\sqrt{c} d},i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} d \sqrt{-a+c x^4}}$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a}+\sqrt{c} x^2}{\sqrt{-a+c x^4}} \, dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{a^{3/4}\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{1/4}\sqrt{-a+c\,x^4}}$$

Result (type 4, 78 leaves):

$$\frac{i\sqrt{c}\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]}{\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{-a+c\,x^4}}$$

Problem 167: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+\sqrt{\frac{c}{a}}\;x^2}{\sqrt{-a+c\,x^4}}\,dx$$

Optimal (type 4, 52 leaves, 3 steps):

$$\frac{\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\left(\frac{c}{a}\right)^{1/4}x\right],-1\right]}{\left(\frac{c}{a}\right)^{1/4}\sqrt{-a+c\,x^4}}$$

Result (type 4, 142 leaves):

$$\frac{1}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{-a+c\,x^4}}$$

$$i\sqrt{1-\frac{c\,x^4}{a}}\left(\sqrt{a}\sqrt{\frac{c}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]+\left(\sqrt{c}-\sqrt{a}\sqrt{\frac{c}{a}}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x^2}{\sqrt{-a-c\,x^4}}\,dx$$

Optimal (type 4, 236 leaves, 3 steps):

$$-\frac{e\,x\,\sqrt{-a-c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4}\,e\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\,\sqrt{-a-c\,x^4}}+$$

$$\frac{a^{1/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,c^{3/4}\,\sqrt{-a-c\,x^4}}$$

Result (type 4, 134 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}\,\sqrt{-a-c\,x^4}}\sqrt{1+\frac{c\,x^4}{a}}\left(\sqrt{a}\,e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\left(-i\sqrt{c}\,d-\sqrt{a}\,e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 169: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)\sqrt{-a-c\,x^4}}\,dx$$

Optimal (type 4, 347 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^2-a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-c\,x^4}}\right]}{2\,\sqrt{d}\,\sqrt{-c\,d^2-a\,e^2}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a-c\,x^4}}-$$

$$\frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,c^{1/4}\,d\left(c\,d^2-a\,e^2\right)\sqrt{-a-c\,x^4}}$$

Result (type 4, 98 leaves):

$$-\frac{i\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\,\sqrt{-a-c\,x^4}}$$

Problem 171: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{4+5 x^4}} \, dx$$

Optimal (type 4, 310 leaves, 3 steps):

$$\frac{\sqrt{b} \, \text{ArcTan}\left[\frac{\sqrt{5 a^2+4 b^2} \, x}{\sqrt{a} \sqrt{b} \sqrt{4+5 x^4}}\right]}{2 \sqrt{a} \sqrt{5 a^2+4 b^2}}+\frac{5^{1/4} \left(\sqrt{5} \, a+2 b\right) \left(2+\sqrt{5} \, x^2\right) \sqrt{\frac{4+5 x^4}{\left(2+\sqrt{5} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\frac{5^{1/4} x}{\sqrt{2}}\right], \frac{1}{2}\right]}{2 \sqrt{2} \left(5 a^2-4 b^2\right) \sqrt{4+5 x^4}}-$$
$$\frac{\left(\sqrt{5} \, a+2 b\right)^2 \left(2+\sqrt{5} \, x^2\right) \sqrt{\frac{4+5 x^4}{\left(2+\sqrt{5} \, x^2\right)^2}} \, \text{EllipticPi}\left[-\frac{\left(\sqrt{5} \, a-2 b\right)^2}{8 \sqrt{5} \, a b}, 2 \, \text{ArcTan}\left[\frac{5^{1/4} x}{\sqrt{2}}\right], \frac{1}{2}\right]}{4 \sqrt{2} \, 5^{1/4} a \left(5 a^2-4 b^2\right) \sqrt{4+5 x^4}}$$

Result (type 4, 50 leaves):

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right) \text{EllipticPi}\left[-\frac{2 i b}{\sqrt{5} \, a}, i \, \text{ArcSinh}\left[\left(\frac{1}{2}+\frac{i}{2}\right) 5^{1/4} x\right], -1\right]}{5^{1/4} a}$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{4-d x^4}} \, dx$$

Optimal (type 4, 40 leaves, 1 step):

$$\frac{\text{EllipticPi}\left[-\frac{2 b}{a \sqrt{d}}, \text{ArcSin}\left[\frac{d^{1/4} x}{\sqrt{2}}\right], -1\right]}{\sqrt{2} \, a \, d^{1/4}}$$

Result (type 4, 59 leaves):

$$-\frac{i \, \text{EllipticPi}\left[-\frac{2 b}{a \sqrt{d}}, i \, \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{d}} \, x}{\sqrt{2}}\right], -1\right]}{\sqrt{2} \, a \sqrt{-\sqrt{d}}}$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+b x^2)\sqrt{4+d x^4}}\,dx$$

Optimal (type 4, 300 leaves, 3 steps):

$$\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{4 b^2+a^2 d}\,x}{\sqrt{a}\,\sqrt{b}\,\sqrt{4+d x^4}}\right]}{2\sqrt{a}\,\sqrt{4 b^2+a^2 d}}-\frac{d^{1/4}\left(2+\sqrt{d}\,x^2\right)\sqrt{\frac{4+d x^4}{\left(2+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{\sqrt{2}}\right],\frac{1}{2}\right]}{2\sqrt{2}\left(2 b-a\sqrt{d}\right)\sqrt{4+d x^4}}+$$

$$\frac{\left(2 b+a\sqrt{d}\right)\left(2+\sqrt{d}\,x^2\right)\sqrt{\frac{4+d x^4}{\left(2+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(2 b-a\sqrt{d}\right)^2}{8 a b\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{\sqrt{2}}\right],\frac{1}{2}\right]}{4\sqrt{2}\,a\left(2 b-a\sqrt{d}\right)d^{1/4}\sqrt{4+d x^4}}$$

Result (type 4, 65 leaves):

$$-\frac{i\,\text{EllipticPi}\left[-\frac{2 i b}{a\sqrt{d}},\,i\,\text{ArcSinh}\left[\frac{\sqrt{i\sqrt{d}}\,x}{\sqrt{2}}\right],-1\right]}{\sqrt{2}\,a\sqrt{i\sqrt{d}}}$$

Problem 174: Unable to integrate problem.

$$\int \frac{\sqrt{a+b x^2}}{\sqrt{1-x^4}}\,dx$$

Optimal (type 4, 112 leaves, ? steps):

$$\frac{a\sqrt{1-x^2}\,\sqrt{\frac{a(1+x^2)}{a+b x^2}}\,\text{EllipticPi}\left[\frac{b}{a+b},\text{ArcSin}\left[\frac{\sqrt{a+b}\,x}{\sqrt{a+b x^2}}\right],-\frac{a-b}{a+b}\right]}{\sqrt{a+b}\,\sqrt{1+x^2}\,\sqrt{\frac{a(1-x^2)}{a+b x^2}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{a+b x^2}}{\sqrt{1-x^4}}\,dx$$

Problem 180: Unable to integrate problem.

$$\int \frac{\left(a+b x^4\right)^p}{c+e x^2} \, dx$$

Optimal (type 6, 123 leaves, 6 steps):

$$\frac{x \left(a+b x^4\right)^p \left(1+\frac{b x^4}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{4},-p,1,\frac{5}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{c}-\frac{e x^3 \left(a+b x^4\right)^p \left(1+\frac{b x^4}{a}\right)^{-p} \text{AppellF1}\left[\frac{3}{4},-p,1,\frac{7}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{3 c^2}$$

Result (type 8, 21 leaves):

$$\int \frac{\left(a+b x^4\right)^p}{c+e x^2} \, dx$$

Problem 181: Unable to integrate problem.

$$\int \frac{\left(a+b x^4\right)^p}{\left(c+e x^2\right)^2} \, dx$$

Optimal (type 6, 189 leaves, 8 steps):

$$\frac{x \left(a+b x^4\right)^p \left(1+\frac{b x^4}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{4},-p,2,\frac{5}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{c^2}-$$

$$\frac{2 e x^3 \left(a+b x^4\right)^p \left(1+\frac{b x^4}{a}\right)^{-p} \text{AppellF1}\left[\frac{3}{4},-p,2,\frac{7}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{3 c^3}+\frac{e^2 x^5 \left(a+b x^4\right)^p \left(1+\frac{b x^4}{a}\right)^{-p} \text{AppellF1}\left[\frac{5}{4},-p,2,\frac{9}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{5 c^4}$$

Result (type 8, 21 leaves):

$$\int \frac{\left(a+b x^4\right)^p}{\left(c+e x^2\right)^2} \, dx$$

Problem 186: Unable to integrate problem.

$$\int \frac{\left(1+b x^4\right)^p}{1-x^2} \, dx$$

Optimal (type 6, 50 leaves, 4 steps):

$$x \, \text{AppellF1}\left[\frac{1}{4},1,-p,\frac{5}{4},x^4,-b x^4\right]+\frac{1}{3} x^3 \, \text{AppellF1}\left[\frac{3}{4},1,-p,\frac{7}{4},x^4,-b x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b x^4\right)^p}{1-x^2}\, dx$$

Problem 187: Unable to integrate problem.

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^2}\, dx$$

Optimal (type 6, 77 leaves, 5 steps):

$$x\,\text{AppellF1}\left[\frac{1}{4},2,-p,\frac{5}{4},x^4,-b x^4\right]+\frac{2}{3}x^3\,\text{AppellF1}\left[\frac{3}{4},2,-p,\frac{7}{4},x^4,-b x^4\right]+\frac{1}{5}x^5\,\text{AppellF1}\left[\frac{5}{4},2,-p,\frac{9}{4},x^4,-b x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^2}\, dx$$

Problem 188: Unable to integrate problem.

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^3}\, dx$$

Optimal (type 6, 101 leaves, 6 steps):

$$x\,\text{AppellF1}\left[\frac{1}{4},3,-p,\frac{5}{4},x^4,-b x^4\right]+x^3\,\text{AppellF1}\left[\frac{3}{4},3,-p,\frac{7}{4},x^4,-b x^4\right]+$$
$$\frac{3}{5}x^5\,\text{AppellF1}\left[\frac{5}{4},3,-p,\frac{9}{4},x^4,-b x^4\right]+\frac{1}{7}x^7\,\text{AppellF1}\left[\frac{7}{4},3,-p,\frac{11}{4},x^4,-b x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^3}\, dx$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{5/2}}{\sqrt{a^2-b^2 x^4}}\, dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$-\frac{9\,a\,x\,\left(a-b\,x^2\right)\,\sqrt{a+b\,x^2}}{8\,\sqrt{a^2-b^2\,x^4}}-\frac{x\,\left(a-b\,x^2\right)\,\left(a+b\,x^2\right)^{3/2}}{4\,\sqrt{a^2-b^2\,x^4}}+\frac{19\,a^2\,\sqrt{a-b\,x^2}\,\sqrt{a+b\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a-b\,x^2}}\right]}{8\,\sqrt{b}\,\sqrt{a^2-b^2\,x^4}}$$

Result (type 3, 98 leaves):

$$-\frac{\left(11\,a\,x+2\,b\,x^3\right)\,\sqrt{a^2-b^2\,x^4}}{8\,\sqrt{a+b\,x^2}}+\frac{19\,i\,a^2\,\text{Log}\left[-2\,i\,\sqrt{b}\,x+\frac{2\,\sqrt{a^2-b^2\,x^4}}{\sqrt{a+b\,x^2}}\right]}{8\,\sqrt{b}}$$

Problem 200: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{x\,\left(a-b\,x^2\right)\,\sqrt{a+b\,x^2}}{2\,\sqrt{a^2-b^2\,x^4}}+\frac{3\,a\,\sqrt{a-b\,x^2}\,\sqrt{a+b\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a-b\,x^2}}\right]}{2\,\sqrt{b}\,\sqrt{a^2-b^2\,x^4}}$$

Result (type 3, 86 leaves):

$$-\frac{x\,\sqrt{a^2-b^2\,x^4}}{2\,\sqrt{a+b\,x^2}}+\frac{3\,i\,a\,\text{Log}\left[-2\,i\,\sqrt{b}\,x+\frac{2\,\sqrt{a^2-b^2\,x^4}}{\sqrt{a+b\,x^2}}\right]}{2\,\sqrt{b}}$$

Problem 201: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$\frac{\sqrt{a-b\,x^2}\,\sqrt{a+b\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a-b\,x^2}}\right]}{\sqrt{b}\,\sqrt{a^2-b^2\,x^4}}$$

Result (type 3, 50 leaves):

$$\frac{i\,\text{Log}\left[-2\,i\,\sqrt{b}\,x+\frac{2\,\sqrt{a^2-b^2\,x^4}}{\sqrt{a+b\,x^2}}\right]}{\sqrt{b}}$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int (1+x^2)^3 \sqrt{1+x^2+x^4}\, dx$$

Optimal (type 4, 183 leaves, 6 steps):

$$\frac{26 x \sqrt{1+x^2+x^4}}{45 (1+x^2)} + \frac{2}{45} x (7+6 x^2) \sqrt{1+x^2+x^4} + \frac{1}{3} x (1+x^2+x^4)^{3/2} + \frac{1}{9} x^3 (1+x^2+x^4)^{3/2} - \frac{26 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{45 \sqrt{1+x^2+x^4}} + \frac{7 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{15 \sqrt{1+x^2+x^4}}$$

Result (type 4, 169 leaves):

$$\frac{1}{45 \sqrt{1+x^2+x^4}} \Big(x (29+61 x^2+81 x^4+57 x^6+25 x^8+5 x^{10}) + 26 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right] + 2 (-1)^{5/6} (9 i + 4 \sqrt{3}) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\Big)$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\, dx$$

Optimal (type 4, 164 leaves, 5 steps):

$$\frac{2 x \sqrt{1+x^2+x^4}}{3 (1+x^2)} + \frac{2}{21} x (4+3 x^2) \sqrt{1+x^2+x^4} + \frac{1}{7} x (1+x^2+x^4)^{3/2} - \frac{2 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{3 \sqrt{1+x^2+x^4}} + \frac{4 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{7 \sqrt{1+x^2+x^4}}$$

Result (type 4, 162 leaves):

$$\frac{1}{21 \sqrt{1+x^2+x^4}} \Big(x (11+20 x^2+23 x^4+12 x^6+3 x^8) + 14 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right] + 2 (-1)^{1/3} (-7+5 (-1)^{1/3}) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\Big)$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{3 x \sqrt{1+x^2+x^4}}{5 \left(1+x^2\right)}+\frac{1}{5} x \left(2+x^2\right) \sqrt{1+x^2+x^4}-\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{5 \sqrt{1+x^2+x^4}}+\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{5 \sqrt{1+x^2+x^4}}$$

Result (type 4, 168 leaves):

$$\frac{1}{5 \sqrt{1+x^2+x^4}}\left(2 x+3 x^3+3 x^5+x^7+3 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\, \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+\right.$$
$$\left.\frac{3}{2} \sqrt{2+\left(1-i \sqrt{3}\right) x^2} \sqrt{2+\left(1+i \sqrt{3}\right) x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2} \left(x+i \sqrt{3} x\right)\right],\frac{1}{2} i \left(i+\sqrt{3}\right)\right]\right)$$

Problem 228: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\, dx$$

Optimal (type 4, 137 leaves, 8 steps):

$$\frac{x \sqrt{1+x^2+x^4}}{1+x^2}+\frac{1}{2} \text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]-\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}+\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{4 \sqrt{1+x^2+x^4}}$$

Result (type 4, 118 leaves):

$$-\frac{1}{\sqrt{1+x^2+x^4}} (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \left(-\text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+\right.$$
$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+(-1)^{1/3} \text{EllipticPi}\left[(-1)^{1/3},-i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]\right)$$

Problem 229: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2}\, dx$$

Optimal (type 4, 49 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}$$

Result (type 4, 164 leaves):

$$\frac{1}{2\sqrt{1+x^2+x^4}}\left(\frac{x+x^3+x^5}{1+x^2}+(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.$$
$$\left.(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)\right)$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx$$

Optimal (type 4, 93 leaves, 23 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{1}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 176 leaves):

$$\frac{1}{4\sqrt{1+x^2+x^4}}\left(\frac{x\left(2+x^2\right)\left(1+x^2+x^4\right)}{\left(1+x^2\right)^2}+\right.$$
$$(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-$$
$$\left.2(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx$$

Optimal (type 4, 166 leaves, 26 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}+\frac{1}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+$$
$$\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{8\sqrt{1+x^2+x^4}}$$

Result (type 4, 240 leaves):

$$\frac{1}{6\sqrt{1+x^2+x^4}}\left(\frac{x\left(1+x^2+x^4\right)\left(4+5x^2+2x^4\right)}{\left(1+x^2\right)^3}-\right.$$
$$2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-$$
$$(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-$$
$$\left.3(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 232: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+x^2\right)^3}{\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 159 leaves, 5 steps):

$$\frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15\left(1+x^2\right)}-$$
$$\frac{14\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{15\sqrt{1+x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{5\sqrt{1+x^2+x^4}}$$

Result (type 4, 157 leaves):

$$\frac{1}{15\sqrt{1+x^2+x^4}}\left(x\left(11+14x^2+14x^4+3x^6\right)+14(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.$$
$$\left.2(-1)^{1/3}\left(-7+2(-1)^{1/3}\right)\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1+x^2\right)^2}{\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$\frac{1}{3} x \sqrt{1+x^2+x^4}+\frac{4 x \sqrt{1+x^2+x^4}}{3 \left(1+x^2\right)}-\frac{4 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{3 \sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 143 leaves):

$$\frac{1}{3 \sqrt{1+x^2+x^4}}\left(x+x^3+x^5+4 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+\right.$$
$$\left.2 (-1)^{1/3} \left(-2+(-1)^{1/3}\right) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\right)$$

Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 115 leaves, 3 steps):

$$\frac{x \sqrt{1+x^2+x^4}}{1+x^2}-\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{1+x^2+x^4}}(-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}$$
$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+\left(-1+(-1)^{1/3}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\right)$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right) \sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 69 leaves, 4 steps):

$$\frac{1}{2}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 73 leaves):

$$-\frac{(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]}{\sqrt{1+x^2+x^4}}$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 118 leaves, 8 steps):

$$\frac{1}{2}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 226 leaves):

$$\frac{1}{2\sqrt{1+x^2+x^4}}\left(\frac{x+x^3+x^5}{1+x^2}-(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.$$
$$(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-$$
$$\left.2\,(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{1}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}$$

Result (type 4, 235 leaves):

$$\frac{1}{4\sqrt{1+x^2+x^4}}\left(\frac{x\left(4+3x^2\right)\left(1+x^2+x^4\right)}{\left(1+x^2\right)^2}-\right.$$
$$3\left(-1\right)^{1/3}\sqrt{1+\left(-1\right)^{1/3}x^2}\sqrt{1-\left(-1\right)^{2/3}x^2}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]\right)-$$
$$2\left(-1\right)^{2/3}\sqrt{1+\left(-1\right)^{1/3}x^2}\sqrt{1-\left(-1\right)^{2/3}x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]-$$
$$\left.2\left(-1\right)^{2/3}\sqrt{1+\left(-1\right)^{1/3}x^2}\sqrt{1-\left(-1\right)^{2/3}x^2}\,\text{EllipticPi}\left[\left(-1\right)^{1/3},-i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]\right)$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+x^2\right)^3}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 144 leaves, 4 steps):

$$-\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 136 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(-x+x^3+2\left(-1\right)^{1/3}\sqrt{1+\left(-1\right)^{1/3}x^2}\sqrt{1-\left(-1\right)^{2/3}x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]+\right.$$
$$\left.2\left(-1\right)^{5/6}\sqrt{3+3\left(-1\right)^{1/3}x^2}\sqrt{1-\left(-1\right)^{2/3}x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{5/6}x\right],\left(-1\right)^{2/3}\right]\right)$$

Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+x^2\right)^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 98 leaves, 2 steps):

$$\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}$$

Result (type 4, 158 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(x+2x^3-2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\mathrm{i}\sqrt{2+\left(1+\mathrm{i}\sqrt{3}\right)x^2}\sqrt{6+\left(3-3\,\mathrm{i}\sqrt{3}\right)x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\left(x+\mathrm{i}\sqrt{3}\,x\right)\right],\frac{1}{2}\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\right]\right)$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 96 leaves, 2 steps):

$$\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}$$

Result (type 4, 160 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(2x+x^3-(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\frac{1}{2}\mathrm{i}\sqrt{2+\left(1+\mathrm{i}\sqrt{3}\right)x^2}\sqrt{6+\left(3-3\,\mathrm{i}\sqrt{3}\right)x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\left(x+\mathrm{i}\sqrt{3}\,x\right)\right],\frac{1}{2}\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\right]\right)$$

Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(1+x^2\right)\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 166 leaves, 9 steps):

$$-\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 204 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(-x-2x^3+2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.$$
$$(-1)^{1/3}\left(-2+(-1)^{1/3}\right)\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-$$
$$\left.3(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 111 leaves, 16 steps):

$$-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{6\sqrt{1+x^2+x^4}}$$

Result (type 4, 168 leaves):

$$\frac{1}{6(1+x^2)\sqrt{1+x^2+x^4}}$$
$$\left(-2x(1+x^2)(2+x^2)+3x(1+x^2+x^4)-(-1)^{1/3}(1+x^2)\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.\right.$$
$$\left.\left.\left(-1+5(-1)^{1/3}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+12(-1)^{1/3}\,\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)\right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 190 leaves, 23 steps):

$$-\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2}-\frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\frac{3}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+$$
$$\frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{12\sqrt{1+x^2+x^4}}-\frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 192 leaves):

$$\frac{1}{12\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\left(4x\left(-1+x^2\right)\left(1+x^2\right)^2+3x\left(1+x^2+x^4\right)+15x\left(1+x^2\right)\left(1+x^2+x^4\right)-\right.$$
$$(-1)^{1/3}\left(1+x^2\right)^2\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(19\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\right.$$
$$\left.\left.\left(-9+10i\sqrt{3}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+18(-1)^{1/3}\,\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)\right)$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 193 leaves, 6 steps):

$$\frac{577x\left(2+x^2\right)}{3\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}-$$
$$\frac{577\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}+\frac{2945\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{21\sqrt{2+3x^2+x^4}}$$

Result (type 4, 119 leaves):

$$\frac{1}{63\sqrt{2+3x^2+x^4}}\left(25548x+61214x^3+57312x^5+28496x^7+7725x^9+875x^{11}-\right.$$
$$\left.12117\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-5553\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 287: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{31x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}-$$
$$\frac{31\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{472\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{21\sqrt{2+3x^2+x^4}}$$

Result (type 4, 114 leaves):

$$\frac{1}{21\sqrt{2+3x^2+x^4}}\left(1114\,x+2349\,x^3+1724\,x^5+564\,x^7+75\,x^9-\right.$$
$$\left.651\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-293\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 288: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$\frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{11\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 109 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}$$
$$\left(20\,x+36\,x^3+19\,x^5+3\,x^7-15\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 289: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 141 leaves, 4 steps):

$$\frac{x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 102 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(2\,x+3\,x^3+x^5-3\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$$

Optimal (type 4, 178 leaves, 8 steps):

$$\frac{x\,(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}} + \frac{3\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 90 leaves):

$$-\frac{1}{175\sqrt{2+3x^2+x^4}} i\sqrt{1+x^2}\sqrt{2+x^2}\left(35\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+21\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-6\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 291: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$$

Optimal (type 4, 209 leaves, 8 steps):

$$-\frac{x\,(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\,(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{140\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{(2+x^2)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{980\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 208 leaves):

$$\left(350\,x+525\,x^3+175\,x^5+35\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(7+5\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-84\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(7+5\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-5\,i\,x^2\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(2450\,\left(7+5\,x^2\right)\,\sqrt{2+3\,x^2+x^4}\right)$$

Problem 292: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+3\,x^2+x^4}}{\left(7+5\,x^2\right)^3}\,dx$$

Optimal (type 4, 237 leaves, 25 steps):

$$-\frac{11\,x\,\left(2+x^2\right)}{11760\,\sqrt{2+3\,x^2+x^4}}+\frac{x\,\sqrt{2+3\,x^2+x^4}}{28\,\left(7+5\,x^2\right)^2}+\frac{11\,x\,\sqrt{2+3\,x^2+x^4}}{2352\,\left(7+5\,x^2\right)}+\frac{11\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5880\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}+\frac{81\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{7840\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}-\frac{1201\,\left(2+x^2\right)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{164640\,\sqrt{2}\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 174 leaves):

$$\frac{1}{411600\,\sqrt{2+3\,x^2+x^4}}\left(\frac{14700\,x\,\left(2+3\,x^2+x^4\right)}{\left(7+5\,x^2\right)^2}+\frac{1925\,x\,\left(2+3\,x^2+x^4\right)}{7+5\,x^2}+385\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-434\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-1201\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 293: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5\,x^2\right)^3\,\left(2+3\,x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 219 leaves, 7 steps):

$$\frac{20884\,x\,(2+x^2)}{65\sqrt{2+3x^2+x^4}}+\frac{x\,(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\,(208212+65345x^2)\,(2+3x^2+x^4)^{3/2}}{3003}+\frac{3825}{143}x\,(2+3x^2+x^4)^{5/2}+$$

$$\frac{125}{13}x^3\,(2+3x^2+x^4)^{5/2}-\frac{20884\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{65\sqrt{2+3x^2+x^4}}+\frac{1171349\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5005\sqrt{2+3x^2+x^4}}$$

Result (type 4, 129 leaves):

$$\frac{1}{15015\sqrt{2+3x^2+x^4}}\Big(13572486\,x+40493455\,x^3+54938052\,x^5+46218643\,x^7+25350660\,x^9+8705725\,x^{11}+1701000\,x^{13}+144375\,x^{15}-$$

$$4824204\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-2203890\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\Big)$$

Problem 294: Result unnecessarily involves imaginary or complex numbers.

$$\int(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}\,dx$$

Optimal (type 4, 198 leaves, 6 steps):

$$\frac{742\,x\,(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{x\,(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155}+\frac{1}{693}x\,(7281+2240x^2)\,(2+3x^2+x^4)^{3/2}+\frac{25}{11}x\,(2+3x^2+x^4)^{5/2}-$$

$$\frac{742\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{15\sqrt{2+3x^2+x^4}}+\frac{13879\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{385\sqrt{2+3x^2+x^4}}$$

Result (type 4, 124 leaves):

$$\frac{1}{3465\sqrt{2+3x^2+x^4}}\Big(429318\,x+1160065\,x^3+1333551\,x^5+892084\,x^7+363480\,x^9+82075\,x^{11}+7875\,x^{13}-$$

$$171402\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-78420\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\Big)$$

Problem 295: Result unnecessarily involves imaginary or complex numbers.

$$\int(7+5x^2)\,(2+3x^2+x^4)^{3/2}\,dx$$

Optimal (type 4, 179 leaves, 5 steps):

$$\frac{116\,x\,(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x\,(519+149x^2)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\,(108+35x^2)\,(2+3x^2+x^4)^{3/2}-$$
$$\frac{116\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{15\sqrt{2+3x^2+x^4}}+\frac{197\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2+3x^2+x^4}}$$

Result (type 4, 119 leaves):

$$\frac{1}{315\sqrt{2+3x^2+x^4}}\left(5274\,x+12\,745\,x^3+12\,018\,x^5+5962\,x^7+1590\,x^9+175\,x^{11}-\right.$$
$$\left.2436\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-1110\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 296: Result unnecessarily involves imaginary or complex numbers.

$$\int(2+3x^2+x^4)^{3/2}\,dx$$

Optimal (type 4, 172 leaves, 5 steps):

$$\frac{6\,x\,(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\,(29+9x^2)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\,(2+3x^2+x^4)^{3/2}-$$
$$\frac{6\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}}+\frac{31\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2+3x^2+x^4}}$$

Result (type 4, 114 leaves):

$$\frac{1}{35\sqrt{2+3x^2+x^4}}$$
$$\left(78\,x+165\,x^3+121\,x^5+39\,x^7+5\,x^9-42\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-20\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 297: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$$

Optimal (type 4, 207 leaves, 13 steps):

$$\frac{24\,x\,(2+x^2)}{125\,\sqrt{2+3\,x^2+x^4}}+\frac{1}{75}\,x\,(11+3\,x^2)\,\sqrt{2+3\,x^2+x^4}-\frac{24\,\sqrt{2}\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{125\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{56\,\sqrt{2}\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{375\,\sqrt{2+3\,x^2+x^4}}-\frac{9\,\sqrt{2}\,(2+x^2)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{875\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 148 leaves):

$$\frac{1}{13125\,\sqrt{2+3\,x^2+x^4}}\left(3850\,x+6825\,x^3+3500\,x^5+525\,x^7-2520\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$
$$\left.1022\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-108\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 298: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2+3\,x^2+x^4)^{3/2}}{(7+5\,x^2)^2}\,dx$$

Optimal (type 4, 222 leaves, 21 steps):

$$\frac{9\,x\,(2+x^2)}{175\,\sqrt{2+3\,x^2+x^4}}+\frac{1}{75}\,x\,\sqrt{2+3\,x^2+x^4}-\frac{3\,x\,\sqrt{2+3\,x^2+x^4}}{175\,(7+5\,x^2)}-\frac{9\,\sqrt{2}\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{175\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{59\,(1+x^2)\,\sqrt{\frac{2+x^2}{2+2\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{1050\,\sqrt{2+3\,x^2+x^4}}+\frac{9\,(1+x^2)\,\sqrt{\frac{2+x^2}{2+2\,x^2}}\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{2450\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 213 leaves):

$$\frac{1}{18375\,(7+5\,x^2)\,\sqrt{2+3\,x^2+x^4}}\left(2800\,x+6650\,x^3+5075\,x^5+1225\,x^7-\right.$$
$$945\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5\,x^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-182\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5\,x^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+$$
$$\left.189\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+135\,i\,x^2\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 299: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(2+3 x^2+x^4\right)^{3/2}}{\left(7+5 x^2\right)^3} \, dx$$

Optimal (type 4, 231 leaves, 27 steps):

$$\frac{3 x \left(2+x^2\right)}{392 \sqrt{2+3 x^2+x^4}}-\frac{3 x \sqrt{2+3 x^2+x^4}}{350 \left(7+5 x^2\right)^2}+\frac{17 x \sqrt{2+3 x^2+x^4}}{9800 \left(7+5 x^2\right)}-\frac{3 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}} \, \text{EllipticE}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{196 \sqrt{2+3 x^2+x^4}}+$$
$$\frac{5 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}} \, \text{EllipticF}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{784 \sqrt{2+3 x^2+x^4}}+\frac{141 \left(2+x^2\right) \text{EllipticPi}\left[\frac{2}{7}, \text{ArcTan}[x], \frac{1}{2}\right]}{27440 \sqrt{2} \sqrt{\frac{2+x^2}{1+x^2}} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 174 leaves):

$$\frac{1}{68600 \sqrt{2+3 x^2+x^4}}\left(-\frac{588 x \left(2+3 x^2+x^4\right)}{\left(7+5 x^2\right)^2}+\frac{119 x \left(2+3 x^2+x^4\right)}{7+5 x^2}-525 i \sqrt{1+x^2} \sqrt{2+x^2} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right]-\right.$$
$$\left.406 i \sqrt{1+x^2} \sqrt{2+x^2} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right]+141 i \sqrt{1+x^2} \sqrt{2+x^2} \, \text{EllipticPi}\left[\frac{10}{7}, i \, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], 2\right]\right)$$

Problem 300: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^3}{\sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 157 leaves, 5 steps):

$$\frac{135 x \left(2+x^2\right)}{\sqrt{2+3 x^2+x^4}}+75 x \sqrt{2+3 x^2+x^4}+25 x^3 \sqrt{2+3 x^2+x^4}-$$
$$\frac{135 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \, \text{EllipticE}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{2+3 x^2+x^4}}+\frac{193 \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \, \text{EllipticF}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{2} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 106 leaves):

$$\frac{1}{\sqrt{2+3x^2+x^4}}\left(25x\left(6+11x^2+6x^4+x^6\right)-135\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-58\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 301: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 142 leaves, 4 steps):

$$\frac{20x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{25}{3}x\sqrt{2+3x^2+x^4}-\frac{20\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{97\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 104 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(25x\left(2+3x^2+x^4\right)-60\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-37\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 302: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 121 leaves, 3 steps):

$$\frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{7\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 69 leaves):

$$-\frac{i\sqrt{1+x^2}\,\sqrt{2+x^2}\left(5\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)}{\sqrt{2+3x^2+x^4}}$$

Problem 303: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 48 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\ \sqrt{2+3x^2+x^4}}$$

Result (type 4, 50 leaves):

$$-\frac{i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{\sqrt{2+3x^2+x^4}}$$

Problem 304: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 106 leaves, 4 steps):

$$\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{2}\ \sqrt{2+3x^2+x^4}}-\frac{5\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{14\sqrt{2}\ \sqrt{\frac{2+x^2}{1+x^2}}\ \sqrt{2+3x^2+x^4}}$$

Result (type 4, 55 leaves):

$$-\frac{i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{7\sqrt{2+3x^2+x^4}}$$

Problem 305: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 209 leaves, 9 steps):

$$\frac{5\,x\,(2+x^2)}{84\,\sqrt{2+3\,x^2+x^4}}-\frac{25\,x\,\sqrt{2+3\,x^2+x^4}}{84\,(7+5\,x^2)}-\frac{5\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{42\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{9\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{56\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}-\frac{65\,(2+x^2)\;\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{1176\,\sqrt{2}\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 208 leaves) :

$$\left(-350\,x-525\,x^3-175\,x^5-35\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5\,x^2)\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$

$$14\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5\,x^2)\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-91\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\;\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-$$

$$\left.65\,i\,x^2\,\sqrt{1+x^2}\,\sqrt{2+x^2}\;\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(588\,(7+5\,x^2)\,\sqrt{2+3\,x^2+x^4}\right)$$

Problem 306: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(7+5\,x^2)^3\,\sqrt{2+3\,x^2+x^4}}\,dx$$

Optimal (type 4, 237 leaves, 10 steps) :

$$\frac{65\,x\,(2+x^2)}{4704\,\sqrt{2+3\,x^2+x^4}}-\frac{25\,x\,\sqrt{2+3\,x^2+x^4}}{168\,(7+5\,x^2)^2}-\frac{325\,x\,\sqrt{2+3\,x^2+x^4}}{4704\,(7+5\,x^2)}-\frac{65\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2352\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{631\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{9408\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}-\frac{2525\,(2+x^2)\;\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{65\,856\,\sqrt{2}\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 186 leaves) :

$$\left(-175\,x\,\left(238+487\,x^2+314\,x^4+65\,x^6\right)-455\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(7+5\,x^2\right)^2\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+14\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(7+5\,x^2\right)^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-505\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(7+5\,x^2\right)^2\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(32928\,\left(7+5\,x^2\right)^2\,\sqrt{2+3\,x^2+x^4}\right)$$

Problem 307: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5\,x^2\right)^5}{\left(2+3\,x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 189 leaves, 6 steps):

$$\frac{7679\,x\,\left(2+x^2\right)}{2\,\sqrt{2+3\,x^2+x^4}}-\frac{x\,\left(115+179\,x^2\right)}{2\,\sqrt{2+3\,x^2+x^4}}+\frac{5000}{3}\,x\,\sqrt{2+3\,x^2+x^4}+625\,x^3\,\sqrt{2+3\,x^2+x^4}-\frac{7679\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}+\frac{15383\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 109 leaves):

$$\frac{1}{6\,\sqrt{2+3\,x^2+x^4}}\left(19655\,x+36963\,x^3+21250\,x^5+3750\,x^7-23037\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7729\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 308: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5\,x^2\right)^4}{\left(2+3\,x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 170 leaves, 5 steps):

$$\frac{637\,x\,(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x\,(145+113x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}-\frac{637\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{1067\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 104 leaves):

$$\frac{1}{6\sqrt{2+3x^2+x^4}}\left(2935\,x+4089\,x^3+1250\,x^5-1911\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-2357\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 309: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$\frac{x\,(5-11x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{261\,x\,(2+x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{261\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves):

$$-\frac{1}{2\sqrt{2+3x^2+x^4}}\left(-5\,x+11\,x^3+261\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+77\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 310: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$-\frac{17\,x\,(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x\,(25+17x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{17\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{6\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves) :

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(25x+17x^3+17\,\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-41\,\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 311: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 145 leaves, 4 steps) :

$$-\frac{x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

Result (type 4, 97 leaves) :

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(5x+x^3+\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-3\,\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 312: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps) :

$$-\frac{3x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves) :

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(5x+3x^3+3\,\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+\mathrm{i}\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 313: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 173 leaves, 9 steps):

$$\frac{x}{6\sqrt{2+3x^2+x^4}}+\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}-$$
$$\frac{9\,(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{2+3x^2+x^4}}+\frac{125\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{84\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 138 leaves):

$$\frac{1}{42\sqrt{2+3x^2+x^4}}\left(35x+14x^3+14\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$
$$\left.7\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+25\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 314: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 235 leaves, 19 steps):

$$-\frac{31x\,(2+x^2)}{56\sqrt{2+3x^2+x^4}}+\frac{x\,(20+11x^2)}{36\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)}+\frac{31\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{28\sqrt{2}\sqrt{2+3x^2+x^4}}-$$
$$\frac{463\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{336\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{375\,(2+x^2)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{784\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 208 leaves):

$$\frac{1}{1176\,(7+5x^2)\,\sqrt{2+3x^2+x^4}}\Bigg(7490\,x+10157\,x^3+3255\,x^5+651\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5x^2)\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]+182\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5x^2)\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]+1575\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\Big[\frac{10}{7},i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]+1125\,i\,x^2\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\Big[\frac{10}{7},i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]\Bigg)$$

Problem 315: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(7+5x^2)^3\,(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 263 leaves, 29 steps):

$$-\frac{5797\,x\,(2+x^2)}{28224\,\sqrt{2+3x^2+x^4}}+\frac{x\,(50+23x^2)}{216\,\sqrt{2+3x^2+x^4}}+\frac{625\,x\,\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}+\frac{41875\,x\,\sqrt{2+3x^2+x^4}}{84672\,(7+5x^2)}+\frac{5797\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\big[\text{ArcTan}[x],\frac{1}{2}\big]}{14112\,\sqrt{2}\,\sqrt{2+3x^2+x^4}}-\frac{49907\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\big[\text{ArcTan}[x],\frac{1}{2}\big]}{56448\,\sqrt{2}\,\sqrt{2+3x^2+x^4}}+\frac{192625\,(2+x^2)\,\text{EllipticPi}\big[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\big]}{395136\,\sqrt{2}\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3x^2+x^4}}$$

Result (type 4, 159 leaves):

$$\frac{1}{197568\,\sqrt{2+3x^2+x^4}}\Bigg(\frac{7\,x\,(550550+1089803\,x^2+698290\,x^4+144925\,x^6)}{(7+5x^2)^2}+40579\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]-742\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]+38525\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\Big[\frac{10}{7},i\,\text{ArcSinh}\Big[\frac{x}{\sqrt{2}}\Big],2\Big]\Bigg)$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int (7+5x^2)^4\,\sqrt{2+x^2-x^4}\,dx$$

Optimal (type 4, 116 leaves, 8 steps):

$$\frac{1}{231} x \left(177953+717372 x^2\right) \sqrt{2+x^2-x^4}-\frac{116100}{77} x \left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33} x^3 \left(2+x^2-x^4\right)^{3/2}-$$
$$\frac{625}{11} x^5 \left(2+x^2-x^4\right)^{3/2}+\frac{3764813}{231} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{539419}{77} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 112 leaves):

$$\frac{1}{231 \sqrt{2+x^2-x^4}}\left(-1037294 x-186503 x^3+1125819 x^5+231228 x^7-105925 x^9-75250 x^{11}-13125 x^{13}+\right.$$
$$\left.3764813 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-4838091 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^3 \sqrt{2+x^2-x^4} \, dx$$

Optimal (type 4, 95 leaves, 7 steps):

$$\frac{1}{63} x \left(5956+14691 x^2\right) \sqrt{2+x^2-x^4}-\frac{1825}{21} x \left(2+x^2-x^4\right)^{3/2}-$$
$$\frac{125}{9} x^3 \left(2+x^2-x^4\right)^{3/2}+\frac{79411}{63} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{8735}{21} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 107 leaves):

$$\frac{1}{63 \sqrt{2+x^2-x^4}}\left(-9988 x+9938 x^3+21660 x^5-1116 x^7-3725 x^9-875 x^{11}+\right.$$
$$\left.79411 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-106014 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 318: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4} \, dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{1}{21} x \left(275+354 x^2\right) \sqrt{2+x^2-x^4}-\frac{25}{7} x \left(2+x^2-x^4\right)^{3/2}+\frac{2045}{21} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{79}{7} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 102 leaves):

$$\frac{1}{21\sqrt{2+x^2-x^4}}\left(250\,x+683\,x^3+304\,x^5-204\,x^7-75\,x^9+2045\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-2949\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 319: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\left(7+5\,x^2\right)\sqrt{2+x^2-x^4}\,dx$$

Optimal (type 4, 46 leaves, 5 steps):

$$x\left(2+x^2\right)\sqrt{2+x^2-x^4}+7\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+3\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{2+x^2-x^4}}\left(4\,x+4\,x^3-x^5-x^7+7\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-12\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 320: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{2+x^2-x^4}\,dx$$

Optimal (type 4, 44 leaves, 5 steps):

$$\frac{1}{3}\,x\sqrt{2+x^2-x^4}+\frac{1}{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 90 leaves):

$$\frac{1}{3\sqrt{2+x^2-x^4}}\left(2\,x+x^3-x^5+i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-3\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 321: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+x^2-x^4}}{7+5\,x^2}\,dx$$

Optimal (type 4, 46 leaves, 7 steps):

$$-\frac{1}{5}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{17}{25}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{34}{175}\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 51 leaves):

$$-\frac{1}{175}\,i\,\sqrt{2}\,\left(35\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+7\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-17\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 322: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{2+x^2-x^4}}{\left(7+5\,x^2\right)^2}\,dx$$

Optimal (type 4, 74 leaves, 7 steps):

$$\frac{x\,\sqrt{2+x^2-x^4}}{14\,\left(7+5\,x^2\right)}+\frac{1}{70}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{6}{175}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{99\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{2450}$$

Result (type 4, 196 leaves):

$$\left(700\,x+350\,x^3-350\,x^5+70\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-\right.$$
$$21\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-693\,i\,\sqrt{2}\,\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-$$
$$\left.495\,i\,\sqrt{2}\,x^2\,\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(4900\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}\right)$$

Problem 323: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{2+x^2-x^4}}{\left(7+5\,x^2\right)^3}\,dx$$

Optimal (type 4, 102 leaves, 21 steps):

$$\frac{x\,\sqrt{2+x^2-x^4}}{28\,\left(7+5\,x^2\right)^2}-\frac{31\,x\,\sqrt{2+x^2-x^4}}{13\,328\,\left(7+5\,x^2\right)}-\frac{31\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{66\,640}-$$
$$\frac{269\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{166\,600}+\frac{16\,601\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{2\,332\,400}$$

Result (type 4, 244 leaves):

$$\frac{1}{4664800\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\left(181300x-17850x^3-144900x^5+54250x^7-2170\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+7021\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-813449\,i\sqrt{2}\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-1162070\,i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-415025\,i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 324: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^4\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{3x\left(2193559+7837383x^2\right)\sqrt{2+x^2-x^4}}{5005}-\frac{x\left(69817-1581440x^2\right)\left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}+\frac{124141422\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}-\frac{50794416\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}$$

Result (type 4, 122 leaves):

$$\frac{1}{15015\sqrt{2+x^2-x^4}}\left(-75836958x+48624305x^3+172881581x^5+32834763x^7-36649955x^9-24642275x^{11}-1556625x^{13}+2646875x^{15}+625625x^{17}+372424266\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-482444775\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 325: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 121 leaves, 8 steps):

$$\frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003}-\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{31072528\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{15015}-\frac{3199778\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}$$

Result (type 4, 117 leaves):

$$\frac{1}{15015\sqrt{2+x^2-x^4}}\left(-872614\,x+11078615\,x^3+13371048\,x^5-1756521\,x^7-4448240\,x^9-1027775\,x^{11}+388500\,x^{13}+144375\,x^{15}+31072528\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-41809125\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 326: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^2\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 100 leaves, 7 steps):

$$\frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}+\frac{85942}{495}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{3392}{165}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 112 leaves):

$$\frac{1}{495\sqrt{2+x^2-x^4}}\left(21254\,x+53435\,x^3+23097\,x^5-19944\,x^7-10760\,x^9+1225\,x^{11}+1125\,x^{13}+85942\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-123825\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 327: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$\frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{4432}{315}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{418}{105}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 107 leaves):

$$\frac{1}{315\sqrt{2+x^2-x^4}}\left(3134\,x+4085\,x^3-438\,x^5-1674\,x^7-110\,x^9+175\,x^{11}+4432\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-7275\,i\sqrt{4+2x^2-2x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(2+x^2-x^4\right)^{3/2} \, dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{1}{35} x \left(19+3 x^2\right) \sqrt{2+x^2-x^4}+\frac{1}{7} x \left(2+x^2-x^4\right)^{3/2}+\frac{34}{35} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{48}{35} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 102 leaves):

$$\frac{1}{35 \sqrt{2+x^2-x^4}}\left(58 x+45 x^3-31 x^5-13 x^7+5 x^9+34 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-75 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{7+5 x^2} \, dx$$

Optimal (type 4, 72 leaves, 13 steps):

$$\frac{1}{75} x \left(13-3 x^2\right) \sqrt{2+x^2-x^4}+\frac{92}{375} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{178}{625} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1156 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{4375}$$

Result (type 4, 130 leaves):

$$\frac{1}{13125 \sqrt{2+x^2-x^4}}\left(4550 x+1225 x^3-2800 x^5+525 x^7+3220 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-\right.$$
$$\left.2961 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-1734 i \sqrt{4+2 x^2-2 x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 330: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5 x^2\right)^2} \, dx$$

Optimal (type 4, 93 leaves, 21 steps):

$$-\frac{1}{75}x\sqrt{2+x^2-x^4}-\frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)}-\frac{97}{525}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+$$
$$\frac{458}{875}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{1241\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{6125}$$

Result (type 4, 201 leaves):

$$\frac{1}{36750\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\left(-14000x-11900x^3+4550x^5+2450x^7-\right.$$
$$6790\,i\sqrt{2}\left(7+5x^2\right)\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+567\,i\sqrt{2}\left(7+5x^2\right)\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+$$
$$\left.26061\,i\sqrt{2}\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+18615\,i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 331: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx$$

Optimal (type 4, 102 leaves, 27 steps):

$$-\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2}+\frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)}+\frac{191\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{9800}-$$
$$\frac{1251\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{24500}+\frac{9879\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{343000}$$

Result (type 4, 244 leaves):

$$\frac{1}{686000\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\left(485100x+636650x^3-45500x^5-197050x^7+13370\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-\right.$$
$$2541\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-484071\,i\sqrt{2}\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-$$
$$\left.691530\,i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-246975\,i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 332: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^3}{\sqrt{2+x^2-x^4}} \, dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{625}{3} x \sqrt{2+x^2-x^4} - 25 x^3 \sqrt{2+x^2-x^4} + \frac{3905}{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right] - 542 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Result (type 4, 97 leaves):

$$\frac{1}{6 \sqrt{2+x^2-x^4}} \left(-2500 x - 1550 x^3 + 1100 x^5 + 150 x^7 + 7810 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x], -\frac{1}{2}\right] - 10089 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x], -\frac{1}{2}\right]\right)$$

Problem 333: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^2}{\sqrt{2+x^2-x^4}} \, dx$$

Optimal (type 4, 46 leaves, 5 steps):

$$-\frac{25}{3} x \sqrt{2+x^2-x^4} + \frac{260}{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right] - 21 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Result (type 4, 92 leaves):

$$\frac{1}{6 \sqrt{2+x^2-x^4}} \left(-100 x - 50 x^3 + 50 x^5 + 520 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x], -\frac{1}{2}\right] - 717 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x], -\frac{1}{2}\right]\right)$$

Problem 334: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5 x^2}{\sqrt{2+x^2-x^4}} \, dx$$

Optimal (type 4, 25 leaves, 4 steps):

$$5 \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right] + 2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Result (type 4, 34 leaves):

$$\frac{i \left(10\, \text{EllipticE}\left[i\, \text{ArcSinh}[x], -\frac{1}{2}\right] - 17\, \text{EllipticF}\left[i\, \text{ArcSinh}[x], -\frac{1}{2}\right]\right)}{\sqrt{2}}$$

Problem 335: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + x^2 - x^4}}\, dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Result (type 4, 19 leaves):

$$-\frac{i\, \text{EllipticF}\left[i\, \text{ArcSinh}[x], -\frac{1}{2}\right]}{\sqrt{2}}$$

Problem 336: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7 + 5 x^2\right) \sqrt{2 + x^2 - x^4}}\, dx$$

Optimal (type 4, 17 leaves, 2 steps):

$$\frac{1}{7}\, \text{EllipticPi}\left[-\frac{10}{7}, \text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Result (type 4, 24 leaves):

$$-\frac{i\, \text{EllipticPi}\left[\frac{5}{7}, i\, \text{ArcSinh}[x], -\frac{1}{2}\right]}{7 \sqrt{2}}$$

Problem 337: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(7 + 5 x^2\right)^2 \sqrt{2 + x^2 - x^4}}\, dx$$

Optimal (type 4, 74 leaves, 8 steps):

$$-\frac{25 x \sqrt{2 + x^2 - x^4}}{476 \left(7 + 5 x^2\right)} - \frac{5}{476}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right] - \frac{1}{238}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right] + \frac{167\, \text{EllipticPi}\left[-\frac{10}{7}, \text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]}{3332}$$

Result (type 4, 196 leaves):

$$\left(-700\,x-350\,x^3+350\,x^5-70\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+119\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-1169\,i\,\sqrt{2}\,\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-835\,i\,\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(6664\,\left(7+5\,x^2\right)\sqrt{2+x^2-x^4}\right)$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5\,x^2\right)^3\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 102 leaves, 9 steps):

$$-\frac{25\,x\,\sqrt{2+x^2-x^4}}{952\,\left(7+5\,x^2\right)^2}-\frac{12\,525\,x\,\sqrt{2+x^2-x^4}}{453\,152\,\left(7+5\,x^2\right)}-\frac{2505\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{453\,152}-\frac{263\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{226\,576}+\frac{58\,915\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{3\,172\,064}$$

Result (type 4, 108 leaves):

$$\frac{1}{6\,344\,128}\left(\frac{350\,x\,\left(-7966-8993\,x^2+1478\,x^4+2505\,x^6\right)}{\left(7+5\,x^2\right)^2\sqrt{2+x^2-x^4}}-35\,070\,i\,\sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+56\,287\,i\,\sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-58\,915\,i\,\sqrt{2}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5\,x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 93 leaves, 7 steps):

$$\frac{x\,\left(1\,419\,985+1\,419\,793\,x^2\right)}{18\,\sqrt{2+x^2-x^4}}+\frac{27\,500}{3}\,x\,\sqrt{2+x^2-x^4}+625\,x^3\,\sqrt{2+x^2-x^4}-\frac{3\,482\,293}{18}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{627\,857}{6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 97 leaves):

$$\frac{1}{18\sqrt{2+x^2-x^4}}\left(1\,749\,985\,x+1\,607\,293\,x^3-153\,750\,x^5-11\,250\,x^7-3\,482\,293\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+4\,281\,654\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 340: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{x\,(83\,585+83\,489\,x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}-\frac{165\,239}{18}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{31\,921}{6}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 92 leaves):

$$\frac{1}{18\sqrt{2+x^2-x^4}}\left(91\,085\,x+87\,239\,x^3-3750\,x^5-165\,239\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+199\,977\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x\,(4945+4897\,x^2)}{18\sqrt{2+x^2-x^4}}-\frac{7147}{18}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1763}{6}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{4945\,x}{\sqrt{2+x^2-x^4}}+\frac{4897\,x^3}{\sqrt{2+x^2-x^4}}-7147\,i\sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+8076\,i\sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 342: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x \left(305+281 x^2\right)}{18 \sqrt{2+x^2-x^4}}-\frac{281}{18} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{139}{6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18} \left(\frac{305 x}{\sqrt{2+x^2-x^4}}+\frac{281 x^3}{\sqrt{2+x^2-x^4}}-281 i \sqrt{2} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+213 i \sqrt{2} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 343: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5 x^2}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x \left(25+13 x^2\right)}{18 \sqrt{2+x^2-x^4}}-\frac{13}{18} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{17}{6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18} \left(\frac{25 x}{\sqrt{2+x^2-x^4}}+\frac{13 x^3}{\sqrt{2+x^2-x^4}}-13 i \sqrt{2} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-6 i \sqrt{2} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 344: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x \left(5-x^2\right)}{18 \sqrt{2+x^2-x^4}}+\frac{1}{18} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1}{6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{5\,x}{\sqrt{2+x^2-x^4}}-\frac{x^3}{\sqrt{2+x^2-x^4}}+i\,\sqrt{2}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-3\,i\,\sqrt{2}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 345: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5\,x^2\right)\,\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 72 leaves, 8 steps):

$$\frac{x\,\left(35-16\,x^2\right)}{306\,\sqrt{2+x^2-x^4}}+\frac{8}{153}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1}{102}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{25}{238}\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 101 leaves):

$$\frac{1}{4284}\left(\frac{490\,x}{\sqrt{2+x^2-x^4}}-\frac{224\,x^3}{\sqrt{2+x^2-x^4}}+224\,i\,\sqrt{2}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-\right.$$
$$\left.357\,i\,\sqrt{2}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+225\,i\,\sqrt{2}\;\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 346: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5\,x^2\right)^2\,\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 100 leaves, 17 steps):

$$\frac{x\,\left(580-287\,x^2\right)}{10\,404\,\sqrt{2+x^2-x^4}}+\frac{625\,x\,\sqrt{2+x^2-x^4}}{16\,184\,\left(7+5\,x^2\right)}+\frac{5143\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{145\,656}+$$
$$\frac{89\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{24\,276}-\frac{10\,825\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{113\,288}$$

Result (type 4, 196 leaves):

$$\frac{1}{2\,039\,184\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}}\left(953\,260\,x+253\,386\,x^3-360\,010\,x^5+\right.$$
$$72\,002\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-111\,741\,i\,\sqrt{2}\,\left(7+5\,x^2\right)\,\sqrt{2+x^2-x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+$$
$$\left.681\,975\,i\,\sqrt{2}\,\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+487\,125\,i\,\sqrt{2}\,x^2\,\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 347: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5 x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 128 leaves, 26 steps):

$$\frac{x \left(9830-4909 x^2\right)}{353736 \sqrt{2+x^2-x^4}}+\frac{625 x \sqrt{2+x^2-x^4}}{32368 \left(7+5 x^2\right)^2}+\frac{645625 x \sqrt{2+x^2-x^4}}{15407168 \left(7+5 x^2\right)}+\frac{3086453 \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{138664512}+$$
$$\frac{60409 \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{23110752}-\frac{6898575 \,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{107850176}$$

Result (type 4, 244 leaves):

$$\frac{1}{1941303168 \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4}}$$
$$\left(3857257460 x+3876617542 x^3-737347940 x^5-1080258550 x^7+43210342 \,i \sqrt{2} \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4} \,\text{EllipticE}\left[i \,\text{ArcSinh}[x],-\frac{1}{2}\right]-\right.$$
$$67352691 \,i \sqrt{2} \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4} \,\text{EllipticF}\left[i \,\text{ArcSinh}[x],-\frac{1}{2}\right]+3042271575 \,i \sqrt{2} \sqrt{2+x^2-x^4} \,\text{EllipticPi}\left[\frac{5}{7},i \,\text{ArcSinh}[x],-\frac{1}{2}\right]+$$
$$\left.4346102250 \,i \sqrt{2} x^2 \sqrt{2+x^2-x^4} \,\text{EllipticPi}\left[\frac{5}{7},i \,\text{ArcSinh}[x],-\frac{1}{2}\right]+1552179375 \,i \sqrt{2} x^4 \sqrt{2+x^2-x^4} \,\text{EllipticPi}\left[\frac{5}{7},i \,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 348: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^4 \sqrt{4+3 x^2+x^4} \, dx$$

Optimal (type 4, 242 leaves, 7 steps):

$$\frac{51665 x \sqrt{4+3 x^2+x^4}}{33 \left(2+x^2\right)}+\frac{1}{33} x \left(18727+4516 x^2\right) \sqrt{4+3 x^2+x^4}+\frac{3050}{11} x \left(4+3 x^2+x^4\right)^{3/2}+\frac{23500}{99} x^3 \left(4+3 x^2+x^4\right)^{3/2}+\frac{625}{11} x^5 \left(4+3 x^2+x^4\right)^{3/2}-$$
$$\frac{51665 \sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{33 \sqrt{4+3 x^2+x^4}}+\frac{33159 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{11 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 354 leaves):

$$\frac{1}{396\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(663924+1257535x^2+1217475x^4+712748x^6+264075x^8+57250x^{10}+5625x^{12}\right)-\right.$$
$$154995\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+$$
$$\left.3\sqrt{2}\left(-36253i+51665\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 349: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}\,dx$$

Optimal (type 4, 221 leaves, 6 steps):

$$\frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)}+\frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}-$$
$$\frac{4717\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{21\sqrt{4+3x^2+x^4}}+\frac{1301\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 349 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(60096+93656x^2+71862x^4+30946x^6+7725x^8+875x^{10}\right)-\right.$$
$$14151\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+$$
$$\left.3\sqrt{2}\left(-3409i+4717\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(252\right.$$
$$\left.\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\, dx$$

Optimal (type 4, 198 leaves, 5 steps):

$$\frac{319\, x\sqrt{4+3x^2+x^4}}{7\left(2+x^2\right)}+\frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}-$$

$$\frac{319\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{7\sqrt{4+3x^2+x^4}}+\frac{81\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 343 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(876+1109x^2+658x^4+188x^6+25x^8\right)-\right.$$

$$319\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-35i+319\sqrt{7}\right)$$

$$\left.\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(28\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\, dx$$

Optimal (type 4, 177 leaves, 4 steps):

$$\frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}-$$

$$\frac{9\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{49\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 338 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\,(40+42x^2+19x^4+3x^6)-27\sqrt{2}\,(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\,(-7i+27\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\bigg/\left(12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{4+3x^2+x^4}\,dx$$

Optimal (type 4, 169 leaves, 4 steps):

$$\frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{7\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 331 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\,(4+3x^2+x^4)-3\sqrt{2}\,(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\,(-7i+3\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\bigg/\left(12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$$

Optimal (type 4, 322 leaves, 7 steps):

$$\frac{x\sqrt{4+3x^2+x^4}}{5\left(2+x^2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-$$

$$\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{5\sqrt{4+3x^2+x^4}}+\frac{9\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{25\sqrt{2}\sqrt{4+3x^2+x^4}}-$$

$$\frac{11\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{75\sqrt{4+3x^2+x^4}}+\frac{187\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{525\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 283 leaves) :

$$-\left(\left(\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\right.\right.$$

$$\left(35\left(3\,i+\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\left(7\,i-35\sqrt{7}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\right.$$

$$\left.\left.\left.88\,i\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\middle/\left(350\sqrt{2}\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,\sqrt{4+3x^2+x^4}\right)\right)$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$$

Optimal (type 4, 284 leaves, 7 steps) :

$$-\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)}+\frac{51\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{280\sqrt{385}}+\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{2}\sqrt{4+3x^2+x^4}}-$$

$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{289\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{9800\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 481 leaves):

$$\frac{1}{9800\sqrt{-\frac{i}{-3i+\sqrt{7}}}\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\left(700\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(4+3x^2+x^4\right)+35\left(3i+\sqrt{7}\right)\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\right.$$
$$\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)-$$
$$98i\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$
$$\left.102i\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 355: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx$$

Optimal (type 4, 312 leaves, 18 steps):

$$-\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{14999\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{344960\sqrt{385}}+\frac{139\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{43120\sqrt{2}\sqrt{4+3x^2+x^4}}-$$
$$\frac{23\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2940\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{254983\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{36220800\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 308 leaves):

$$\frac{1}{12\,073\,600\,\sqrt{4+3\,x^2+x^4}}\left(\frac{700\,x\,\left(1589+695\,x^2\right)\,\left(4+3\,x^2+x^4\right)}{\left(7+5\,x^2\right)^2}+\right.$$

$$i\,\sqrt{6+2\,i\,\sqrt{7}}\,\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\,\left(4865\,\left(3-i\,\sqrt{7}\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\left(-9597+4865\,i\,\sqrt{7}\right)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]-29\,998\,\text{EllipticPi}\left[\frac{5}{14}\,\left(3+i\,\sqrt{7}\right),\,i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)$$

Problem 356: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5\,x^2\right)^4\,\left(4+3\,x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 268 leaves, 8 steps):

$$\frac{12\,665\,086\,x\,\sqrt{4+3\,x^2+x^4}}{2145\,\left(2+x^2\right)}+\frac{7\,x\,\left(661\,429+174\,989\,x^2\right)\,\sqrt{4+3\,x^2+x^4}}{2145}+$$

$$\frac{x\,\left(452\,001+131\,080\,x^2\right)\,\left(4+3\,x^2+x^4\right)^{3/2}}{1287}+\frac{92\,150}{429}\,x\,\left(4+3\,x^2+x^4\right)^{5/2}+\frac{2250}{13}\,x^3\,\left(4+3\,x^2+x^4\right)^{5/2}+\frac{125}{3}\,x^5\,\left(4+3\,x^2+x^4\right)^{5/2}-$$

$$\frac{12\,665\,086\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\,\frac{1}{8}\right]}{2145\,\sqrt{4+3\,x^2+x^4}}+\frac{2\,383\,556\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\,\frac{1}{8}\right]}{429\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 364 leaves):

$$\frac{1}{12\,870\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,\sqrt{4+3\,x^2+x^4}}\left(2\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,x\right.$$

$$\left(180\,184\,116+391\,419\,623\,x^2+472\,235\,001\,x^4+377\,574\,349\,x^6+212\,188\,905\,x^8+83\,076\,275\,x^{10}+21\,862\,875\,x^{12}+3\,526\,875\,x^{14}+268\,125\,x^{16}\right)-$$

$$18\,997\,629\,\sqrt{2}\,\left(3\,i+\sqrt{7}\right)\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+$$

$$\left.21\,\sqrt{2}\,\left(-477\,617\,i+904\,649\,\sqrt{7}\right)\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)$$

Problem 357: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} dx$$

Optimal (type 4, 247 leaves, 7 steps):

$$\frac{4525662\, x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)}+\frac{x\left(1653701+435441\, x^2\right)\sqrt{4+3x^2+x^4}}{5005}+\frac{x\left(53504+15365\, x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001}+\frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}-\frac{4525662\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{5005\sqrt{4+3x^2+x^4}}+\frac{121826\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{143\sqrt{4+3x^2+x^4}}$$

Result (type 4, 358 leaves):

$$\frac{1}{10010\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575x^{10}+567000x^{12}+48125x^{14}\right)-2262831\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-1215823i+2262831\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 358: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} dx$$

Optimal (type 4, 226 leaves, 6 steps):

$$\frac{175346\,x\sqrt{4+3x^2+x^4}}{1155\,(2+x^2)}+\frac{x\,(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\,(6831+2240x^2)\,(4+3x^2+x^4)^{3/2}+\frac{25}{11}x\,(4+3x^2+x^4)^{5/2}-\frac{175346\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{1155\sqrt{4+3x^2+x^4}}+\frac{4628\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{33\sqrt{4+3x^2+x^4}}$$

Result (type 4, 354 leaves):

$$\frac{1}{6930\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\,(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+7875x^{12})-263019\sqrt{2}\,(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+3\sqrt{2}\,(-34209i+87673\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 359: Result unnecessarily involves imaginary or complex numbers.

$$\int (7+5x^2)\,(4+3x^2+x^4)^{3/2}\,dx$$

Optimal (type 4, 207 leaves, 5 steps):

$$\frac{2798\,x\sqrt{4+3x^2+x^4}}{105\,(2+x^2)}+\frac{1}{105}x\,(1029+289x^2)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\,(108+35x^2)\,(4+3x^2+x^4)^{3/2}-\frac{2798\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{105\sqrt{4+3x^2+x^4}}+\frac{74\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{4+3x^2+x^4}}$$

Result (type 4, 349 leaves):

$$\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(20988+28489x^2+19068x^4+7082x^6+1590x^8+175x^{10}\right)-4197\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+3\sqrt{2}\left(-567i+1399\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(630\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 360: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(4+3x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 198 leaves, 5 steps):

$$\frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)}+\frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{138\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{4+3x^2+x^4}}+\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}$$

Result (type 4, 343 leaves):

$$\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(276+303x^2+161x^4+39x^6+5x^8\right)-69\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-77i+69\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(70\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(4+3 x^2+x^4\right)^{3/2}}{7+5 x^2} \, dx$$

Optimal (type 4, 284 leaves, 12 steps):

$$\frac{94 x \sqrt{4+3 x^2+x^4}}{125 \left(2+x^2\right)}+\frac{1}{75} x \left(11+3 x^2\right) \sqrt{4+3 x^2+x^4}+\frac{44}{125} \sqrt{\frac{11}{35}} \text{ArcTan}\left[\frac{2 \sqrt{\frac{11}{35}} x}{\sqrt{4+3 x^2+x^4}}\right]-\frac{94 \sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{125 \sqrt{4+3 x^2+x^4}}+$$
$$\frac{54 \sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{125 \sqrt{4+3 x^2+x^4}}+\frac{4114 \sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticPi}\left[-\frac{9}{280},2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{13125 \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 477 leaves):

$$\frac{1}{26250 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} \sqrt{4+3 x^2+x^4}}\left(350 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} x \left(44+45 x^2+20 x^4+3 x^6\right)-\right.$$
$$4935 \sqrt{2} \left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+$$
$$7 \sqrt{2} \left(-241 i+705 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]-$$
$$\left.5808 i \sqrt{2} \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticPi}\left[\frac{5}{14} \left(3+i \sqrt{7}\right),i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(4+3 x^2+x^4\right)^{3/2}}{\left(7+5 x^2\right)^2} \, dx$$

Optimal (type 4, 305 leaves, 19 steps):

$$\frac{1}{75} x \sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+$$

$$\frac{13}{350}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{175\sqrt{4+3x^2+x^4}}+$$

$$\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{175\sqrt{4+3x^2+x^4}}+\frac{2431\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{36750\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 309 leaves) :

$$\frac{1}{18375\sqrt{4+3x^2+x^4}}\left(\frac{175x\left(23+7x^2\right)\left(4+3x^2+x^4\right)}{7+5x^2}-i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\right.$$

$$\left(105\left(3-i\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+7\left(158+15i\sqrt{7}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\right.$$

$$\left.\left.429\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\right)$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx$$

Optimal (type 4, 440 leaves, 22 steps) :

$$\frac{9x\sqrt{4+3x^2+x^4}}{1960\,(2+x^2)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\,(7+5x^2)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\,(7+5x^2)}+\frac{1347\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{7840\sqrt{385}}+\frac{111\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{24500\sqrt{2}\sqrt{4+3x^2+x^4}}-$$

$$\frac{6\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{875\sqrt{4+3x^2+x^4}}-\frac{817\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{91875\sqrt{2}\sqrt{4+3x^2+x^4}}-$$

$$\frac{22\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{13125\sqrt{4+3x^2+x^4}}+\frac{7633\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{274400\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 309 leaves):

$$\frac{1}{274400\sqrt{4+3x^2+x^4}}\left(\frac{140x\,(357+167x^2)\,(4+3x^2+x^4)}{(7+5x^2)^2}-i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\right.$$

$$\left(315\,(3-i\sqrt{7})\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+7\,(103+45i\sqrt{7})\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\right.$$

$$\left.\left.2694\,\text{EllipticPi}\left[\frac{5}{14}\,(3+i\sqrt{7}),\,i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\right)$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 187 leaves, 5 steps):

$$75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}+$$

$$\frac{15\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{13\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 337 leaves):

$$\left(100\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(12+13x^2+6x^4+x^6\right)+15\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-\sqrt{2}\left(131i+15\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{167\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{6\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 331 leaves):

$$\left(50\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(4+3x^2+x^4\right)-30\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(43i+30\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(6\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 366: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 151 leaves, 3 steps):

$$\frac{5x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{5\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{17\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 214 leaves):

$$\left(\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\left(-5\left(3\,i+\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\left(i+5\sqrt{7}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(2\sqrt{2}\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 367: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 64 leaves, 1 step):

$$\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 142 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right],\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right]}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Problem 368: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 168 leaves, 3 steps):

$$\frac{1}{4}\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{6\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{84\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 159 leaves):

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticPi}\left[-\frac{5}{14}\left(-3-i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right],\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right]}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Problem 369: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 286 leaves, 6 steps):

$$-\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)}+\frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)}+\frac{37\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{2464}+\frac{5\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{308\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{42\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{629\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{51744\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 481 leaves):

$$\frac{1}{17248\sqrt{-\frac{i}{-3i+\sqrt{7}}}\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\left(700\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(4+3x^2+x^4\right)+35\left(3i+\sqrt{7}\right)\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\right.$$
$$\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)+$$
$$98i\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$
$$\left.74i\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 370: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 314 leaves, 7 steps):

$$-\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)}+\frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2}+\frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)}-$$
$$\frac{3285\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{3035648}+\frac{555\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{379456\sqrt{2}\sqrt{4+3x^2+x^4}}-$$
$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{8624\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{18615\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{21249536\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 308 leaves):

$$\frac{1}{21\,249\,536\sqrt{4+3x^2+x^4}}\left(\frac{700\,x\left(1393+555x^2\right)\left(4+3x^2+x^4\right)}{\left(7+5x^2\right)^2}+\right.$$
$$i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(3885\left(3-i\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\left(-9401+3885i\sqrt{7}\right)\right.$$
$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+6570\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\right)$$

Problem 371: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^5}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 219 leaves, 6 steps):

$$\frac{x\left(99\,493+45\,779x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}-\frac{220\,779\,x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}+$$
$$\frac{220\,779\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{130\,729\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{12\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 339 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(858\,479+767\,337x^2+297\,500x^4+52\,500x^6\right)+\right.$$
$$662\,337\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$
$$\left.\sqrt{2}\left(975\,947i+662\,337\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(336\right.$$
$$\left.\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 372: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^4}{\left(4+3 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 200 leaves, 5 steps):

$$\frac{x \left(2719-4023 x^2\right)}{28 \sqrt{4+3 x^2+x^4}}+\frac{625}{3} x \sqrt{4+3 x^2+x^4}+\frac{14523 x \sqrt{4+3 x^2+x^4}}{28 \left(2+x^2\right)}-\frac{14523 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{14 \sqrt{2} \sqrt{4+3 x^2+x^4}}+\frac{4243 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{12 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 333 leaves):

$$\left(4 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} x \left(78157+40431 x^2+17500 x^4\right)-43569 \sqrt{2} \left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+\sqrt{2} \left(186179 i+43569 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right) \Bigg/ \left(336 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} \sqrt{4+3 x^2+x^4}\right)$$

Problem 373: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^3}{\left(4+3 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(2323+949 x^2\right)}{28 \sqrt{4+3 x^2+x^4}}+\frac{4449 x \sqrt{4+3 x^2+x^4}}{28\left(2+x^2\right)}-\frac{4449\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14 \sqrt{2} \sqrt{4+3 x^2+x^4}}+\frac{973\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 328 leaves):

$$\left(-4 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\, x\left(2323+949 x^2\right)-4449 \sqrt{2}\left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+\sqrt{2}\left(3899 i+4449 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right) \Bigg/ \left(112 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} \sqrt{4+3 x^2+x^4}\right)$$

Problem 374: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^2}{\left(4+3 x^2+x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(9-113 x^2\right)}{28 \sqrt{4+3 x^2+x^4}}-\frac{113 x \sqrt{4+3 x^2+x^4}}{28\left(2+x^2\right)}+\frac{113\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14 \sqrt{2} \sqrt{4+3 x^2+x^4}}+\frac{9\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 329 leaves):

$$\left(4\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\;x\left(-9+113\,x^2\right)+\right.$$
$$113\sqrt{2}\left(3\,i+\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]-\sqrt{2}\left(1043\,i+113\sqrt{7}\right)$$
$$\left.\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\Big/\left(112\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3\,x^2+x^4}\right)$$

Problem 375: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5\,x^2}{\left(4+3\,x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$\frac{x\left(53+19\,x^2\right)}{28\sqrt{4+3\,x^2+x^4}}-\frac{19\,x\sqrt{4+3\,x^2+x^4}}{28\left(2+x^2\right)}+\frac{19\left(2+x^2\right)\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3\,x^2+x^4}}-\frac{3\left(2+x^2\right)\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4\sqrt{2}\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 329 leaves):

$$\left(4\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\;x\left(53+19\,x^2\right)+\right.$$
$$19\sqrt{2}\left(3\,i+\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]-\sqrt{2}\left(49\,i+19\sqrt{7}\right)$$
$$\left.\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\Big/\left(112\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3\,x^2+x^4}\right)$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(4+3\,x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{3\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 328 leaves):

$$\left(-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(1+3x^2\right)-3\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-7i+3\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Bigg/\left(112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 284 leaves, 8 steps):

$$-\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}}+\frac{x\sqrt{4+3x^2+x^4}}{77\left(2+x^2\right)}+\frac{25}{176}\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{77\sqrt{4+3x^2+x^4}}-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{12\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{425\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 483 leaves):

$$\frac{1}{616\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(-26\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x-8\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x^3-\right.$$

$$2\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+$$

$$\sqrt{2}\left(7i+2\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$

$$\left.25i\sqrt{2}\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 312 leaves, 15 steps):

$$\frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}+$$

$$\frac{575\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{108416}+\frac{199\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{13552\sqrt{2}\sqrt{4+3x^2+x^4}}-$$

$$\frac{2\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{231\sqrt{4+3x^2+x^4}}+\frac{9775\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2276736\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 311 leaves):

$$\frac{1}{758912\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\left(28x\left(2836+2633x^2+995x^4\right)+i\sqrt{6+2i\sqrt{7}}\left(7+5x^2\right)\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\right.$$
$$\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(1393\left(3-i\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+7\left(101+199i\sqrt{7}\right)\right.$$
$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-1150\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\right)$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 340 leaves, 22 steps):

$$\frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}}-\frac{18159x\sqrt{4+3x^2+x^4}}{33392128\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2}+\frac{51875x\sqrt{4+3x^2+x^4}}{33392128\left(7+5x^2\right)}-$$
$$\frac{529425\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{133568512}+\frac{18159\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{16696064\sqrt{2}\sqrt{4+3x^2+x^4}}+$$
$$\frac{843\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{379456\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{3000075\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{934979584\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 320 leaves):

$$\frac{1}{934979584\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\left(28x\left(4496212+5811451x^2+2838330x^4+453975x^6\right)+3\,\mathrm{i}\sqrt{6+2\,\mathrm{i}\sqrt{7}}\left(7+5x^2\right)^2\sqrt{1-\frac{2\,\mathrm{i}\,x^2}{-3\,\mathrm{i}+\sqrt{7}}}\sqrt{1+\frac{2\,\mathrm{i}\,x^2}{3\,\mathrm{i}+\sqrt{7}}}\right.$$
$$\left(42371\left(3-\mathrm{i}\sqrt{7}\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{2\,\mathrm{i}}{-3\,\mathrm{i}+\sqrt{7}}}\,x\right],\frac{3\,\mathrm{i}-\sqrt{7}}{3\,\mathrm{i}+\sqrt{7}}\right]+7\,\mathrm{i}\left(23633\,\mathrm{i}+6053\sqrt{7}\right)\right.$$
$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{2\,\mathrm{i}}{-3\,\mathrm{i}+\sqrt{7}}}\,x\right],\frac{3\,\mathrm{i}-\sqrt{7}}{3\,\mathrm{i}+\sqrt{7}}\right]+352950\,\text{EllipticPi}\left[\frac{5}{14}\left(3+\mathrm{i}\sqrt{7}\right),\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{2\,\mathrm{i}}{-3\,\mathrm{i}+\sqrt{7}}}\,x\right],\frac{3\,\mathrm{i}-\sqrt{7}}{3\,\mathrm{i}+\sqrt{7}}\right]\right)\right)$$

Problem 380: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\left(d+e\,x^2\right)^3}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 467 leaves, 5 steps):

$$\frac{e^2\left(15\,c\,d-4\,b\,e\right)x\sqrt{a+b\,x^2+c\,x^4}}{15\,c^2}+\frac{e^3x^3\sqrt{a+b\,x^2+c\,x^4}}{5\,c}+\frac{e\left(45\,c^2d^2+8\,b^2e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)x\sqrt{a+b\,x^2+c\,x^4}}{15\,c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{15\,c^{11/4}\sqrt{a+b\,x^2+c\,x^4}}$$
$$a^{1/4}e\left(45\,c^2d^2+8\,b^2e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]+$$
$$\frac{1}{30\,c^{11/4}\sqrt{a+b\,x^2+c\,x^4}}a^{1/4}\left(\frac{\sqrt{c}\left(15\,c^2d^3-15\,a\,c\,d\,e^2+4\,a\,b\,e^3\right)}{\sqrt{a}}+e\left(45\,c^2d^2+8\,b^2e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)\right)$$
$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 1825 leaves):

$$\left(-\frac{e^2\left(-15\,c\,d+4\,b\,e\right)x}{15\,c^2}+\frac{e^3x^3}{5\,c}\right)\sqrt{a+b\,x^2+c\,x^4}+$$
$$\frac{1}{15\,c^2}\left(\left(45\,\mathrm{i}\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.\right.\right.\right.$$

$$\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]-\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Big(15\,\mathrm{i}\,b\left(-b+\sqrt{b^2-4ac}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4ac}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4ac}}}$$

$$\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Big(9\,\mathrm{i}\,a\left(-b+\sqrt{b^2-4ac}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4ac}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4ac}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]-\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big/$$

$$\left(2\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\Big(2\,\mathrm{i}\,\sqrt{2}\,b^2\left(-b+\sqrt{b^2-4ac}\right)e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4ac}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4ac}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big/\left(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Big(15\,\mathrm{i}\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4ac}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\Big],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(15\,i\,a\,c\,d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$
$$\left(2\,i\,\sqrt{2}\,a\,b\,e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 356 leaves, 4 steps):

$$\frac{e^2\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c}+\frac{2\,e\,(3\,c\,d-b\,e)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$
$$\frac{2\,a^{1/4}\,e\,(3\,c\,d-b\,e)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{3\,c^{7/4}\sqrt{a+b\,x^2+c\,x^4}}+\frac{1}{6\,c^{7/4}\sqrt{a+b\,x^2+c\,x^4}}$$
$$a^{1/4}\left(2\,e\,(3\,c\,d-b\,e)+\frac{\sqrt{c}\,\left(3\,c\,d^2-a\,e^2\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 488 leaves):

$$\frac{1}{6\,c^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}\left(2\,c\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,e^2\,x\,\left(a+b\,x^2+c\,x^4\right)-i\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\,\left(-3\,c\,d+b\,e\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+i\,\left(-3\,c^2\,d^2+b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^2+c\,e\,\left(3\,b\,d-3\,\sqrt{b^2-4\,a\,c}\,d+a\,e\right)\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 382: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{e\,x\,\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4}\,e\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{a^{1/4}\,\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 302 leaves):

$$\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.$$
$$\left(\left(-b+\sqrt{b^2-4ac}\right)e\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)\right.$$
$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\right)\Bigg/\left(2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 401 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right]}{2\sqrt{d}\sqrt{cd^2-bde+ae^2}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+bx^2+cx^4}}-$$
$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\sqrt{c}\,de},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Bigg/$$
$$\left(4c^{1/4}d\left(cd^2-ae^2\right)\sqrt{a+bx^2+cx^4}\right)$$

Result (type 4, 214 leaves):

$$-\left(\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\Bigg/\right.$$
$$\left.\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right)\right)$$

Problem 384: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^2 \sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 718 leaves, 6 steps):

$$-\frac{\sqrt{c}\, e\, x \sqrt{a+b x^2+c x^4}}{2 d \left(c d^2-b d e+a e^2\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{e^2 x \sqrt{a+b x^2+c x^4}}{2 d \left(c d^2-b d e+a e^2\right) \left(d+e x^2\right)}+\frac{\sqrt{e}\, \left(3 c d^2-e \left(2 b d-a e\right)\right) \text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\, x}{\sqrt{d} \sqrt{e} \sqrt{a+b x^2+c x^4}}\right]}{4 d^{3/2} \left(c d^2-b d e+a e^2\right)^{3/2}}+$$

$$\frac{a^{1/4} c^{1/4} e \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{2 d \left(c d^2-b d e+a e^2\right) \sqrt{a+b x^2+c x^4}}+$$

$$\frac{c^{1/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{2 a^{1/4} d \left(\sqrt{c}\, d-\sqrt{a}\, e\right) \sqrt{a+b x^2+c x^4}}-$$

$$\left(\left(\sqrt{c}\, d+\sqrt{a}\, e\right) \left(3 c d^2-e \left(2 b d-a e\right)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4 \sqrt{a} \sqrt{c}\, d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(8 a^{1/4} c^{1/4} d^2 \left(\sqrt{c}\, d-\sqrt{a}\, e\right) \left(c d^2-b d e+a e^2\right) \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 1069 leaves):

$$
\begin{aligned}
&\frac{1}{8\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;d\left(c\,d^3+d\,e\left(-b\,d+a\,e\right)\right)\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}} \\
&\left(4\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;d\,e^2\,x\left(a+b\,x^2+c\,x^4\right)+i\sqrt{2}\left(b-\sqrt{b^2-4\,a\,c}\right)d\,e\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(d+e\,x^2\right)\right. \\
&\quad\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+ \\
&\quad 2\,i\sqrt{2}\,c\,d^2\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(d+e\,x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]- \\
&\quad 6\,i\sqrt{2}\,c\,d^2\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(d+e\,x^2\right)\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\right. \\
&\quad\left. i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+4\,i\sqrt{2}\,b\,d\,e\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(d+e\,x^2\right) \\
&\quad\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-2\,i\sqrt{2}\,a\,e^2\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}} \\
&\quad\left.\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(d+e\,x^2\right)\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)
\end{aligned}
$$

Problem 385: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^3}{\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 553 leaves, 6 steps):

$$-\frac{e^2 (15 c d+4 b e) x \sqrt{a+b x^2-c x^4}}{15 c^2}-\frac{e^3 x^3 \sqrt{a+b x^2-c x^4}}{5 c}-$$

$$\left(\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}}\; e \left(45 c^2 d^2+8 b^2 e^2+3 c e (10 b d+3 a e)\right) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\right.$$

$$\left.\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(30 \sqrt{2}\, c^{7/2} \sqrt{a+b x^2-c x^4}\right)+$$

$$\left(\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}} \left(\frac{2 c \left(15 c^2 d^3+15 a c d e^2+4 a b e^3\right)}{b-\sqrt{b^2+4 a c}}+e \left(45 c^2 d^2+8 b^2 e^2+3 c e (10 b d+3 a e)\right)\right)\right.$$

$$\left.\sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(30 \sqrt{2}\, c^{7/2} \sqrt{a+b x^2-c x^4}\right)$$

Result (type 4, 596 leaves):

$$\frac{1}{60 c^3 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}}$$

$$\left(-4 c \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; e^2 x \left(a+b x^2-c x^4\right) \left(4 b e+3 c \left(5 d+e x^2\right)\right)-i \sqrt{2} \left(-b+\sqrt{b^2+4 a c}\right) e \left(45 c^2 d^2+8 b^2 e^2+3 c e (10 b d+3 a e)\right)\right.$$

$$\sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+i \sqrt{2} \left(-30 c^3 d^3+\right.$$

$$\left.8 b^2 \left(-b+\sqrt{b^2+4 a c}\right) e^3+15 c^2 d e \left(-3 b d+3 \sqrt{b^2+4 a c}\; d-2 a e\right)+c e^2 \left(-30 b^2 d+30 b \sqrt{b^2+4 a c}\; d-17 a b e+9 a \sqrt{b^2+4 a c}\; e\right)\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^2}{\sqrt{a+b x^2-c x^4}}\, dx$$

Optimal (type 4, 454 leaves, 5 steps):

$$-\frac{e^2 x \sqrt{a+b x^2-c x^4}}{3 c}-\frac{1}{3 \sqrt{2}\, c^{5/2} \sqrt{a+b x^2-c x^4}}\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}}\; e\,(3 c d+b e) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+\frac{1}{3 \sqrt{2}\, c^{5/2} \sqrt{a+b x^2-c x^4}} \sqrt{b+\sqrt{b^2+4 a c}}\left(3 c^2 d^2+b\left(b-\sqrt{b^2+4 a c}\right) e^2+c e\left(3 b d-3 \sqrt{b^2+4 a c}\, d+a e\right)\right) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]$$

Result (type 4, 503 leaves):

$$\frac{1}{6 c^2 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}}\left(2 c \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; e^2 x\left(-a-b x^2+c x^4\right)-i \sqrt{2}\left(-b+\sqrt{b^2+4 a c}\right) e\,(3 c d+b e) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+i \sqrt{2}\left(-3 c^2 d^2+b\left(-b+\sqrt{b^2+4 a c}\right) e^2-c e\left(3 b d-3 \sqrt{b^2+4 a c}\, d+a e\right)\right) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{a+b x^2-c x^4}}\, dx$$

Optimal (type 4, 385 leaves, 4 steps):

$$-\frac{1}{2\sqrt{2}\,c^{3/2}\sqrt{a+b\,x^2-c\,x^4}}\left(b-\sqrt{b^2+4\,a\,c}\right)\sqrt{b+\sqrt{b^2+4\,a\,c}}\;e\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]+\frac{1}{2\sqrt{2}\,c^{3/2}\sqrt{a+b\,x^2-c\,x^4}}\sqrt{b+\sqrt{b^2+4\,a\,c}}\left(2\,c\,d+\left(b-\sqrt{b^2+4\,a\,c}\right)e\right)\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]$$

Result (type 4, 293 leaves):

$$-\left(\left(i\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\left(\left(-b+\sqrt{b^2+4\,a\,c}\right)e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]+\left(2\,c\,d+\left(b-\sqrt{b^2+4\,a\,c}\right)e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\right)\Big/\left(2\sqrt{2}\,c\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{a+b\,x^2-c\,x^4}\right)\right)$$

Problem 388: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 197 leaves, 2 steps):

$$\frac{1}{\sqrt{2}\,\sqrt{c}\,d\sqrt{a+b\,x^2-c\,x^4}}\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\;\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4\,a\,c}\right)e}{2\,c\,d},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]$$

Result (type 4, 205 leaves):

$$-\left(\left(i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\,-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right]\right)\Big/\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,d\sqrt{a+bx^2-cx^4}\right)\right)$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2-cx^4}}\,dx$$

Optimal (type 4, 718 leaves, 8 steps):

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+bde-ae^2\right)\left(d+ex^2\right)}+$$
$$\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\;e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/$$
$$\left(4\sqrt{2}\sqrt{c}\,d\left(cd^2+e\left(bd-ae\right)\right)\sqrt{a+bx^2-cx^4}\right)-\left(\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\right.$$
$$\left.\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(4\sqrt{2}\sqrt{c}\,d\left(cd^2+e\left(bd-ae\right)\right)\sqrt{a+bx^2-cx^4}\right)+$$
$$\left(\sqrt{b+\sqrt{b^2+4ac}}\left(3cd^2+e\left(2bd-ae\right)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right.$$
$$\left.\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(2\sqrt{2}\sqrt{c}\,d^2\left(cd^2+e\left(bd-ae\right)\right)\sqrt{a+bx^2-cx^4}\right)$$

Result (type 4, 1341 leaves):

$$
\begin{aligned}
&-\frac{e^2 x \sqrt{a+b x^2-c x^4}}{2 d \left(c d^2+b d e-a e^2\right) \left(d+e x^2\right)}+\frac{1}{2 c d^2+2 b d e-a e^2-2 c d e x^2-c e^2 x^4} \left(d+e x^2\right) \sqrt{a+b x^2-c x^4} \\
&\quad \left(-\frac{c}{2 \left(c d^2+b d e-a e^2\right) \sqrt{a+b x^2-c x^4}}-\frac{c e x^2}{2 d \left(c d^2+b d e-a e^2\right) \sqrt{a+b x^2-c x^4}}+\frac{3 c d^2+2 b d e-a e^2}{2 d \left(c d^2+b d e-a e^2\right) \left(d+e x^2\right) \sqrt{a+b x^2-c x^4}}\right) \\
&\quad \Bigg(\Bigg(\mathbb{i} \left(-b+\sqrt{b^2+4 a c}\right) e \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}} \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]- \\
&\qquad \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\Bigg)\Bigg)\Bigg/\left(2 \sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)+ \\
&\quad \frac{\mathbb{i}\, c d \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\, \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]}{\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}}-\Bigg(3\, \mathbb{i}\, c d \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \\
&\qquad \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\, \text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4 a c}\right) e}{2 c d},\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\Bigg)\Bigg/ \\
&\qquad \left(\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)-\Bigg(\mathbb{i} \sqrt{2}\, b e \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}} \\
&\qquad \text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4 a c}\right) e}{2 c d},\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\Bigg)\Bigg/\left(\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)+ \\
&\quad \Bigg(\mathbb{i}\, a e^2 \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\, \text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4 a c}\right) e}{2 c d},\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],\right. \\
&\qquad \left.-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\Bigg)\Bigg/\left(\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, d \sqrt{a+b x^2-c x^4}\right)\Bigg)
\end{aligned}
$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{-a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 479 leaves, 5 steps):

$$\frac{\left(b-\sqrt{b^2+4 a c}\right) e x \left(1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}\right)}{2 c \sqrt{-a+b x^2+c x^4}}-\frac{\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}}\ e \left(1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{2} \sqrt{c}\ x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],-\frac{2 \sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]}{2 \sqrt{2}\ c^{3/2} \sqrt{\frac{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}} \sqrt{-a+b x^2+c x^4}}+$$

$$\frac{\sqrt{b+\sqrt{b^2+4 a c}}\ d \left(1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{2} \sqrt{c}\ x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],-\frac{2 \sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]}{\sqrt{2} \sqrt{c} \sqrt{\frac{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}} \sqrt{-a+b x^2+c x^4}}$$

Result (type 4, 304 leaves):

$$\left(i \sqrt{\frac{b+\sqrt{b^2+4 a c}+2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\right.$$
$$\left(\left(-b+\sqrt{b^2+4 a c}\right) e \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+\left(-2 c d+\left(b-\sqrt{b^2+4 a c}\right) e\right)\right.$$
$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right],\frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)\right) \Big/ \left(2 \sqrt{2}\ c \sqrt{\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{-a+b x^2+c x^4}\right)$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right) \sqrt{-a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 204 leaves, 2 steps):

$$\left(\sqrt{-b+\sqrt{b^2+4ac}}\ \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\ \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\ \text{EllipticPi}\left[\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\ \text{ArcSin}\left[\frac{\sqrt{2}\ \sqrt{c}\ x}{\sqrt{-b+\sqrt{b^2+4ac}}}\right],\ \frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right]\right)\Big/\left(\sqrt{2}\ \sqrt{c}\ d\sqrt{-a+bx^2+cx^4}\right)$$

Result (type 4, 216 leaves) :

$$-\left(\left(i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\ \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\ \text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\ i\ \text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\ x\right],\ \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(\sqrt{2}\ \sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\ d\sqrt{-a+bx^2+cx^4}\right)\right)$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Optimal (type 4, 293 leaves, 3 steps) :

$$-\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\ x^2\right)}-\frac{a^{1/4}e\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\ \frac{1}{4}\left(2+\frac{b}{\sqrt{a}\ \sqrt{c}}\right)\right]}{c^{3/4}\sqrt{-a+bx^2-cx^4}}+$$

$$\frac{a^{1/4}\left(\frac{\sqrt{c}\ d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\ \frac{1}{4}\left(2+\frac{b}{\sqrt{a}\ \sqrt{c}}\right)\right]}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

Result (type 4, 295 leaves) :

$$-\left(\left(i\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.\right.$$

$$\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\left(2\,c\,d+\left(b-\sqrt{b^2-4\,a\,c}\right)e\right)\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\middle/\left(2\,\sqrt{2}\,c\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{-a+b\,x^2-c\,x^4}\right)\right)$$

Problem 393: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)\sqrt{-a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 412 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^2-e\,(b\,d+a\,e)}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a+b\,x^2-c\,x^4}}\right]}{2\,\sqrt{d}\,\sqrt{-c\,d^2-e\,(b\,d+a\,e)}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a+b\,x^2-c\,x^4}}-$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(4\,c^{1/4}\,d\left(c\,d^2-a\,e^2\right)\sqrt{-a+b\,x^2-c\,x^4}\right)$$

Result (type 4, 207 leaves):

$$-\left(\left(i\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],-\frac{b+\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\middle/\right.$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,d\,\sqrt{-a+b\,x^2-c\,x^4}\right)\right)$$

Problem 394: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^3}{\sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 229 leaves, 5 steps):

$$\frac{3 e \left(5 d^2-10 d e+6 e^2\right) x \left(2+x^2\right)}{5 \sqrt{2+3 x^2+x^4}}+\frac{1}{5} (5 d-4 e) e^2 x \sqrt{2+3 x^2+x^4}+\frac{1}{5} e^3 x^3 \sqrt{2+3 x^2+x^4}-$$
$$\frac{3 \sqrt{2} e \left(5 d^2-10 d e+6 e^2\right) \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5 \sqrt{2+3 x^2+x^4}}+\frac{\left(5 d^3-10 d e^2+8 e^3\right) \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5 \sqrt{2} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 154 leaves):

$$\frac{1}{5 \sqrt{2+3 x^2+x^4}}\left(e^2 x \left(2+3 x^2+x^4\right) \left(5 d+e \left(-4+x^2\right)\right)-3 i e \left(5 d^2-10 d e+6 e^2\right) \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$
$$\left.5 i \left(d^3-3 d^2 e+4 d e^2-2 e^3\right) \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^2}{\sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 168 leaves, 4 steps):

$$\frac{2 (d-e) e x \left(2+x^2\right)}{\sqrt{2+3 x^2+x^4}}+\frac{1}{3} e^2 x \sqrt{2+3 x^2+x^4}-$$
$$\frac{2 \sqrt{2} (d-e) e \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3 x^2+x^4}}+\frac{\left(3 d^2-2 e^2\right) \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3 \sqrt{2} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 127 leaves):

$$\frac{1}{3 \sqrt{2+3 x^2+x^4}}\left(e^2 x \left(2+3 x^2+x^4\right)-6 i (d-e) e \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$
$$\left.i \left(3 d^2-6 d e+4 e^2\right) \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 122 leaves, 3 steps):

$$\frac{e x \left(2+x^2\right)}{\sqrt{2+3 x^2+x^4}}-\frac{\sqrt{2}\, e \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3 x^2+x^4}}+\frac{d \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 73 leaves):

$$-\frac{i \sqrt{1+x^2} \sqrt{2+x^2} \left(e\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+(d-e)\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)}{\sqrt{2+3 x^2+x^4}}$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right) \sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$\frac{\left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\, (d-e) \sqrt{2+3 x^2+x^4}}-\frac{e \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticPi}\left[1-\frac{e}{d},\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\, d\, (d-e) \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 59 leaves):

$$-\frac{i \sqrt{1+x^2} \sqrt{2+x^2}\, \text{EllipticPi}\left[\frac{2 e}{d},i\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{d \sqrt{2+3 x^2+x^4}}$$

Problem 398: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^2 \sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 316 leaves, 9 steps):

$$-\frac{e\,x\,(2+x^2)}{2\,d\,(d^2-3\,d\,e+2\,e^2)\,\sqrt{2+3\,x^2+x^4}}+\frac{e^2\,x\,\sqrt{2+3\,x^2+x^4}}{2\,d\,(d^2-3\,d\,e+2\,e^2)\,(d+e\,x^2)}+\frac{e\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,d\,(d-2\,e)\,(d-e)\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{(2\,d-e)\,(1+x^2)\,\sqrt{\frac{2+x^2}{2+2\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\,d\,(d-e)^2\,\sqrt{2+3\,x^2+x^4}}-\frac{e\,(3\,d^2-6\,d\,e+2\,e^2)\,(2+x^2)\,\text{EllipticPi}\left[1-\frac{e}{d},\text{ArcTan}[x],\frac{1}{2}\right]}{2\,\sqrt{2}\,d^2\,(d-2\,e)\,(d-e)^2\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 175 leaves):

$$\frac{1}{2\,d\,\sqrt{2+3\,x^2+x^4}}\left(\frac{e^2\,x\,(2+3\,x^2+x^4)}{(d^2-3\,d\,e+2\,e^2)\,(d+e\,x^2)}+\frac{1}{d\,(d-2\,e)\,(d-e)}\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\right.$$
$$\left.\left(d\,e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+d\,(d-e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+(-3\,d^2+6\,d\,e-2\,e^2)\,\text{EllipticPi}\left[\frac{2\,e}{d},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\right)$$

Problem 400: Result more than twice size of optimal antiderivative.

$$\int (c+e\,x^2)^3\,(a+c\,x^2+b\,x^4)^p\,dx$$

Optimal (type 6, 498 leaves, 8 steps):

$$\frac{c\,e^2\,(21\,b-5\,e+12\,b\,p-2\,e\,p)\,x\,(a+c\,x^2+b\,x^4)^{1+p}}{b^2\,(5+4\,p)\,(7+4\,p)}+\frac{e^3\,x^3\,(a+c\,x^2+b\,x^4)^{1+p}}{b\,(7+4\,p)}+\frac{1}{b^2\,(5+4\,p)\,(7+4\,p)}$$
$$c\,(a\,e^3\,(5+2\,p)-3\,a\,b\,e^2\,(7+4\,p)+b^2\,c^2\,(35+48\,p+16\,p^2))\,x\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}$$
$$(a+c\,x^2+b\,x^4)^p\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]+\frac{1}{3\,b^2\,(5+4\,p)\,(7+4\,p)}$$
$$e\,(c^2\,e^2\,(15+16\,p+4\,p^2)+3\,b^2\,c^2\,(35+48\,p+16\,p^2)-3\,b\,e\,(a\,e\,(5+4\,p)+c^2\,(21+26\,p+8\,p^2)))\,x^3\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p}$$
$$\left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}(a+c\,x^2+b\,x^4)^p\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]$$

Result (type 6, 1871 leaves):

$$\left(3\times 4^{-1-p}\,c^3\,\left(c+\sqrt{-4\,a\,b+c^2}\right)\,x\,\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c+\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(\frac{c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{-1+p}\right.$$

$$\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2\left(a+c\,x^2+b\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg/$$

$$\Bigg(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\Bigg(-3\,a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+$$

$$p\,x^2\Bigg(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-$$

$$\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(5\times 2^{-2-p}\,b\,c^2\left(c+\sqrt{-4\,a\,b+c^2}\right)e\,x^3\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2$$

$$\left(a+c\,x^2+b\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg/$$

$$\Bigg(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\left(c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2\right)\Bigg(-5\,a\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+$$

$$p\,x^2\Bigg(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-$$

$$\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{5}{2},-p,1-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(21\times 2^{-2-p}\,b\,c\left(c+\sqrt{-4\,a\,b+c^2}\right)e^2\,x^5\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2$$

$$\left(a+c\,x^2+b\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg/$$

$$\Bigg(5\left(-c+\sqrt{-4\,a\,b+c^2}\right)\left(c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2\right)\Bigg(-7\,a\,\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+$$

$$p\,x^2\Bigg(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{7}{2},1-p,-p,\frac{9}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-$$

$$\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{7}{2},-p,1-p,\frac{9}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(9\times 2^{-2-p}\,b\left(c+\sqrt{-4\,a\,b+c^2}\right)e^3\,x^7\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2$$

$$\left(a+c x^2+b x^4\right)^{-1+p} \text{AppellF1}\left[\frac{7}{2},-p,-p,\frac{9}{2},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}},\frac{2 b x^2}{-c+\sqrt{-4 a b+c^2}}\right]\Bigg)\Bigg/$$

$$\left(7\left(-c+\sqrt{-4 a b+c^2}\right)\left(c+\sqrt{-4 a b+c^2}+2 b x^2\right)\left(-9 a \text{AppellF1}\left[\frac{7}{2},-p,-p,\frac{9}{2},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}},\frac{2 b x^2}{-c+\sqrt{-4 a b+c^2}}\right]+\right.\right.$$

$$p x^2\left(\left(-c+\sqrt{-4 a b+c^2}\right) \text{AppellF1}\left[\frac{9}{2},1-p,-p,\frac{11}{2},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}},\frac{2 b x^2}{-c+\sqrt{-4 a b+c^2}}\right]-\right.$$

$$\left.\left.\left.\left(c+\sqrt{-4 a b+c^2}\right) \text{AppellF1}\left[\frac{9}{2},-p,1-p,\frac{11}{2},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}},\frac{2 b x^2}{-c+\sqrt{-4 a b+c^2}}\right]\right)\right)\right)$$

Problem 401: Result more than twice size of optimal antiderivative.

$$\int\left(c+e x^2\right)^2\left(a+c x^2+b x^4\right)^p \, dx$$

Optimal (type 6, 358 leaves, 7 steps):

$$\frac{e^2 x\left(a+c x^2+b x^4\right)^{1+p}}{b(5+4 p)}-\frac{1}{b(5+4 p)}\left(a e^2-b c^2(5+4 p)\right) x\left(1+\frac{2 b x^2}{c-\sqrt{-4 a b+c^2}}\right)^{-p}\left(1+\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}}\right)^{-p}$$

$$\left(a+c x^2+b x^4\right)^p \text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2 b x^2}{c-\sqrt{-4 a b+c^2}},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}}\right]+\frac{1}{3 b(5+4 p)} c e(10 b-3 e+8 b p-2 e p) x^3$$

$$\left(1+\frac{2 b x^2}{c-\sqrt{-4 a b+c^2}}\right)^{-p}\left(1+\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}}\right)^{-p}\left(a+c x^2+b x^4\right)^p \text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2 b x^2}{c-\sqrt{-4 a b+c^2}},-\frac{2 b x^2}{c+\sqrt{-4 a b+c^2}}\right]$$

Result (type 6, 1001 leaves):

$$\frac{1}{15}\times 2^{-3-p}\left(c+\sqrt{-4ab+c^2}\right)x\left(\frac{c-\sqrt{-4ab+c^2}}{2b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{b}\right)^{1+p}\left(-2a+\left(-c+\sqrt{-4ab+c^2}\right)x^2\right)$$

$$(a+cx^2+bx^4)^{-1+p}\Bigg(-\Bigg(\Bigg(45c^2\,\text{AppellF1}\Big[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg/$$

$$\Bigg(3a\,\text{AppellF1}\Big[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]+px^2\Bigg(\left(c-\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{3}{2},1-p,-p,\frac{5}{2},$$

$$-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]+\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$ex^2\Bigg(-\Bigg(\Bigg(50c\,\text{AppellF1}\Big[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg/\Bigg(5a\,\text{AppellF1}\Big[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},$$

$$\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]+px^2\Bigg(\left(c-\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]+$$

$$\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{5}{2},-p,1-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(21ex^2\,\text{AppellF1}\Big[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg/\Bigg(-7a\,\text{AppellF1}\Big[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},$$

$$\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]+px^2\Bigg(\left(-c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{7}{2},1-p,-p,\frac{9}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]-$$

$$\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\Big[\frac{7}{2},-p,1-p,\frac{9}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 402: Result more than twice size of optimal antiderivative.

$$\int (c+ex^2)\,(a+cx^2+bx^4)^p\,dx$$

Optimal (type 6, 274 leaves, 6 steps):

$$cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\text{AppellF1}\Big[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\Big]+$$

$$\frac{1}{3}ex^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\text{AppellF1}\Big[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\Big]$$

Result (type 6, 706 leaves):

$$\frac{1}{3}\times 2^{-3-p}\left(c+\sqrt{-4ab+c^2}\right)x\left(\frac{c-\sqrt{-4ab+c^2}}{2b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{b}\right)^{1+p}\left(-2a+\left(-c+\sqrt{-4ab+c^2}\right)x^2\right)$$
$$\left(a+cx^2+bx^4\right)^{-1+p}\left(-\left(\left(9c\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\Big/\right.\right.$$
$$\left(3a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+px^2\left(\left(c-\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)+$$
$$\left(5ex^2\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\Big/\left(-5a\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\right.\right.$$
$$\left.\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+px^2\left(\left(-c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]-\right.$$
$$\left.\left.\left.\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{5}{2},-p,1-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)$$

Problem 403: Result more than twice size of optimal antiderivative.

$$\int\left(a+cx^2+bx^4\right)^p\,dx$$

Optimal (type 6, 133 leaves, 2 steps):

$$x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right]$$

Result (type 6, 487 leaves):

$$\left(3\times 4^{-1-p}\left(c+\sqrt{-4ab+c^2}\right)x\left(\frac{c-\sqrt{-4ab+c^2}}{2b}+x^2\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}}{2b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{b}\right)^{1+p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{b}\right)^{-1+p}\right.$$
$$\left.\left(-2a+\left(-c+\sqrt{-4ab+c^2}\right)x^2\right)^2\left(a+cx^2+bx^4\right)^{-1+p}\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\Big/$$
$$\left(\left(-c+\sqrt{-4ab+c^2}\right)\left(-3a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\right.\right.$$
$$px^2\left(\left(-c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]-\right.$$
$$\left.\left.\left.\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)$$

Problem 406: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$$

Optimal (type 4, 446 leaves, 8 steps):

$$\frac{(ef-dg)\,\text{ArcTan}\left[\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right]}{2\sqrt{-cd^4-ae^4}}-\frac{(ef-dg)\,\text{ArcTanh}\left[\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{a+cx^4}}\right]}{2\sqrt{cd^4+ae^4}}+$$
$$\frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2a^{1/4}c^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+cx^4}}-$$
$$\left(\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(4a^{1/4}c^{1/4}de\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+cx^4}\right)$$

Result (type 4, 275 leaves):

$$\frac{1}{2\,e\,\sqrt{a+c\,x^4}}\left(-\frac{2\,\mathrm{i}\,g\,\sqrt{1+\frac{c\,x^4}{a}}\;\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]}{\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}}+\frac{1}{c^{1/4}\,d\,\sqrt{c\,d^4+a\,e^4}}\right.$$

$$(-e\,f+d\,g)\left(2\,(-1)^{1/4}\,a^{1/4}\,\sqrt{c\,d^4+a\,e^4}\,\sqrt{1+\frac{c\,x^4}{a}}\;\mathrm{EllipticPi}\left[\frac{\mathrm{i}\,\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\mathrm{ArcSin}\left[\frac{(-1)^{3/4}\,c^{1/4}\,x}{a^{1/4}}\right],-1\right]+\right.$$

$$\left.\left.c^{1/4}\,d\,e\,\sqrt{a+c\,x^4}\,\left(-\mathrm{Log}\left[-d^2+e^2\,x^2\right]+\mathrm{Log}\left[a\,e^2+c\,d^2\,x^2+\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}\right]\right)\right)\right)$$

Problem 407: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{f+g\,x}{(d+e\,x)\,\sqrt{-a+c\,x^4}}\,dx$$

Optimal (type 4, 218 leaves, 10 steps):

$$\frac{(e\,f-d\,g)\,\mathrm{ArcTanh}\left[\frac{a\,e^2-c\,d^2\,x^2}{\sqrt{c\,d^4-a\,e^4}\,\sqrt{-a+c\,x^4}}\right]}{2\,\sqrt{c\,d^4-a\,e^4}}+\frac{a^{1/4}\,g\,\sqrt{1-\frac{c\,x^4}{a}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{1/4}\,e\,\sqrt{-a+c\,x^4}}+$$

$$\frac{a^{1/4}\,(e\,f-d\,g)\,\sqrt{1-\frac{c\,x^4}{a}}\;\mathrm{EllipticPi}\left[\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\mathrm{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{1/4}\,d\,e\,\sqrt{-a+c\,x^4}}$$

Result (type 4, 719 leaves):

$$\frac{1}{\sqrt{-a+c\,x^4}}\left(-\frac{\mathrm{i}\,g\,\sqrt{1-\frac{c\,x^4}{a}}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ e}+\left(\mathrm{i}\,f\,\left(a^{1/4}-\mathrm{i}\,c^{1/4}\,x\right)^2\sqrt{-\frac{(1-\mathrm{i})\left(a^{1/4}-c^{1/4}\,x\right)}{\mathrm{i}\,a^{1/4}+c^{1/4}\,x}}\ \sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+\mathrm{i}\,c^{1/4}\,x\right)\left(a^{1/4}+c^{1/4}\,x\right)}{\left(a^{1/4}-\mathrm{i}\,c^{1/4}\,x\right)^2}}\right.\right.$$

$$\left(\left(-c^{1/4}\,d+a^{1/4}\,e\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+c^{1/4}\,x\right)}{2\,\mathrm{i}\,a^{1/4}+2\,c^{1/4}\,x}}\right],2\right]-\right.$$

$$\left.\left.(1-\mathrm{i})\,a^{1/4}\,e\,\text{EllipticPi}\left[\frac{(1-\mathrm{i})\left(c^{1/4}\,d-\mathrm{i}\,a^{1/4}\,e\right)}{c^{1/4}\,d-a^{1/4}\,e},\text{ArcSin}\left[\sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+c^{1/4}\,x\right)}{2\,\mathrm{i}\,a^{1/4}+2\,c^{1/4}\,x}}\right],2\right]\right)\right)\Big/\left(a^{1/4}\left(-c^{1/4}\,d+a^{1/4}\,e\right)\left(\mathrm{i}\,c^{1/4}\,d+a^{1/4}\,e\right)\right)+$$

$$\left(d\,g\,\left(a^{1/4}-\mathrm{i}\,c^{1/4}\,x\right)^2\sqrt{-\frac{(1-\mathrm{i})\left(a^{1/4}-c^{1/4}\,x\right)}{\mathrm{i}\,a^{1/4}+c^{1/4}\,x}}\ \sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+\mathrm{i}\,c^{1/4}\,x\right)\left(a^{1/4}+c^{1/4}\,x\right)}{\left(a^{1/4}-\mathrm{i}\,c^{1/4}\,x\right)^2}}\right.$$

$$\left(\mathrm{i}\left(c^{1/4}\,d-a^{1/4}\,e\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+c^{1/4}\,x\right)}{2\,\mathrm{i}\,a^{1/4}+2\,c^{1/4}\,x}}\right],2\right]+(1+\mathrm{i})\,a^{1/4}\,e\right.$$

$$\left.\left.\left.\text{EllipticPi}\left[\frac{(1-\mathrm{i})\left(c^{1/4}\,d-\mathrm{i}\,a^{1/4}\,e\right)}{c^{1/4}\,d-a^{1/4}\,e},\text{ArcSin}\left[\sqrt{\frac{(1+\mathrm{i})\left(a^{1/4}+c^{1/4}\,x\right)}{2\,\mathrm{i}\,a^{1/4}+2\,c^{1/4}\,x}}\right],2\right]\right)\right)\Big/\left(a^{1/4}\,e\left(-c^{1/4}\,d+a^{1/4}\,e\right)\left(\mathrm{i}\,c^{1/4}\,d+a^{1/4}\,e\right)\right)\right)$$

Problem 408: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\ \mathrm{ArcTanh}\left[\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\ \sqrt{-4+4\sqrt{3}\ x^2+x^4}}\right]$$

Result (type 4, 685 leaves) :

$$\left((-1+\sqrt{3}+x)^2\sqrt{2\left(1+\sqrt{3}\right)-2\left(2+\sqrt{3}\right)x+\left(-1+\sqrt{3}\right)x^2-x^3}\sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+x}}{3+\sqrt{3}+\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(\mathrm{i}\left(-1+\sqrt{3}+\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}\right)+\frac{2\left(2\,\mathrm{i}\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+x}\right)\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+\mathrm{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right.$$

$$\texttt{EllipticF}\left[\texttt{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathrm{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\ \frac{2\,\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}}\right]+$$

$$2\sqrt{6}\sqrt{\frac{4+2\sqrt{3}+x^2}{\left(-1+\sqrt{3}+x\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathrm{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}$$

$$\left.\left.\texttt{EllipticPi}\left[\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+\mathrm{i}\left(3+\sqrt{3}\right)},\ \texttt{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathrm{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\ \frac{2\,\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathrm{i}\sqrt{2\left(2+\sqrt{3}\right)}}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{2\left(2+\sqrt{3}\right)}+\mathrm{i}\left(3+\sqrt{3}\right)\right)\sqrt{1+\sqrt{3}-\left(2+\sqrt{3}\right)x+\frac{1}{2}\left(-1+\sqrt{3}\right)x^2-\frac{x^3}{2}}\sqrt{-4+4\sqrt{3}\,x^2+x^4}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathrm{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right)$$

Problem 409: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right]$$

Result (type 4, 1137 leaves):

$$-\left(\left(\left(-1-\sqrt{3}+x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\sqrt{-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+x\right)^2+\left(1-\sqrt{3}+x\right)^3}\right)\right.$$

$$\Big/\left(\left(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+\right.\right.$$

$$\left.\sqrt{-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\frac{1}{-1-\sqrt{3}+x}\right.$$

$$2\left(2i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+\sqrt{6}\sqrt{-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\right.$$

$$\left.\left.\left.\sqrt{-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}\right)\right)\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]+$$

$$2\sqrt{6}\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+x\right)^2}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+x}}$$

$$\left.\left.\text{EllipticPi}\left[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)},\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}\right.$$

$$\sqrt{8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)+2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)^2+\frac{1}{2}\left(-1-\sqrt{3}+x\right)^3}$$

$$\sqrt{\left(48-32\sqrt{3}-64\left(1-\sqrt{3}+x\right)+32\sqrt{3}\left(1-\sqrt{3}+x\right)+24\left(1-\sqrt{3}+x\right)^2-\right.}$$

$$\left.\left.16\sqrt{3}\left(1-\sqrt{3}+x\right)^2-4\left(1-\sqrt{3}+x\right)^3+4\sqrt{3}\left(1-\sqrt{3}+x\right)^3+\left(1-\sqrt{3}+x\right)^4\right)\right)$$

Problem 410: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\,dx$$

Optimal (type 3, 72 leaves, 2 steps) :

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\ \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\ \sqrt{-1+4\sqrt{3}\ x^2+4x^4}}\right]$$

Result (type 4, 623 leaves) :

$$\left(\left(-1+\sqrt{3}+2\,x\right)^2\sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+2\,x}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(i\left(-1+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}\right)+\frac{2\left(2\,i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+2\,x}\right)\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]+$$

$$4\sqrt{3}\sqrt{\frac{2+\sqrt{3}+2\,x^2}{\left(-1+\sqrt{3}+2\,x\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}$$

$$\left.\left.\text{EllipticPi}\left[\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)},\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]\right)\right)\Bigg/$$

$$\left(\left(\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)\right)\sqrt{-2+8\sqrt{3}\,x^2+8\,x^4}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}\right)$$

Problem 411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx$$

Optimal (type 3, 70 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\;\text{ArcTan}\left[\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\right]$$

Result (type 4, 1198 leaves):

$$-\Bigg(\Bigg(\left(-1-\sqrt{3}+2x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+2x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\;\sqrt{-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+2x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+2x\right)^2+\left(1-\sqrt{3}+2x\right)^3}$$

$$\Bigg(\Bigg(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+$$

$$\sqrt{-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}}+\frac{1}{-1-\sqrt{3}+2x}$$

$$2\Bigg(2i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+\sqrt{6}\sqrt{-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}}+$$

$$\sqrt{-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}}\Bigg)\Bigg)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+2x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]+$$

$$2\sqrt{6}\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+2x}}\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+2x\right)^2}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+2x}}$$

$$\left.\left.\text{EllipticPi}\left[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)},\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+2x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]\right)\right)\Big/$$

$$\left(2\left(\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+2x}}\right.$$

$$\sqrt{8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+2x\right)+2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+2x\right)^2+\frac{1}{2}\left(-1-\sqrt{3}+2x\right)^3}$$

$$\sqrt{\left(12-8\sqrt{3}-16\left(1-\sqrt{3}+2x\right)+8\sqrt{3}\left(1-\sqrt{3}+2x\right)+6\left(1-\sqrt{3}+2x\right)^2-\right.}$$

$$\left.\left.4\sqrt{3}\left(1-\sqrt{3}+2x\right)^2-\left(1-\sqrt{3}+2x\right)^3+\sqrt{3}\left(1-\sqrt{3}+2x\right)^3+\frac{1}{4}\left(1-\sqrt{3}+2x\right)^4\right)\right)$$

Problem 412: Unable to integrate problem.

$$\int\frac{f+g x}{(d+e x)\sqrt{a+b x^2+c x^4}}\,dx$$

Optimal (type 4, 560 leaves, 8 steps) :

$$\frac{(e f-d g) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4-b d^2 e^2-a e^4}\, x}{d e \sqrt{a+b x^2+c x^4}}\right]}{2 \sqrt{-c d^4-e^2 (b d^2+a e^2)}}-\frac{(e f-d g) \operatorname{ArcTanh}\left[\frac{b d^2+2 a e^2+(2 c d^2+b e^2) x^2}{2 \sqrt{c d^4+b d^2 e^2+a e^4} \sqrt{a+b x^2+c x^4}}\right]}{2 \sqrt{c d^4+b d^2 e^2+a e^4}}+$$

$$\frac{(\sqrt{c}\, d f+\sqrt{a}\, e g)(\sqrt{a}+\sqrt{c}\, x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c}\, x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{2 a^{1/4} c^{1/4} (\sqrt{c}\, d^2+\sqrt{a}\, e^2) \sqrt{a+b x^2+c x^4}}-$$

$$\left((\sqrt{c}\, d^2-\sqrt{a}\, e^2)(e f-d g)(\sqrt{a}+\sqrt{c}\, x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c}\, x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c}\, d^2+\sqrt{a}\, e^2)^2}{4 \sqrt{a} \sqrt{c}\, d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(4 a^{1/4} c^{1/4} d e (\sqrt{c}\, d^2+\sqrt{a}\, e^2) \sqrt{a+b x^2+c x^4}\right)$$

Result (type 8, 31 leaves):

$$\int \frac{f+g x}{(d+e x) \sqrt{a+b x^2+c x^4}}\, dx$$

Problem 413: Unable to integrate problem.

$$\int \frac{f+g x}{(d+e x) \sqrt{-a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 527 leaves, 10 steps):

$$-\frac{(e f-d g) \operatorname{ArcTanh}\left[\frac{b d^2-2 a e^2+(2 c d^2+b e^2) x^2}{2 \sqrt{c d^4+b d^2 e^2-a e^4} \sqrt{-a+b x^2+c x^4}}\right]}{2 \sqrt{c d^4+b d^2 e^2-a e^4}}+\frac{\sqrt{b+\sqrt{b^2+4 a c}}\, g \left(1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], -\frac{2 \sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]}{\sqrt{2} \sqrt{c}\, e \sqrt{\frac{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}} \sqrt{-a+b x^2+c x^4}}+$$

$$\left(\sqrt{-b+\sqrt{b^2+4 a c}}\, (e f-d g) \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\right.$$

$$\left.\operatorname{EllipticPi}\left[-\frac{\left(b-\sqrt{b^2+4 a c}\right) e^2}{2 c d^2}, \operatorname{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{-b+\sqrt{b^2+4 a c}}}\right], \frac{b-\sqrt{b^2+4 a c}}{b+\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{c}\, d e \sqrt{-a+b x^2+c x^4}\right)$$

Result (type 8, 33 leaves):

$$\int \frac{f+g x}{(d+e x) \sqrt{-a+b x^2+c x^4}} \, dx$$

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(2+3 x^2\right) \sqrt{5+x^4} \, dx$$

Optimal (type 4, 208 leaves, 6 steps):

$$\frac{20}{21} x \sqrt{5+x^4}+\frac{2}{3} x^3 \sqrt{5+x^4}-\frac{10 x \sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21} x^5 \left(6+7 x^2\right) \sqrt{5+x^4}+$$
$$\frac{10\times 5^{1/4} \left(\sqrt{5}+x^2\right) \sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}}-\frac{5\times 5^{1/4} \left(21+2 \sqrt{5}\right) \left(\sqrt{5}+x^2\right) \sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{21 \sqrt{5+x^4}}$$

Result (type 4, 105 leaves):

$$\frac{1}{21} \left(\frac{x \left(100+70 x^2+50 x^4+49 x^6+6 x^8+7 x^{10}\right)}{\sqrt{5+x^4}}+\right.$$
$$\left.210 (-1)^{3/4} 5^{1/4} \text{EllipticE}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]+10 (-5)^{1/4} \left(-21 i+2 \sqrt{5}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4} x\right], -1\right]\right)$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(2+3 x^2\right) \sqrt{5+x^4} \, dx$$

Optimal (type 4, 192 leaves, 5 steps):

$$\frac{10}{7} x \sqrt{5+x^4}+\frac{4 x \sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35} x^3 \left(14+15 x^2\right) \sqrt{5+x^4}-$$
$$\frac{4\times 5^{1/4} \left(\sqrt{5}+x^2\right) \sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{5^{1/4} \left(14-5 \sqrt{5}\right) \left(\sqrt{5}+x^2\right) \sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{5^{1/4}}\right], \frac{1}{2}\right]}{7 \sqrt{5+x^4}}$$

Result (type 4, 101 leaves):

$$\frac{x\left(250+70x^2+125x^4+14x^6+15x^8\right)}{35\sqrt{5+x^4}}-4(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\frac{2}{7}(-5)^{1/4}\left(14i+5\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+3x^2\right)\sqrt{5+x^4}\,dx$$

Optimal (type 4, 176 leaves, 4 steps):

$$\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\frac{6\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{5^{1/4}\left(9+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{5+x^4}}$$

Result (type 4, 96 leaves):

$$\frac{x\left(50+45x^2+10x^4+9x^6\right)}{15\sqrt{5+x^4}}-6(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\frac{2}{3}(-5)^{1/4}\left(9i-2\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^2}\,dx$$

Optimal (type 4, 171 leaves, 4 steps):

$$-\frac{\left(2-x^2\right)\sqrt{5+x^4}}{x}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{4\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{5^{1/4}\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}$$

Result (type 4, 108 leaves):

$$\frac{1}{x\sqrt{5+x^4}}\left(-10+5x^2-2x^4+x^6-4(-1)^{3/4}5^{1/4}x\sqrt{5+x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]-2(-5)^{1/4}\left(-2i+\sqrt{5}\right)x\sqrt{5+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$$

Optimal (type 4, 192 leaves, 5 steps):

$$-\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}+\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{6\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{3\times5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 98 leaves):

$$\frac{1}{15}\left(-\frac{5\left(10+45x^2+2x^4+9x^6\right)}{x^3\sqrt{5+x^4}}-90(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+2(-5)^{1/4}\left(45i-2\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx$$

Optimal (type 4, 235 leaves, 7 steps):

$$\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}-\frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}+\frac{300\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{13\sqrt{5+x^4}}-\frac{50\times5^{1/4}\left(231+26\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{1001\sqrt{5+x^4}}$$

Result (type 4, 115 leaves):

$$\frac{1}{1001}\left(\frac{x\left(13000+7700x^2+11050x^4+11165x^6+2600x^8+3080x^{10}+182x^{12}+231x^{14}\right)}{\sqrt{5+x^4}}+23100(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+100(-5)^{1/4}\left(-231i+26\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx$$

Optimal (type 4, 219 leaves, 6 steps):

$$\frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{40\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{5+x^4}}+\frac{10\times5^{1/4}\left(154-45\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{231\sqrt{5+x^4}}$$

Result (type 4, 110 leaves):

$$\frac{1}{693}\left(\frac{x\left(13500+8470x^2+11475x^4+2464x^6+2700x^8+154x^{10}+189x^{12}\right)}{\sqrt{5+x^4}}-9240(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+60(-5)^{1/4}\left(154i+45\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx$$

Optimal (type 4, 197 leaves, 5 steps):

$$\frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\frac{20\times5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{10\times5^{1/4}\left(7+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{7\sqrt{5+x^4}}$$

Result (type 4, 106 leaves):

$$\frac{x\left(450+385x^2+120x^4+112x^6+6x^8+7x^{10}\right)}{21\sqrt{5+x^4}}-$$
$$20\,(-1)^{3/4}\,5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\frac{20}{7}\,(-5)^{1/4}\left(7\,i-2\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 199 leaves, 5 steps):

$$\frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4}-\frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x}-$$
$$\frac{24\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{6\times 5^{1/4}\left(14+5\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{7\sqrt{5+x^4}}$$

Result (type 4, 125 leaves):

$$\frac{1}{35x\sqrt{5+x^4}}\left(-1750+1125x^2-280x^4+300x^6+14x^8+15x^{10}-\right.$$
$$\left.840\,(-1)^{3/4}\,5^{1/4}\,x\sqrt{5+x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+60\,(-5)^{1/4}\left(14\,i-5\sqrt{5}\right)x\sqrt{5+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 201 leaves, 5 steps):

$$-\frac{2\left(27-2x^2\right)\sqrt{5+x^4}}{3x}+\frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{\left(10-9x^2\right)\left(5+x^4\right)^{3/2}}{15x^3}-$$
$$\frac{36\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{2\times 5^{1/4}\left(27+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{5+x^4}}$$

Result (type 4, 124 leaves) :

$$\frac{1}{15\,x^3\,\sqrt{5+x^4}}\left(-250-1125\,x^2-180\,x^6+10\,x^8+9\,x^{10}-540\,(-1)^{3/4}\,5^{1/4}\,x^3\,\sqrt{5+x^4}\;\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+20\,(-5)^{1/4}\left(27\,\text{i}-2\,\sqrt{5}\right)x^3\,\sqrt{5+x^4}\;\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(2+3\,x^2\right)}{\sqrt{5+x^4}}\,dx$$

Optimal (type 4, 185 leaves, 5 steps) :

$$\frac{2}{3}\,x\,\sqrt{5+x^4}+\frac{3}{5}\,x^3\,\sqrt{5+x^4}-\frac{9\,x\,\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{9\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}-\frac{5^{1/4}\left(27+2\,\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{6\,\sqrt{5+x^4}}$$

Result (type 4, 96 leaves) :

$$9\,(-1)^{3/4}\,5^{1/4}\,\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\frac{1}{15}\left(\frac{x\left(50+45\,x^2+10\,x^4+9\,x^6\right)}{\sqrt{5+x^4}}+5\,(-5)^{1/4}\left(-27\,\text{i}+2\,\sqrt{5}\right)\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(2+3\,x^2\right)}{\sqrt{5+x^4}}\,dx$$

Optimal (type 4, 166 leaves, 4 steps) :

$$x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{2\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+$$
$$\frac{5^{1/4}\left(2-\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{2\sqrt{5+x^4}}$$

Result (type 4, 71 leaves):

$$x\sqrt{5+x^4}-2\,(-1)^{3/4}\,5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+(-5)^{1/4}\left(2\,i+\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{\sqrt{5+x^4}}\,dx$$

Optimal (type 4, 155 leaves, 3 steps):

$$\frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{3\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{2\times 5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 62 leaves):

$$\left(-\frac{1}{5}\right)^{1/4}\left(-3\,i\sqrt{5}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\left(-2+3\,i\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$-\frac{2\sqrt{5+x^4}}{5x}+\frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{5^{3/4}\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{2\times 5^{3/4}\sqrt{5+x^4}}$$

Result (type 4, 81 leaves):

$$\frac{1}{5}\left(-\frac{2\sqrt{5+x^4}}{x}-2(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]-(-5)^{1/4}\left(-2i+3\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$$

Optimal (type 4, 189 leaves, 5 steps):

$$-\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{30\times 5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 97 leaves):

$$\frac{1}{75}\left(-\frac{5\left(10+45x^2+2x^4+9x^6\right)}{x^3\sqrt{5+x^4}}-45(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+(-5)^{1/4}\left(45i+2\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$-\frac{x^3\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)}-\frac{9\times 5^{1/4}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{2\sqrt{5+x^4}}+\frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{4\times 5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 85 leaves):

$$\frac{1}{10}\left(-\frac{5x\left(2+3x^2\right)}{\sqrt{5+x^4}}-45\,(-1)^{3/4}\,5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+(-5)^{1/4}\left(45\,i-2\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 177 leaves, 4 steps):

$$-\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}+\frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{4\times 5^{3/4}\sqrt{5+x^4}}$$

Result (type 4, 85 leaves):

$$\frac{1}{10}\left(\frac{x\left(-15+2x^2\right)}{\sqrt{5+x^4}}+2\,(-1)^{3/4}\,5^{1/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]-(-5)^{1/4}\left(2\,i+3\sqrt{5}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 180 leaves, 4 steps):

$$\frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}}-\frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)}+\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{2\times 5^{3/4}\sqrt{5+x^4}}+$$
$$\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{20\times 5^{1/4}\sqrt{5+x^4}}$$

Result (type 4, 86 leaves):

$$\frac{1}{50}\left(\frac{5x\left(2+3x^2\right)}{\sqrt{5+x^4}}+15(-1)^{3/4}5^{1/4}\,\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]-(-5)^{1/4}\left(15\,\mathbb{i}+2\sqrt{5}\right)\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$\frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{5\times 5^{3/4}\sqrt{5+x^4}}+$$
$$\frac{3\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{20\times 5^{3/4}\sqrt{5+x^4}}$$

Result (type 4, 108 leaves):

$$-\frac{1}{50x\sqrt{5+x^4}}\left(20-15x^2+6x^4+6(-1)^{3/4}5^{1/4}x\sqrt{5+x^4}\,\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]+\right.$$
$$\left.3(-5)^{1/4}\left(-2\,\mathbb{i}+\sqrt{5}\right)x\sqrt{5+x^4}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 214 leaves, 6 steps):

$$\frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}+\frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)}-$$
$$\frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{10\times 5^{3/4}\sqrt{5+x^4}}+\frac{\left(27-2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{5^{1/4}}\right],\frac{1}{2}\right]}{60\times 5^{3/4}\sqrt{5+x^4}}$$

Result (type 4, 119 leaves):

$$-\frac{1}{150x^3\sqrt{5+x^4}}\left(20+90x^2+10x^4+27x^6+27(-1)^{3/4}5^{1/4}x^3\sqrt{5+x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]-\right.$$
$$\left.(-5)^{1/4}\left(27i+2\sqrt{5}\right)x^3\sqrt{5+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-\frac{1}{5}\right)^{1/4}x\right],-1\right]\right)$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int x^5\left(d+e x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Optimal (type 1, 63 leaves, 4 steps):

$$\frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e\left(1+x^2\right)^{14}$$

Result (type 1, 153 leaves):

$$\frac{dx^6}{6}+\frac{1}{8}(10d+e)x^8+\frac{1}{2}(9d+2e)x^{10}+\frac{5}{4}(8d+3e)x^{12}+\frac{15}{7}(7d+4e)x^{14}+\frac{21}{8}(6d+5e)x^{16}+$$
$$\frac{7}{3}(5d+6e)x^{18}+\frac{3}{2}(4d+7e)x^{20}+\frac{15}{22}(3d+8e)x^{22}+\frac{5}{24}(2d+9e)x^{24}+\frac{1}{26}(d+10e)x^{26}+\frac{ex^{28}}{28}$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int x^3\left(d+e x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Optimal (type 1, 45 leaves, 4 steps):

$$-\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}$$

Result (type 1, 151 leaves):

$$\frac{d x^4}{4}+\frac{1}{6}(10 d+e) x^6+\frac{5}{8}(9 d+2 e) x^8+\frac{3}{2}(8 d+3 e) x^{10}+\frac{5}{2}(7 d+4 e) x^{12}+3(6 d+5 e) x^{14}+\frac{21}{8}(5 d+6 e) x^{16}+\frac{5}{3}(4 d+7 e) x^{18}+\frac{3}{4}(3 d+8 e) x^{20}+\frac{5}{22}(2 d+9 e) x^{22}+\frac{1}{24}(d+10 e) x^{24}+\frac{e x^{26}}{26}$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int x\left(d+e x^2\right)\left(1+2 x^2+x^4\right)^5 \, dx$$

Optimal (type 1, 29 leaves, 4 steps):

$$\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24} e\left(1+x^2\right)^{12}$$

Result (type 1, 149 leaves):

$$\frac{d x^2}{2}+\frac{1}{4}(10 d+e) x^4+\frac{5}{6}(9 d+2 e) x^6+\frac{15}{8}(8 d+3 e) x^8+3(7 d+4 e) x^{10}+\frac{7}{2}(6 d+5 e) x^{12}+3(5 d+6 e) x^{14}+\frac{15}{8}(4 d+7 e) x^{16}+\frac{5}{6}(3 d+8 e) x^{18}+\frac{1}{4}(2 d+9 e) x^{20}+\frac{1}{22}(d+10 e) x^{22}+\frac{e x^{24}}{24}$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int x^5\left(1+x^2\right)\left(1+2 x^2+x^4\right)^5 \, dx$$

Optimal (type 1, 34 leaves, 4 steps):

$$\frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}$$

Result (type 1, 85 leaves):

$$\frac{x^6}{6}+\frac{11 x^8}{8}+\frac{11 x^{10}}{2}+\frac{55 x^{12}}{4}+\frac{165 x^{14}}{7}+\frac{231 x^{16}}{8}+\frac{77 x^{18}}{3}+\frac{33 x^{20}}{2}+\frac{15 x^{22}}{2}+\frac{55 x^{24}}{24}+\frac{11 x^{26}}{26}+\frac{x^{28}}{28}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int x^3\left(1+x^2\right)\left(1+2 x^2+x^4\right)^5 \, dx$$

Optimal (type 1, 23 leaves, 4 steps):

$$-\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}$$

Result (type 1, 83 leaves):

$$\frac{x^4}{4}+\frac{11\,x^6}{6}+\frac{55\,x^8}{8}+\frac{33\,x^{10}}{2}+\frac{55\,x^{12}}{2}+33\,x^{14}+\frac{231\,x^{16}}{8}+\frac{55\,x^{18}}{3}+\frac{33\,x^{20}}{4}+\frac{5\,x^{22}}{2}+\frac{11\,x^{24}}{24}+\frac{x^{26}}{26}$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\left(2+3\,x^2\right)\sqrt{3+5\,x^2+x^4}\;dx$$

Optimal (type 4, 322 leaves, 6 steps):

$$-\frac{1924\,x\left(5+\sqrt{13}+2\,x^2\right)}{105\,\sqrt{3+5\,x^2+x^4}}+\frac{13}{3}\,x\,\sqrt{3+5\,x^2+x^4}-\frac{26}{35}\,x^3\,\sqrt{3+5\,x^2+x^4}+\frac{1}{21}\,x^5\left(11+7\,x^2\right)\sqrt{3+5\,x^2+x^4}+\frac{1}{105\,\sqrt{3+5\,x^2+x^4}}$$

$$962\,\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]-$$

$$\frac{13\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]}{\sqrt{6\left(5+\sqrt{13}\right)}\,\sqrt{3+5\,x^2+x^4}}$$

Result (type 4, 237 leaves):

$$\frac{1}{210\,\sqrt{3+5\,x^2+x^4}}\left(2730\,x+4082\,x^3+460\,x^5+604\,x^7+460\,x^9+70\,x^{11}-\right.$$

$$1924\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]+$$

$$\left.13\,i\,\sqrt{2}\,\left(-635+148\,\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\left(2+3\,x^2\right)\sqrt{3+5\,x^2+x^4}\;dx$$

Optimal (type 4, 305 leaves, 5 steps):

$$\frac{1247\,x\left(5+\sqrt{13}+2\,x^2\right)}{210\,\sqrt{3+5\,x^2+x^4}}-\frac{4}{3}\,x\,\sqrt{3+5\,x^2+x^4}+\frac{1}{35}\,x^3\,\left(29+15\,x^2\right)\,\sqrt{3+5\,x^2+x^4}-\frac{1}{210\,\sqrt{3+5\,x^2+x^4}}$$

$$1247\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}2\,\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 234 leaves):

$$\frac{1}{420\,\sqrt{3+5\,x^2+x^4}}\Bigg(4\,x\,\left(-420-439\,x^2+430\,x^4+312\,x^6+45\,x^8\right)+$$

$$1247\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-$$

$$i\,\sqrt{2}\,\left(-5395+1247\,\sqrt{13}\right)\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\Bigg)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+3\,x^2\right)\,\sqrt{3+5\,x^2+x^4}\,dx$$

Optimal (type 4, 279 leaves, 4 steps):

$$-\frac{23\,x\left(5+\sqrt{13}+2\,x^2\right)}{15\,\sqrt{3+5\,x^2+x^4}}+\frac{1}{15}\,x\left(25+9\,x^2\right)\sqrt{3+5\,x^2+x^4}+\frac{1}{15\,\sqrt{3+5\,x^2+x^4}}$$

$$23\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]}{\sqrt{6\left(5+\sqrt{13}\right)}\,\sqrt{3+5\,x^2+x^4}}$$

Result (type 4, 229 leaves) :

$$\frac{1}{30\,\sqrt{3+5\,x^2+x^4}}$$

$$\left(2\,x\left(75+152\,x^2+70\,x^4+9\,x^6\right)-23\,\mathrm{i}\,\sqrt{2}\,\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]+\right.$$

$$\left.\mathrm{i}\,\sqrt{2}\,\left(-130+23\,\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3\,x^2\right)\sqrt{3+5\,x^2+x^4}}{x^2}\,dx$$

Optimal (type 4, 284 leaves, 4 steps) :

$$\frac{9\,x\left(5+\sqrt{13}+2\,x^2\right)}{2\,\sqrt{3+5\,x^2+x^4}}-\frac{\left(2-x^2\right)\sqrt{3+5\,x^2+x^4}}{x}-\frac{1}{2\,\sqrt{3+5\,x^2+x^4}}$$

$$3\,\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}8\,\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 231 leaves):

$$\frac{1}{4\,x\,\sqrt{3+5\,x^2+x^4}}$$

$$\left(4\left(-6-7\,x^2+3\,x^4+x^6\right)+9\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)x\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-\right.$$

$$\left.i\,\sqrt{2}\,\left(-13+9\,\sqrt{13}\right)x\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3\,x^2\right)\sqrt{3+5\,x^2+x^4}}{x^4}\,dx$$

Optimal (type 4, 305 leaves, 5 steps):

$$\frac{32\,x\left(5+\sqrt{13}+2\,x^2\right)}{9\sqrt{3+5\,x^2+x^4}}-\frac{64\sqrt{3+5\,x^2+x^4}}{9\,x}-\frac{\left(2-9\,x^2\right)\sqrt{3+5\,x^2+x^4}}{3\,x^3}-\frac{1}{9\sqrt{3+5\,x^2+x^4}}$$

$$16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]+$$

$$\frac{49\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5\,x^2+x^4}}$$

Result (type 4, 237 leaves) :

$$\frac{1}{18\,x^3\sqrt{3+5\,x^2+x^4}}$$

$$\left(-2\left(18+141\,x^2+191\,x^4+37\,x^6\right)+32\,i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-\right.$$

$$\left.i\sqrt{2}\left(-13+32\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\left(2+3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 356 leaves, 7 steps) :

$$\frac{176723\,x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}}-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-$$

$$\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{4290\sqrt{3+5x^2+x^4}}$$

$$176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]+$$

$$\frac{1}{143\sqrt{3+5x^2+x^4}}2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]$$

Result (type 4, 249 leaves) :

$$\frac{1}{8580\sqrt{3+5x^2+x^4}}\left(4x\left(-63150-93991x^2+3055x^4+29003x^6+39650x^8+24635x^{10}+6015x^{12}+495x^{14}\right)+\right.$$

$$176723\,i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-$$

$$\left.i\sqrt{2}\left(-757315+176723\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 331 leaves, 6 steps) :

$$-\frac{49949\,x\left(5+\sqrt{13}+2\,x^2\right)}{3465\,\sqrt{3+5\,x^2+x^4}}+\frac{353}{99}\,x\,\sqrt{3+5\,x^2+x^4}-\frac{x^3\left(911+890\,x^2\right)\sqrt{3+5\,x^2+x^4}}{1155}+\frac{1}{99}\,x^3\left(67+27\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}+\frac{1}{3465\,\sqrt{3+5\,x^2+x^4}}$$

$$49949\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]-$$

$$\frac{353\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]}{33\,\sqrt{6\left(5+\sqrt{13}\right)}\,\sqrt{3+5\,x^2+x^4}}$$

Result (type 4, 244 leaves):

$$\frac{1}{6930\,\sqrt{3+5\,x^2+x^4}}\left(2\,x\left(37065+74681\,x^2+69535\,x^4+84962\,x^6+50075\,x^8+11795\,x^{10}+945\,x^{12}\right)-\right.$$

$$49949\,i\,\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]+$$

$$\left.i\,\sqrt{2}\left(-212680+49949\,\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 308 leaves, 5 steps):

$$\frac{203\,x\left(5+\sqrt{13}+2\,x^2\right)}{30\,\sqrt{3+5\,x^2+x^4}}-\frac{1}{15}\,x\left(5+12\,x^2\right)\sqrt{3+5\,x^2+x^4}+\frac{1}{3}\,x\left(3+x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}-\frac{1}{30\,\sqrt{3+5\,x^2+x^4}}$$

$$203\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}5\,\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 239 leaves) :

$$\frac{1}{60\,\sqrt{3+5\,x^2+x^4}}\left(4\,x\left(120+434\,x^2+550\,x^4+293\,x^6+65\,x^8+5\,x^{10}\right)+\right.$$

$$203\,i\,\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-$$

$$\left.i\,\sqrt{2}\left(-715+203\,\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 312 leaves, 5 steps) :

$$\frac{412\,x\left(5+\sqrt{13}+2\,x^2\right)}{35\,\sqrt{3+5\,x^2+x^4}}+\frac{1}{35}\,x\left(655+129\,x^2\right)\sqrt{3+5\,x^2+x^4}-\frac{\left(14-3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}}{7\,x}-\frac{1}{35\,\sqrt{3+5\,x^2+x^4}}$$

$$206\,\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}19\,\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 235 leaves):

$$\frac{1}{70\,x\,\sqrt{3+5\,x^2+x^4}}\left(-1260+3884\,x^4+2130\,x^6+418\,x^8+30\,x^{10}+\right.$$

$$412\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)x\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-$$

$$\left.i\,\sqrt{2}\,\left(-65+412\,\sqrt{13}\right)x\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 167: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$\frac{949\,x\left(5+\sqrt{13}+2\,x^2\right)}{30\,\sqrt{3+5\,x^2+x^4}}-\frac{13\left(24-5\,x^2\right)\sqrt{3+5\,x^2+x^4}}{15\,x}-\frac{\left(10-9\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}}{15\,x^3}-\frac{1}{30\,\sqrt{3+5\,x^2+x^4}}$$

$$949\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}65\,\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 247 leaves):

$$\frac{1}{60\,x^3\,\sqrt{3+5\,x^2+x^4}}\left(4\left(-90-1155\,x^2-1405\,x^4+192\,x^6+145\,x^8+9\,x^{10}\right)+\right.$$

$$949\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)x^3\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-$$

$$\left.13\,i\,\sqrt{2}\,\left(-65+73\,\sqrt{13}\right)x^3\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3\,x^2\right)\left(3+5\,x^2+x^4\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$\frac{361\,x\left(5+\sqrt{13}+2\,x^2\right)}{15\,\sqrt{3+5\,x^2+x^4}}-\frac{722\,\sqrt{3+5\,x^2+x^4}}{15\,x}-\frac{\left(40-87\,x^2\right)\,\sqrt{3+5\,x^2+x^4}}{5\,x^3}-\frac{\left(2-5\,x^2\right)\,\left(3+5\,x^2+x^4\right)^{3/2}}{5\,x^5}-\frac{1}{15\,\sqrt{3+5\,x^2+x^4}}$$

$$\Bigg(361\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$103\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]\Bigg)\Bigg/\left(\sqrt{6\left(5+\sqrt{13}\right)}\,\sqrt{3+5\,x^2+x^4}\right)$$

Result (type 4, 244 leaves) :

$$\frac{1}{30\,x^5\,\sqrt{3+5\,x^2+x^4}}\Bigg(-108-810\,x^2-3438\,x^4-4040\,x^6-634\,x^8+30\,x^{10}+$$

$$361\,i\,\sqrt{2}\,\left(-5+\sqrt{13}\right)\,x^5\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-$$

$$i\,\sqrt{2}\,\left(-260+361\,\sqrt{13}\right)\,x^5\,\sqrt{\frac{-5+\sqrt{13}-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}+2\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\Bigg)$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(A+B\,x^2\right)}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 403 leaves, 5 steps) :

$$-\frac{(4bB-5Ac)\,x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}+\frac{(8b^2B-10Abc-9aBc)\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$a^{1/4}\left(8b^2B-10Abc-9aBc\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]+\frac{1}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$a^{1/4}\left(8b^2B-10Abc-9aBc+\sqrt{a}\sqrt{c}\,(4bB-5Ac)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 532 leaves):

$$\frac{1}{60c^3\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

$$\Bigg(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(-4bB+5Ac+3Bcx^2\right)\left(a+bx^2+cx^4\right)+i\left(8b^2B-10Abc-9aBc\right)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}$$

$$\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-$$

$$i\left(-8b^3B+bc\left(17aB-10A\sqrt{b^2-4ac}\right)+2b^2\left(5Ac+4B\sqrt{b^2-4ac}\right)-ac\left(10Ac+9B\sqrt{b^2-4ac}\right)\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}$$

$$\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\Bigg)$$

Problem 177: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 336 leaves, 4 steps):

$$\frac{B\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c}-\frac{(2\,b\,B-3\,A\,c)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\frac{a^{1/4}\,(2\,b\,B-3\,A\,c)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{3\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{1}{6\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$a^{1/4}\,\left(2\,b\,B+\sqrt{a}\,B\,\sqrt{c}-3\,A\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 479 leaves) :

$$\frac{1}{12\,c^2\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,B\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\left(a+b\,x^2+c\,x^4\right)-i\,(2\,b\,B-3\,A\,c)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+i\,\left(-2\,b^2\,B+3\,A\,b\,c+2\,a\,B\,c+2\,b\,B\,\sqrt{b^2-4\,a\,c}-3\,A\,c\,\sqrt{b^2-4\,a\,c}\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 178: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps) :

$$\frac{B x \sqrt{a+b x^2+c x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{a^{1/4} B\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{c^{3/4}\sqrt{a+b x^2+c x^4}}+$$

$$\frac{a^{1/4}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2 c^{3/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 302 leaves) :

$$\left(i \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\left(B\left(-b+\sqrt{b^2-4 a c}\right)\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+\right.\right.$$

$$\left.\left.\left(b B-2 A c-B\sqrt{b^2-4 a c}\right)\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right)\Big/\left(2\sqrt{2}\, c\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{a+b x^2+c x^4}\right)$$

Problem 179: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{x^2\sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 312 leaves, 4 steps) :

$$-\frac{A\sqrt{a+b x^2+c x^4}}{a x}+\frac{A\sqrt{c}\, x\sqrt{a+b x^2+c x^4}}{a\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{A c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{a^{3/4}\sqrt{a+b x^2+c x^4}}+$$

$$\frac{\left(\sqrt{a}\, B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2 a^{3/4} c^{1/4}\sqrt{a+b x^2+c x^4}}$$

Result (type 4, 448 leaves) :

$$\frac{1}{4\,a\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(-4\,A\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(a+b\,x^2+c\,x^4\right)+i\,A\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,x\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-i\,\left(2\,a\,B+A\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\right)\,x$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 180: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^4\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 376 leaves, 5 steps):

$$-\frac{A\,\sqrt{a+b\,x^2+c\,x^4}}{3\,a\,x^3}+\frac{\left(2\,A\,b-3\,a\,B\right)\,\sqrt{a+b\,x^2+c\,x^4}}{3\,a^2\,x}-\frac{\left(2\,A\,b-3\,a\,B\right)\,\sqrt{c}\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,a^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\frac{\left(2\,A\,b-3\,a\,B\right)\,c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{3\,a^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{1}{6\,a^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(2\,A\,b-3\,a\,B+\sqrt{a}\,A\,\sqrt{c}\right)\,c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 373 leaves):

$$\frac{1}{12\, a^2 \sqrt{a+b\,x^2+c\,x^4}}\left(-\frac{4\left(a+b\,x^2+c\,x^4\right)\left(-2\,A\,b\,x^2+a\left(A+3\,B\,x^2\right)\right)}{x^3}+\frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}}\,\mathrm{i}\,\sqrt{2}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(-\left(2\,A\,b-3\,a\,B\right)\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left(3\,a\,B\left(b-\sqrt{b^2-4\,a\,c}\right)+2\,A\left(-b^2+a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\left(2+3\,x^2\right)}{\sqrt{3+5\,x^2+x^4}}\,dx$$

Optimal (type 4, 298 leaves, 5 steps):

$$\frac{419\,x\left(5+\sqrt{13}+2\,x^2\right)}{30\,\sqrt{3+5\,x^2+x^4}}-\frac{10}{3}\,x\,\sqrt{3+5\,x^2+x^4}+\frac{3}{5}\,x^3\,\sqrt{3+5\,x^2+x^4}-\frac{1}{30\,\sqrt{3+5\,x^2+x^4}}$$

$$419\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]+$$

$$\frac{1}{\sqrt{3+5\,x^2+x^4}}5\,\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,\left(6+\left(5+\sqrt{13}\right)x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]$$

Result (type 4, 229 leaves):

$$\frac{1}{60\sqrt{3+5x^2+x^4}}\left(4x\left(-150-223x^2-5x^4+9x^6\right)+419\,i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-\right.$$
$$\left.i\sqrt{2}\left(-1795+419\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$-\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}+\frac{1}{\sqrt{3+5x^2+x^4}}$$
$$2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]-$$
$$\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]}{\sqrt{3+5x^2+x^4}}$$

Result (type 4, 222 leaves):

$$\frac{1}{2\sqrt{3+5x^2+x^4}}\left(2x\left(3+5x^2+x^4\right)-4\,i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]+\right.$$
$$\left.i\sqrt{2}\left(-17+4\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3 x^2}{\sqrt{3+5 x^2+x^4}} \, dx$$

Optimal (type 4, 257 leaves, 3 steps):

$$\frac{3 x \left(5+\sqrt{13}+2 x^2\right)}{2 \sqrt{3+5 x^2+x^4}}-\frac{1}{2 \sqrt{3+5 x^2+x^4}}\sqrt{\frac{3}{2} \left(5+\sqrt{13}\right)} \sqrt{\frac{6+\left(5-\sqrt{13}\right) x^2}{6+\left(5+\sqrt{13}\right) x^2}} \left(6+\left(5+\sqrt{13}\right) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5+\sqrt{13}\right)} x\right],\frac{1}{6} \left(-13+5 \sqrt{13}\right)\right]+$$

$$\frac{\sqrt{\frac{2}{3 \left(5+\sqrt{13}\right)}} \sqrt{\frac{6+\left(5-\sqrt{13}\right) x^2}{6+\left(5+\sqrt{13}\right) x^2}} \left(6+\left(5+\sqrt{13}\right) x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5+\sqrt{13}\right)} x\right],\frac{1}{6} \left(-13+5 \sqrt{13}\right)\right]}{\sqrt{3+5 x^2+x^4}}$$

Result (type 4, 159 leaves):

$$\frac{1}{2 \sqrt{2} \sqrt{3+5 x^2+x^4}} i \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2}$$

$$\left(3 \left(-5+\sqrt{13}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right],\frac{19}{6}+\frac{5 \sqrt{13}}{6}\right]+\left(11-3 \sqrt{13}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right],\frac{19}{6}+\frac{5 \sqrt{13}}{6}\right]\right)$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3 x^2}{x^2 \sqrt{3+5 x^2+x^4}} \, dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{1}{3\sqrt{3+5x^2+x^4}}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]+\right.$$

$$\left.\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]}{\sqrt{3+5x^2+x^4}}\right)$$

Result (type 4, 224 leaves):

$$\frac{1}{6x\sqrt{3+5x^2+x^4}}\left(-4\left(3+5x^2+x^4\right)+i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-\right.$$

$$\left.i\sqrt{2}\left(4+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 193: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$

Optimal (type 4, 302 leaves, 5 steps):

$$\frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{1}{54\sqrt{3+5x^2+x^4}}\left(7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]-\right.$$

$$\left.\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]}{9\sqrt{3+5x^2+x^4}}\right)$$

Result (type 4, 237 leaves):

$$\frac{1}{108\,x^3\sqrt{3+5x^2+x^4}}\left(-4\left(18+51x^2+41x^4+7x^6\right)+7\,i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-\right.$$
$$\left.i\sqrt{2}\left(-47+7\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 307 leaves, 5 steps):

$$\frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{1}{13\sqrt{3+5x^2+x^4}}$$
$$43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]+$$
$$\frac{1}{13\sqrt{3+5x^2+x^4}}11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]$$

Result (type 4, 219 leaves):

$$\frac{1}{26\sqrt{3+5x^2+x^4}}\left(-2x\left(33+47x^2\right)+43\,i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-\right.$$
$$\left.i\sqrt{2}\left(-182+43\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\right)$$

Problem 200: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(2+3 x^2\right)}{\left(3+5 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$-\frac{11 x \left(5+\sqrt{13}+2 x^2\right)}{26 \sqrt{3+5 x^2+x^4}}+\frac{x \left(8+11 x^2\right)}{13 \sqrt{3+5 x^2+x^4}}+\frac{1}{26 \sqrt{3+5 x^2+x^4}} 11 \sqrt{\frac{1}{6} \left(5+\sqrt{13}\right)} \sqrt{\frac{6+\left(5-\sqrt{13}\right) x^2}{6+\left(5+\sqrt{13}\right) x^2}} \left(6+\left(5+\sqrt{13}\right) x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5+\sqrt{13}\right)} x\right], \frac{1}{6} \left(-13+5 \sqrt{13}\right)\right]-\frac{1}{13 \sqrt{3+5 x^2+x^4}} 4 \sqrt{\frac{2}{3 \left(5+\sqrt{13}\right)}} \sqrt{\frac{6+\left(5-\sqrt{13}\right) x^2}{6+\left(5+\sqrt{13}\right) x^2}} \left(6+\left(5+\sqrt{13}\right) x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6} \left(5+\sqrt{13}\right)} x\right], \frac{1}{6} \left(-13+5 \sqrt{13}\right)\right]$$

Result (type 4, 219 leaves):

$$\frac{1}{52 \sqrt{3+5 x^2+x^4}} \left(4 x \left(8+11 x^2\right)-11 i \sqrt{2} \left(-5+\sqrt{13}\right) \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right], \frac{19}{6}+\frac{5 \sqrt{13}}{6}\right]+i \sqrt{2} \left(-39+11 \sqrt{13}\right) \sqrt{\frac{-5+\sqrt{13}-2 x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2 x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}} x\right], \frac{19}{6}+\frac{5 \sqrt{13}}{6}\right]\right)$$

Problem 201: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3 x^2}{\left(3+5 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{1}{39\sqrt{3+5x^2+x^4}}\Bigg(2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]\Bigg)+$$

$$\frac{11\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Result (type 4, 219 leaves):

$$\frac{1}{78\sqrt{3+5x^2+x^4}}\Bigg(-2x\left(7+8x^2\right)+4\,i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]-$$

$$i\sqrt{2}\left(13+4\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\Bigg)$$

Problem 202: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 309 leaves, 5 steps):

$$\frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}-\frac{1}{234\sqrt{3+5x^2+x^4}}\Bigg(19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]\Bigg)-$$

$$\frac{1}{39\sqrt{3+5x^2+x^4}}4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\frac{1}{6}\left(-13+5\sqrt{13}\right)\right]$$

Result (type 4, 228 leaves):

$$\frac{1}{468\,x\,\sqrt{3+5\,x^2+x^4}}\left(-4\,\left(78+119\,x^2+19\,x^4\right)+19\,\mathrm{i}\,\sqrt{2}\,\left(-5+\sqrt{13}\right)\,x\,\sqrt{\frac{-5+\sqrt{13}\,-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}\,+2\,x^2}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]-\mathrm{i}\,\sqrt{2}\,\left(-143+19\,\sqrt{13}\right)\,x\,\sqrt{\frac{-5+\sqrt{13}\,-2\,x^2}{-5+\sqrt{13}}}\,\sqrt{5+\sqrt{13}\,+2\,x^2}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\,\frac{19}{6}+\frac{5\,\sqrt{13}}{6}\right]\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+3\,x^2}{x^4\,\left(3+5\,x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 326 leaves, 6 steps):

$$-\frac{133\,x\,\left(5+\sqrt{13}\,+2\,x^2\right)}{1053\,\sqrt{3+5\,x^2+x^4}}-\frac{7+8\,x^2}{39\,x^3\,\sqrt{3+5\,x^2+x^4}}-\frac{5\,\sqrt{3+5\,x^2+x^4}}{351\,x^3}+\frac{266\,\sqrt{3+5\,x^2+x^4}}{1053\,x}+\frac{1}{1053\,\sqrt{3+5\,x^2+x^4}}133\,\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)\,x^2}{6+\left(5+\sqrt{13}\right)\,x^2}}\,\left(6+\left(5+\sqrt{13}\right)\,x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]-$$

$$\frac{5\,\sqrt{\frac{6+\left(5-\sqrt{13}\right)\,x^2}{6+\left(5+\sqrt{13}\right)\,x^2}}\,\left(6+\left(5+\sqrt{13}\right)\,x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\,x\right],\,\frac{1}{6}\left(-13+5\,\sqrt{13}\right)\right]}{351\,\sqrt{6\left(5+\sqrt{13}\right)}\,\sqrt{3+5\,x^2+x^4}}$$

Result (type 4, 234 leaves):

$$\frac{1}{2106\,x^3\sqrt{3+5x^2+x^4}}\Bigg(-468+1014x^2+2630x^4+532x^6-$$

$$133\,i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]+$$

$$i\sqrt{2}\left(-650+133\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right],\frac{19}{6}+\frac{5\sqrt{13}}{6}\right]\Bigg)$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int (f\,x)^{3/2}\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\frac{2\,d\,(f\,x)^{5/2}\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,e\,(f\,x)^{9/2}\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{9\,f^3\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 2835 leaves):

$$\frac{(f\,x)^{3/2}\sqrt{a+b\,x^2+c\,x^4}\left(\frac{4\left(13\,b\,c\,d-7\,b^2\,e+18\,a\,c\,e\right)\sqrt{x}}{585\,c^2}+\frac{2\left(13\,c\,d+2\,b\,e\right)x^{5/2}}{117\,c}+\frac{2}{13}\,e\,x^{9/2}\right)}{x^{3/2}}+$$

$$\left(4\,a^3\,b\,d\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Bigg(9\,c\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)x\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\Bigg(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg)-$$

$$\Big(28\,a^3\,b^2\,e\,(f\,x)^{3/2}\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(117\,c^2\,\big(b-\sqrt{b^2-4\,a\,c}\big)\,\big(b+\sqrt{b^2-4\,a\,c}\big)\,x\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-5\,a\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$

$$\Big(8\,a^4\,e\,(f\,x)^{3/2}\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(13\,c\,\big(b-\sqrt{b^2-4\,a\,c}\big)\,\big(b+\sqrt{b^2-4\,a\,c}\big)\,x\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-5\,a\,\text{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(8\,a^3\,d\,x\,(f\,x)^{3/2}\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(5\,\big(b-\sqrt{b^2-4\,a\,c}\big)\,\big(b+\sqrt{b^2-4\,a\,c}\big)\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$

$$\Big(12\,a^2\,b^2\,d\,x\,(f\,x)^{3/2}\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(25\,c\,\big(b-\sqrt{b^2-4\,a\,c}\big)\,\big(b+\sqrt{b^2-4\,a\,c}\big)\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\big(b+\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\big(b-\sqrt{b^2-4\,a\,c}\big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(84\,a^2\,b^3\,e\,x\,(f\,x)^{3/2}\,\big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\left(325\,c^2\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(316\,a^3\,b\,e\,x\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(325\,c\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int\sqrt{f\,x}\,\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\frac{2\,d\,(f\,x)^{3/2}\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,f\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,e\,(f\,x)^{7/2}\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{7\,f^3\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1717 leaves):

$$\frac{1}{1617\,c^2\left(a+b\,x^2+c\,x^4\right)^{3/2}}x\sqrt{f\,x}\left(42\,c\left(11\,c\,d+2\,b\,e+7\,c\,e\,x^2\right)\left(a+b\,x^2+c\,x^4\right)^2+\right.$$

$$\left(1078\,a^2\,c\,d\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Bigg(7\,a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)-$$

$$\Bigg(147\,a^2\,b\,e\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(7\,a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)+$$

$$\Bigg(363\,a\,b\,c\,d\,x^2\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)+$$

$$\Bigg(462\,a^2\,c\,e\,x^2\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)+$$

$$\Bigg(165\,a\,b^2\,e\,x^2\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(d+e\,x^2\right)\,\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{f\,x}}\,dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\frac{2\,d\,\sqrt{f\,x}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,e\,(f\,x)^{5/2}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f^3\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1717 leaves):

$$\frac{1}{225\,c^2\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}x\,\Bigg(10\,c\,\left(9\,c\,d+2\,b\,e+5\,c\,e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)^2+$$

$$\Bigg(450\,a^2\,c\,d\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\Bigg(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)-$$

$$\Bigg(25\,a^2\,b\,e\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\Bigg(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)+$$

$$\Bigg(81\,a\,b\,c\,d\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(90\,a^2\,c\,e\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(27\,a\,b^2\,e\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}}{(f\,x)^{3/2}}\,dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$-\frac{2\,d\,\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,\sqrt{f\,x}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,e\,(f\,x)^{3/2}\,\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,f^3\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 1383 leaves) :

$$
\begin{aligned}
&\frac{1}{147\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,x\,\Bigg(42\,\left(-7\,d+e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)^2+\\
&\left(343\,a\,b\,d\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\\
&\left(c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.\\
&\qquad\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+\\
&\left(98\,a^2\,e\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\\
&\left(c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.\\
&\qquad\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+\\
&\left(462\,a\,d\,x^4\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\\
&\left(11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.\\
&\qquad x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\\
&\qquad\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+\\
&\left(33\,a\,b\,e\,x^4\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\\
&\left(c\,\left(11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.\\
&\qquad\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Bigg)
\end{aligned}
$$

Problem 208: Result more than twice size of optimal antiderivative.

$$\int (f\,x)^{3/2}\,\left(d+e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\,dx$$

Optimal (type 6, 299 leaves, 6 steps):

$$\frac{2\,a\,d\,(f\,x)^{5/2}\,\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,a\,e\,(f\,x)^{9/2}\,\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\left[\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{9\,f^3\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 4499 leaves):

$$\frac{1}{x^{3/2}}(f\,x)^{3/2}\,\sqrt{a+b\,x^2+c\,x^4}\,\left(\frac{8\,\left(-147\,b^3\,c\,d+924\,a\,b\,c^2\,d+77\,b^4\,e-501\,a\,b^2\,c\,e+612\,a^2\,c^2\,e\right)\,\sqrt{x}}{69\,615\,c^3}+\right.$$

$$\left.\frac{2\,\left(84\,b^2\,c\,d+1911\,a\,c^2\,d-44\,b^3\,e+240\,a\,b\,c\,e\right)\,x^{5/2}}{13\,923\,c^2}+\frac{2\,\left(399\,b\,c\,d+12\,b^2\,e+425\,a\,c\,e\right)\,x^{9/2}}{4641\,c}+\frac{2}{357}\,\left(21\,c\,d+23\,b\,e\right)\,x^{13/2}+\frac{2}{21}\,c\,e\,x^{17/2}\right)-$$

$$\left(56\,a^3\,b^3\,d\,(f\,x)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(663\,c^2\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(352\,a^4\,b\,d\,(f\,x)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(663\,c\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(88\,a^3\,b^4\,e\,(f\,x)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(1989\,c^3\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\Bigg(-5\,a\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{5}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{9}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(1336\,a^4\,b^2\,e\,(f\,x)^{3/2}\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{5}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(4641\,c^2\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,x\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-5\,a\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{5}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{9}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(32\,a^5\,e\,(f\,x)^{3/2}\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{5}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(91\,c\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,x\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-5\,a\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{5}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{9}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(96\,a^4\,d\,x\,(f\,x)^{3/2}\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(85\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{13}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(504\,a^2\,b^4\,d\,x\,(f\,x)^{3/2}\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(5525\,c^2\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{9}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{9}{4},\ \frac{3}{2},\ \frac{1}{2},\ \frac{13}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(3768\,a^3\,b^2\,d\,x\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg/$$

$$\Bigg(5525\,c\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\Bigg(\Bigg(b+\sqrt{b^2-4\,a\,c}\Bigg)\text{AppellF1}\Big[\frac{9}{4},\ \frac{1}{2},\ \frac{3}{2},\ \frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{9}{4},\ \frac{3}{2},\ \frac{1}{2},\ \frac{13}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(264\,a^2\,b^5\,e\,x\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg/$$

$$\Bigg(5525\,c^3\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\Bigg(\Bigg(b+\sqrt{b^2-4\,a\,c}\Bigg)\text{AppellF1}\Big[\frac{9}{4},\ \frac{1}{2},\ \frac{3}{2},\ \frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{9}{4},\ \frac{3}{2},\ \frac{1}{2},\ \frac{13}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(2032\,a^3\,b^3\,e\,x\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg/$$

$$\Bigg(5525\,c^2\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\Bigg(\Bigg(b+\sqrt{b^2-4\,a\,c}\Bigg)\text{AppellF1}\Big[\frac{9}{4},\ \frac{1}{2},\ \frac{3}{2},\ \frac{13}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{9}{4},\ \frac{3}{2},\ \frac{1}{2},\ \frac{13}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(26\,688\,a^4\,b\,e\,x\,(f\,x)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg/$$

$$\Bigg(38\,675\,c\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\Bigg(-9\,a\,\text{AppellF1}\Big[\frac{5}{4},\ \frac{1}{2},\ \frac{1}{2},\ \frac{9}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\Bigg(\Bigg(b+\sqrt{b^2-4\,a\,c}\Bigg)\text{AppellF1}\Big[\frac{9}{4},\ \frac{1}{2},\ \frac{3}{2},\ \frac{13}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)\Big)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \sqrt{f x}\,\left(d+e x^2\right)\left(a+b x^2+c x^4\right)^{3/2}\,dx$$

Optimal (type 6, 299 leaves, 6 steps):

$$\frac{2 a d (f x)^{3/2} \sqrt{a+b x^2+c x^4}\,\text{AppellF1}\left[\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{3 f \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\,\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}}+$$

$$\frac{2 a e (f x)^{7/2} \sqrt{a+b x^2+c x^4}\,\text{AppellF1}\left[\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{7 f^3 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\,\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}}$$

Result (type 6, 3656 leaves):

$$\frac{1}{\sqrt{x}}\sqrt{f x}\,\sqrt{a+b x^2+c x^4}$$

$$\Big(\frac{2\left(228 b^2 c d+3971 a c^2 d-108 b^3 e+624 a b c e\right) x^{3/2}}{21945 c^2}+\frac{2\left(323 b c d+12 b^2 e+345 a c e\right) x^{7/2}}{3135 c}+\frac{2}{285}\left(19 c d+21 b e\right) x^{11/2}+\frac{2}{19} c e x^{15/2}\Big)-$$

$$\Big(32 a^4 d x \sqrt{f x}\,\Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(15\Big(b-\sqrt{b^2-4 a c}\Big)\Big(b+\sqrt{b^2-4 a c}\Big)\left(a+b x^2+c x^4\right)^{3/2}$$

$$\Big(-7 a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\Big(b-\sqrt{b^2-4 a c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)+$$

$$\Big(8 a^3 b^2 d x \sqrt{f x}\,\Big(b-\sqrt{b^2-4 a c}+2 c x^2\Big)\Big(b+\sqrt{b^2-4 a c}+2 c x^2\Big)\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(55 c\Big(b-\sqrt{b^2-4 a c}\Big)\Big(b+\sqrt{b^2-4 a c}\Big)\left(a+b x^2+c x^4\right)^{3/2}$$

$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{7}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{11}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Big)-$$

$$\left(72\,a^3\,b^3\,e\,x\,\sqrt{f\,x}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{7}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(1045\,c^2\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{7}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{11}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Big)+$$

$$\left(416\,a^4\,b\,e\,x\,\sqrt{f\,x}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{7}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(1045\,c\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{7}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{11}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Big)-$$

$$\left(288\,a^3\,b\,d\,x^3\,\sqrt{f\,x}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(245\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-11\,a\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{15}{4},\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\,\frac{3}{2},\,\frac{1}{2},\,\frac{15}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Big)+$$

$$\left(8\,a^2\,b^3\,d\,x^3\,\sqrt{f\,x}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(49\,c\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-11\,a\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,\frac{1}{2},\,\frac{11}{4},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{15}{4},\right.\right.\right.$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)\Big)-$$

$$\Big(96 a^4 e x^3 \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(133\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3/2}$$

$$\Big(-11 a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)\Big)-$$

$$\Big(72 a^2 b^4 e x^3 \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(931 c^2\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3/2}$$

$$\Big(-11 a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)\Big)+$$

$$\Big(2472 a^3 b^2 e x^3 \sqrt{f x}\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big/$$

$$\Big(4655 c\left(b-\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}\right)\left(a+b x^2+c x^4\right)^{3/2}$$

$$\Big(-11 a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4 a c}\Big)\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big)\Big)\Big)$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(d+e x^2\right)\left(a+b x^2+c x^4\right)^{3/2}}{\sqrt{f x}}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\frac{2\,a\,d\,\sqrt{f\,x}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{2\,a\,e\,(f\,x)^{5/2}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f^3\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 3656 leaves):

$$\frac{1}{\sqrt{f\,x}}$$

$$\sqrt{x}\,\sqrt{a+b\,x^2+c\,x^4}\,\left(\frac{2\,\left(68\,b^2\,c\,d+867\,a\,c^2\,d-28\,b^3\,e+176\,a\,b\,c\,e\right)\sqrt{x}}{3315\,c^2}+\frac{2\,\left(85\,b\,c\,d+4\,b^2\,e+91\,a\,c\,e\right)\,x^{5/2}}{663\,c}+\frac{2}{221}\,\left(17\,c\,d+19\,b\,e\right)\,x^{9/2}+\frac{2}{17}\,c\,e\,x^{13/2}\right)-$$

$$\left(96\,a^4\,d\,x\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(13\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(8\,a^3\,b^2\,d\,x\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(39\,c\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$

$$\left(56\,a^3\,b^3\,e\,x\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(663\,c^2\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\Big)\right)+$$

$$\left(352\,a^4\,b\,e\,x\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(663\,c\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\right)\Big)-$$

$$\left(672\,a^3\,b\,d\,x^3\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(325\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\right)\Big)+$$

$$\left(72\,a^2\,b^3\,d\,x^3\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(325\,c\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.$$

$$\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Big)\Big)\right)\Big)-$$

$$\left(96\,a^4\,e\,x^3\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\Big(85\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.$$

$$-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big)\Big)-$$

$$\left(504a^2b^4ex^3\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]\right)\Big/$$

$$\Big(5525c^2\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)\sqrt{fx}\,\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big(-9a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4ac}\Big)\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big)+$$

$$\left(3768a^3b^2ex^3\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]\right)\Big/$$

$$\Big(5525c\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)\sqrt{fx}\,\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big(-9a\,\text{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4ac}\Big)\text{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},$$

$$-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\Big]\Big)\Big)\Big)$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$-\frac{2ad\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\Big[-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\Big]}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+$$

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\Big[\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\Big]}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Result (type 6, 2839 leaves):

$$\frac{x^{3/2}\sqrt{a+b\,x^2+c\,x^4}\,\left(-\frac{2\,a\,d}{\sqrt{x}}+\frac{2\,\left(195\,b\,c\,d+12\,b^2\,e+209\,a\,c\,e\right)\,x^{3/2}}{1155\,c}+\frac{2}{165}\,\left(15\,c\,d+17\,b\,e\right)\,x^{7/2}+\frac{2}{15}\,c\,e\,x^{11/2}\right)}{(f\,x)^{3/2}}-$$

$$\left(128\,a^3\,b\,d\,x^3\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(11\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$

$$\left(32\,a^4\,e\,x^3\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(15\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(8\,a^3\,b^2\,e\,x^3\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(55\,c\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$

$$\left(24\,a^2\,b^2\,d\,x^5\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(49\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}\right.$$
$$\left(-11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},\right.\right.\right.$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(96\,a^3\,c\,d\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(7\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\Big(288\,a^3\,b\,e\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(245\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$

$$\Big(8\,a^2\,b^3\,e\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(49\,c\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Big(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int\frac{(f\,x)^{3/2}\,\big(d+e\,x^2\big)}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\frac{2\,d\,(f\,x)^{5/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{2\,e\,(f\,x)^{9/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{9\,f^3\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 1037 leaves) :

$$\frac{1}{50\,c^2\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,f\,\sqrt{f\,x}\,\Bigg(20\,c\,e\,\left(a+b\,x^2+c\,x^4\right)^2+$$
$$\left(25\,a^2\,e\left(-b+\sqrt{b^2-4\,a\,c}-2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$
$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$
$$\left(45\,a\,c\,d\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$
$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$
$$\left(27\,a\,b\,e\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.$$
$$\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg)$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{f\,x}\,\left(d+e\,x^2\right)}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 297 leaves, 6 steps):

$$\frac{2\,d\,(f\,x)^{3/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,f\,\sqrt{a+b\,x^2+c\,x^4}}+\frac{2\,e\,(f\,x)^{7/2}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{7\,f^3\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 642 leaves):

$$\frac{1}{42\,c\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,a\,x\,\sqrt{f\,x}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(-\left(\left(49\,d\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(-7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-\left(33\,e\,x^2\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(-11\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e\,x^2}{\sqrt{f\,x}\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 295 leaves, 6 steps):

$$\frac{2\,d\,\sqrt{f\,x}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,\sqrt{a+b\,x^2+c\,x^4}}+$$
$$\frac{2\,e\,(f\,x)^{5/2}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{5\,f^3\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 642 leaves) :

$$\frac{1}{10\,c\,\sqrt{f\,x}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}$$
$$a\,x\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(-\left(\left(25\,d\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/\right.$$
$$\left(-5\,a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$
$$\left(9\,e\,x^2\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(-9\,a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e\,x^2}{(f\,x)^{3/2}\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 6, 295 leaves, 6 steps) :

$$-\frac{2\,d\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,\sqrt{f\,x}\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{2\,e\,(f\,x)^{3/2}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{3\,f^3\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 1049 leaves) :

$$\frac{1}{21\,a\,(f\,x)^{3/2}\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,2\,x\,\Bigg(-21\,d\,\left(a+b\,x^2+c\,x^4\right)^2+$$

$$\left(49\,a\,b\,d\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(4\,c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(49\,a^2\,e\,x^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(4\,c\,\left(7\,a\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(99\,a\,d\,x^4\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(44\,a\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$4\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\Bigg)$$

Problem 216: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^{3/2} \left(d + e x^2\right)}{\left(a + b x^2 + c x^4\right)^{3/2}} \, dx$$

Optimal (type 6, 303 leaves, 6 steps):

$$\frac{2 d (f x)^{5/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \,\text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]}{5 a f \sqrt{a + b x^2 + c x^4}} +$$
$$\frac{2 e (f x)^{9/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \,\text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]}{9 a f^3 \sqrt{a + b x^2 + c x^4}}$$

Result (type 6, 1404 leaves):

$$\frac{1}{5\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} f\sqrt{fx}\,\Bigg(5\left(-bd+2ae-2cdx^2+bex^2\right)\left(a+bx^2+cx^4\right)+$$

$$\left(25a^2e\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(2c\left(5a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)+$$

$$\left(25abd\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(4c\left(5a\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)+$$

$$\left(9adx^2\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(18a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$2x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\right.$$

$$\left.\left.\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)-$$

$$\left(9abex^2\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(b+\sqrt{b^2-4ac}+2cx^2\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(4c\left(9a\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]-x^2\left(\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]+\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)\Bigg)$$

Problem 217: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{fx}\,\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 6, 303 leaves, 6 steps):

$$\frac{2 d (f x)^{3/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{3 a f \sqrt{a+b x^2+c x^4}}+$$
$$\frac{2 e (f x)^{7/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{7 a f^3 \sqrt{a+b x^2+c x^4}}$$

Result (type 6, 1740 leaves):

$$\frac{1}{84 a \left(-b^2+4 a c\right) \left(a+b x^2+c x^4\right)^{3/2}} x \sqrt{f x} \left(84 \left(a+b x^2+c x^4\right) \left(-b^2 d+b \left(a e-c d x^2\right)+2 a c \left(d+e x^2\right)\right)+\right.$$
$$\left(196 a^2 d \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(14 a \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-2 x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+$$
$$\left(49 a b^2 d \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(c \left(7 a \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)-$$
$$\left(147 a^2 b e \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(c \left(7 a \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)-$$
$$\left(99 a b d x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(-11 a \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+$$

$$\left(198 a^2 e x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$

$$\left(-11 a \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{\sqrt{f x} \left(a+b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 6, 301 leaves, 6 steps):

$$\frac{2 d \sqrt{f x} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{a f \sqrt{a+b x^2+c x^4}}+\frac{2 e (f x)^{5/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{5 a f^3 \sqrt{a+b x^2+c x^4}}$$

Result (type 6, 1740 leaves):

$$\frac{1}{20 a \left(-b^2+4 a c\right) \sqrt{f x} \left(a+b x^2+c x^4\right)^{3/2}} \times \left(20 \left(a+b x^2+c x^4\right) \left(-b^2 d+b \left(a e-c d x^2\right)+2 a c \left(d+e x^2\right)\right)+\right.$$

$$\left(300 a^2 d \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$

$$\left(10 a \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-2 x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},\right.\right.\right.$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\mathrm{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)-$$

$$\left(25\,a\,b^2\,d\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\mathrm{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\right)\Big/$$

$$\Big(c\,\Big(5\,a\,\mathrm{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},$$

$$\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\left(25\,a^2\,b\,e\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\mathrm{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\right)\Big/$$

$$\Big(c\,\Big(5\,a\,\mathrm{AppellF1}\Big[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},$$

$$\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-$$

$$\left(9\,a\,b\,d\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\right)\Big/$$

$$\Big(-9\,a\,\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\left(b-\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)+$$

$$\left(18\,a^2\,e\,x^2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\right)\Big/$$

$$\Big(-9\,a\,\mathrm{AppellF1}\Big[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$x^2\,\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$

$$\left(b-\sqrt{b^2-4\,a\,c}\right)\mathrm{AppellF1}\Big[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)$$

Problem 219: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{(f x)^{3/2} \left(a+b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 6, 301 leaves, 6 steps):

$$-\frac{2 d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{a f \sqrt{f x} \sqrt{a+b x^2+c x^4}}+$$
$$\frac{2 e (f x)^{3/2} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{3 a f^3 \sqrt{a+b x^2+c x^4}}$$

Result (type 6, 2959 leaves):

$$\frac{x^{3/2} \sqrt{a+b x^2+c x^4} \left(-\frac{2 d}{a^2 \sqrt{x}}+\frac{b^3 d x^{3/2}-3 a b c d x^{3/2}-a b^2 e x^{3/2}+2 a^2 c e x^{3/2}+b^2 c d x^{7/2}-2 a c^2 d x^{7/2}-a b c e x^{7/2}}{a^2 \left(-b^2+4 a c\right) \left(a+b x^2+c x^4\right)}\right)}{(f x)^{3/2}}+$$
$$\left(7 b^3 d x^3 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(\left(-b^2+4 a c\right) \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) (f x)^{3/2} \left(a+b x^2+c x^4\right)^{3/2}\right.$$
$$\left(-7 a \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)-$$
$$\left(21 a b c d x^3 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(\left(-b^2+4 a c\right) \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) (f x)^{3/2} \left(a+b x^2+c x^4\right)^{3/2}\right.$$
$$\left(-7 a \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)-$$
$$\left(7 a b^2 e x^3 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(3 \left(-b^2+4 a c\right) \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) (f x)^{3/2} \left(a+b x^2+c x^4\right)^{3/2}\right.$$

$$\Bigg(-7\,a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(14\,a^2\,c\,e\,x^3\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(3\,\big(-b^2+4\,a\,c\big)\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-7\,a\,\text{AppellF1}\Big[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(99\,b^2\,c\,d\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(7\,\big(-b^2+4\,a\,c\big)\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(330\,a\,c^2\,d\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(7\,\big(-b^2+4\,a\,c\big)\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)-$$

$$\Bigg(33\,a\,b\,c\,e\,x^5\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\Big)\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg(7\,\big(-b^2+4\,a\,c\big)\,\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,(f\,x)^{3/2}\,\big(a+b\,x^2+c\,x^4\big)^{3/2}$$

$$\Bigg(-11\,a\,\text{AppellF1}\Big[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\Big[\frac{11}{4},\ \frac{3}{2},\ \frac{1}{2},\ \frac{15}{4},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\ \frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)$$

Problem 223: Result is not expressed in closed-form.

$$\int\frac{(f\,x)^m\,\left(d+e\,x^2\right)}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 5, 194 leaves, 3 steps):

$$\frac{\left(e+\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\,(f\,x)^{1+m}\,\text{Hypergeometric2F1}\Big[1,\ \frac{1+m}{2},\ \frac{3+m}{2},\ -\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\Big]}{\left(b-\sqrt{b^2-4\,a\,c}\right)\,f\,(1+m)}+\frac{\left(e-\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\,(f\,x)^{1+m}\,\text{Hypergeometric2F1}\Big[1,\ \frac{1+m}{2},\ \frac{3+m}{2},\ -\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,f\,(1+m)}$$

Result (type 7, 316 leaves):

$$\frac{d\,(f\,x)^m\,\text{RootSum}\Big[a+b\,\sharp1^2+c\,\sharp1^4\,\&,\ \frac{\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\sharp1}{x-\sharp1}\right]\,\left(\frac{x}{x-\sharp1}\right)^{-m}}{b\,\sharp1+2\,c\,\sharp1^3}\,\&\Big]}{2\,m}+$$
$$\frac{1}{2\,m\,(1+m)\,(2+m)}e\,(f\,x)^m\,\text{RootSum}\Big[a+b\,\sharp1^2+c\,\sharp1^4\,\&,\ \frac{1}{b\,\sharp1+2\,c\,\sharp1^3}\Big(m\,x^2+m^2\,x^2+2\,m\,x\,\sharp1+m^2\,x\,\sharp1+$$
$$2\,\text{Hypergeometric2F1}\Big[-m,\ -m,\ 1-m,\ -\frac{\sharp1}{x-\sharp1}\Big]\,\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+3\,m\,\text{Hypergeometric2F1}\Big[-m,\ -m,\ 1-m,\ -\frac{\sharp1}{x-\sharp1}\Big]\,\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+$$
$$m^2\,\text{Hypergeometric2F1}\Big[-m,\ -m,\ 1-m,\ -\frac{\sharp1}{x-\sharp1}\Big]\,\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+m\,\left(\frac{x}{\sharp1}\right)^{-m}\sharp1^2\Big)\,\&\Big]$$

Problem 224: Result unnecessarily involves higher level functions.

$$\int\frac{(f\,x)^m\,\left(d+e\,x^2\right)}{\left(a+b\,x^2+c\,x^4\right)^2}\,dx$$

Optimal (type 5, 392 leaves, 4 steps):

$$\frac{(f x)^{1+m}\left(b^2 d-2 a c d-a b e+c\left(b d-2 a e\right) x^2\right)}{2 a\left(b^2-4 a c\right) f\left(a+b x^2+c x^4\right)}+$$
$$\left(c\left(b\left(4 a e+\sqrt{b^2-4 a c}\; d(1-m)\right)-2 a\left(\sqrt{b^2-4 a c}\; e(1-m)+2 c d(3-m)\right)+b^2(d-d m)\right)(f x)^{1+m}\right.$$
$$\left.\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}\right]\right)\Big/\left(2 a\left(b^2-4 a c\right)^{3/2}\left(b-\sqrt{b^2-4 a c}\right) f(1+m)\right)-$$
$$\left(c\left(b\left(4 a e-\sqrt{b^2-4 a c}\; d(1-m)\right)+2 a\left(\sqrt{b^2-4 a c}\; e(1-m)-2 c d(3-m)\right)+b^2 d(1-m)\right)(f x)^{1+m}\right.$$
$$\left.\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(2 a\left(b^2-4 a c\right)^{3/2}\left(b+\sqrt{b^2-4 a c}\right) f(1+m)\right)$$

Result (type 6, 692 leaves):

$$\frac{1}{4 c(3+m)\left(a+b x^2+c x^4\right)^3}$$
$$a x(f x)^m\left(b-\sqrt{b^2-4 a c}+2 c x^2\right)\left(b+\sqrt{b^2-4 a c}+2 c x^2\right)\left(\left(d(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\right.$$
$$\left((1+m)\left(a(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]-2 x^2\left(\left(b+\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{3+m}{2},2,3,\frac{5+m}{2},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)+$$
$$\left(e(5+m) x^2\,\text{AppellF1}\left[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(a(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},2,2,\frac{5+m}{2},\right.\right.$$
$$-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\Big]-2 x^2\left(\left(b+\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{5+m}{2},2,3,\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right)\text{AppellF1}\left[\frac{5+m}{2},3,2,\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int (f x)^m\left(d+e x^2\right)\left(a+b x^2+c x^4\right)^{3/2} dx$$

Optimal (type 6, 319 leaves, 6 steps):

$$\frac{a\,d\,(f\,x)^{1+m}\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\Big[\frac{1+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{f\,(1+m)\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{a\,e\,(f\,x)^{3+m}\sqrt{a+b\,x^2+c\,x^4}\;\text{AppellF1}\Big[\frac{3+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{f^3\,(3+m)\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 2559 leaves) :

$$\Big(a\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)d\,(3+m)\,x\,(f\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)$$
$$\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\text{AppellF1}\Big[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$
$$\Big(8\,c^2\,(1+m)\sqrt{a+b\,x^2+c\,x^4}\,\Big(2\,a\,(3+m)\,\text{AppellF1}\Big[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$
$$x^2\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{3+m}{2},-\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$
$$\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{3+m}{2},\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$
$$\Big(b\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)d\,(5+m)\,x^3\,(f\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)$$
$$\text{AppellF1}\Big[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$
$$\Big(8\,c^2\,(3+m)\sqrt{a+b\,x^2+c\,x^4}\,\Big(2\,a\,(5+m)\,\text{AppellF1}\Big[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$
$$x^2\Big(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{5+m}{2},-\frac{1}{2},\frac{1}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+$$
$$\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{5+m}{2},\frac{1}{2},-\frac{1}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)+$$
$$\Big(a\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)e\,(5+m)\,x^3\,(f\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)$$
$$\text{AppellF1}\Big[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\left(8 c^2 (3+m) \sqrt{a+b x^2+c x^4} \left(2 a (5+m) \text{AppellF1}\left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) +$$

$$\left(\left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) d (7+m) x^5 (f x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(8 c (5+m) \sqrt{a+b x^2+c x^4} \left(2 a (7+m) \text{AppellF1}\left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) +$$

$$\left(b \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) e (7+m) x^5 (f x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(8 c^2 (5+m) \sqrt{a+b x^2+c x^4} \left(2 a (7+m) \text{AppellF1}\left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) +$$

$$\left(\left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) e (9+m) x^7 (f x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(8 c (7+m) \sqrt{a+b x^2+c x^4} \left(2 a (9+m) \text{AppellF1}\left[\frac{7+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{9+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{11+m}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] +\right.\right.\right.$$

$$\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{9+m}{2},\frac{1}{2},-\frac{1}{2},\frac{11+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 226: Result more than twice size of optimal antiderivative.

$$\int (f\,x)^m\,\left(d+e\,x^2\right)\,\sqrt{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 6, 317 leaves, 6 steps):

$$\frac{d\,(f\,x)^{1+m}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f\,(1+m)\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}+$$

$$\frac{e\,(f\,x)^{3+m}\,\sqrt{a+b\,x^2+c\,x^4}\,\text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{f^3\,(3+m)\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}$$

Result (type 6, 755 leaves):

$$\frac{1}{8 c^2 (3+m) \sqrt{a+b x^2+c x^4}} \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) x (f x)^m \left(b-\sqrt{b^2-4 a c}+2 c x^2\right)$$
$$\left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \left(\left(d (3+m)^2 \text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \right.$$
$$\left((1+m) \left(2 a (3+m) \text{AppellF1}\left[\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$
$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{3+m}{2},\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)+$$
$$\left(e (5+m) x^2 \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(2 a (5+m) \text{AppellF1}\left[\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$
$$x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2},-\frac{1}{2},\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{5+m}{2},\frac{1}{2},-\frac{1}{2},\frac{7+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^m \left(d+e x^2\right)}{\sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 6, 317 leaves, 6 steps):

$$\frac{d (f x)^{1+m} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{f (1+m) \sqrt{a+b x^2+c x^4}}+$$
$$\frac{e (f x)^{3+m} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{AppellF1}\left[\frac{3+m}{2},\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]}{f^3 (3+m) \sqrt{a+b x^2+c x^4}}$$

Result (type 6, 728 leaves):

$$\frac{1}{2\,c\,(3+m)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,a\,x\,(f\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\Bigg(\Big(d\,(3+m)^2\,\text{AppellF1}\Big[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big((1+m)\Big(2\,a\,(3+m)\,\text{AppellF1}\Big[\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-x^2\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{3+m}{2},\frac{1}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{3+m}{2},\frac{3}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big)-\Big(e\,(5+m)\,x^2\,\text{AppellF1}\Big[\frac{3+m}{2},\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/\Big(-2\,a\,(5+m)\,\text{AppellF1}\Big[\frac{3+m}{2},\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+x^2\Big(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5+m}{2},\frac{1}{2},\frac{3}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,\text{AppellF1}\Big[\frac{5+m}{2},\frac{3}{2},\frac{1}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Bigg)$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int\frac{(f\,x)^m\left(d+e\,x^2\right)}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 323 leaves, 6 steps):

$$\frac{d\,(f\,x)^{1+m}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\text{AppellF1}\Big[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{a\,f\,(1+m)\,\sqrt{a+b\,x^2+c\,x^4}}+\frac{e\,(f\,x)^{3+m}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\text{AppellF1}\Big[\frac{3+m}{2},\frac{3}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\Big]}{a\,f^3\,(3+m)\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 6, 728 leaves):

$$\frac{1}{2\,c\,(3+m)\,\left(a+b\,x^2+c\,x^4\right)^{5/2}}$$

$$a\,x\,(f\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(\left(d\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left((1+m)\,\left(2\,a\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.\right.$$

$$3\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{3+m}{2},\frac{3}{2},\frac{5}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{3+m}{2},\frac{5}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(e\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},\frac{3}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left(2\,a\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},\frac{3}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$3\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5+m}{2},\frac{3}{2},\frac{5}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$

$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5+m}{2},\frac{5}{2},\frac{3}{2},\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

Problem 259: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(1+x^2\right)\,\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{2\,\sqrt{2}}+\frac{\left(1+x^2\right)\,\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\,\sqrt{1+x^4}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4}\,\left(-\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]+\text{EllipticPi}\left[-i,\,i\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]\right)$$

Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 70 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-\text{EllipticPi}\left[i,\text{ArcSin}\left[(-1)^{3/4}x\right],-1\right]\right)$$

Problem 265: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(1+x^2\right)\sqrt{-1-x^4}}\,dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right]}{2\sqrt{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{-1-x^4}}$$

Result (type 4, 60 leaves):

$$\frac{(-1)^{1/4}\sqrt{1+x^4}\left(-\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]+\text{EllipticPi}\left[-i,i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)}{\sqrt{-1-x^4}}$$

Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(1-x^2\right)\sqrt{-1-x^4}}\,dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right]}{2\sqrt{2}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{-1-x^4}}$$

Result (type 4, 56 leaves):

$$\frac{(-1)^{1/4}\sqrt{1+x^4}\left(\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-\text{EllipticPi}\left[\text{i},\text{ArcSin}\left[(-1)^{3/4}x\right],-1\right]\right)}{\sqrt{-1-x^4}}$$

Problem 310: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{f\,x}\,\left(d+e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 866 leaves, 19 steps):

$$\frac{c^{3/4}\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{f\,x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}\sqrt{f}}\right]}{2^{1/4}\sqrt{b^2-4\,a\,c}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}-$$

$$\frac{c^{3/4}\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e\right)\text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{f\,x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}\sqrt{f}}\right]}{2^{1/4}\sqrt{b^2-4\,a\,c}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}-\frac{e^{7/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,e^{1/4}\sqrt{f\,x}}{d^{1/4}\sqrt{f}}\right]}{\sqrt{2}\,d^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}+\frac{e^{7/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,e^{1/4}\sqrt{f\,x}}{d^{1/4}\sqrt{f}}\right]}{\sqrt{2}\,d^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}+$$

$$\frac{c^{3/4}\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{f\,x}}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}\sqrt{f}}\right]}{2^{1/4}\sqrt{b^2-4\,a\,c}\,\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}-\frac{c^{3/4}\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e\right)\text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{f\,x}}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}\sqrt{f}}\right]}{2^{1/4}\sqrt{b^2-4\,a\,c}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}-$$

$$\frac{e^{7/4}\,\text{Log}\left[\sqrt{d}\,\sqrt{f}+\sqrt{e}\,\sqrt{f}\,x-\sqrt{2}\,d^{1/4}e^{1/4}\sqrt{f\,x}\right]}{2\sqrt{2}\,d^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}+\frac{e^{7/4}\,\text{Log}\left[\sqrt{d}\,\sqrt{f}+\sqrt{e}\,\sqrt{f}\,x+\sqrt{2}\,d^{1/4}e^{1/4}\sqrt{f\,x}\right]}{2\sqrt{2}\,d^{3/4}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{f}}$$

Result (type 7, 267 leaves):

$$\Bigg(\sqrt{x}\,\Bigg(\sqrt{2}\,e^{7/4}$$

$$\left(-2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,e^{1/4}\sqrt{x}}{d^{1/4}}\right]+2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,e^{1/4}\sqrt{x}}{d^{1/4}}\right]-\text{Log}\left[\sqrt{d}-\sqrt{2}\,d^{1/4}e^{1/4}\sqrt{x}+\sqrt{e}\,x\right]+\text{Log}\left[\sqrt{d}+\sqrt{2}\,d^{1/4}e^{1/4}\sqrt{x}+\sqrt{e}\,x\right]\right)-$$

$$2\,d^{3/4}\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\frac{-c\,d\,\text{Log}\left[\sqrt{x}-\#1\right]+b\,e\,\text{Log}\left[\sqrt{x}-\#1\right]+c\,e\,\text{Log}\left[\sqrt{x}-\#1\right]\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]\Bigg)\Bigg)\Big/\left(4\,d^{3/4}\left(c\,d^2+e\left(-b\,d+a\,e\right)\right)\sqrt{f\,x}\right)$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \sqrt{1+2x^2+2x^4}}{3+2x^2} \, dx$$

Optimal (type 4, 424 leaves, 17 steps):

$$-\frac{1}{60} x \left(13-6x^2\right) \sqrt{1+2x^2+2x^4} + \frac{109 x \sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}x^2\right)} +$$
$$\frac{3}{16}\sqrt{15}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right] - \frac{109\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{60\times 2^{3/4}\sqrt{1+2x^2+2x^4}} +$$
$$\frac{\left(-70+263\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{60\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} +$$
$$\frac{15\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{16\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 209 leaves):

$$\frac{1}{240\sqrt{1+2x^2+2x^4}}\left(-52x-80x^3-56x^5+48x^7-218\,i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$(199-417\,i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.225\,(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \sqrt{1+2x^2+2x^4}}{3+2x^2} \, dx$$

Optimal (type 4, 417 leaves, 13 steps):

$$\frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{8}\sqrt{15}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]+$$

$$\frac{7\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{6\times 2^{3/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{\left(-4+17\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{6\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{8\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 204 leaves):

$$\frac{1}{24\sqrt{1+2x^2+2x^4}}\left(4x+8x^3+8x^5+14\,i\sqrt{1-i}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+\right.$$
$$\left(13-27\,i\right)\sqrt{1-i}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$\left.15\,(1-i)^{3/2}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 318: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$

Optimal (type 4, 381 leaves, 7 steps):

$$\frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{2^{3/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}+$$

$$\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{12\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 127 leaves) :

$$-\frac{1}{6\sqrt{1-i}\,\sqrt{1+2x^2+2x^4}}\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}$$
$$\left((3+3i)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-(3+6i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+5\,i\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 319: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3+2x^2\right)}\,dx$$

Optimal (type 4, 399 leaves, 8 steps) :

$$-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{6}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-$$

$$\frac{2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{3\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{21\times 2^{1/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{252\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 208 leaves):

$$\frac{1}{18\,x\sqrt{1+2x^2+2x^4}}\left(-6-12x^2-12x^4-6\,i\sqrt{1-i}\,x\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+\right.$$
$$(9-3\,i)\sqrt{1-i}\,x\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$\left.5\,(1-i)^{3/2}\,x\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 320: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx$$

Optimal (type 4, 360 leaves, 7 steps):

$$-\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\,\text{ArcTan}\Big[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\Big]-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\Big[2\,\text{ArcTan}\big[2^{1/4}\,x\big],\frac{1}{4}\left(2-\sqrt{2}\right)\Big]}{9\times 2^{1/4}\sqrt{1+2x^2+2x^4}}+$$
$$\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\Big[2\,\text{ArcTan}\big[2^{1/4}\,x\big],\frac{1}{4}\left(2-\sqrt{2}\right)\Big]}{63\times 2^{1/4}\sqrt{1+2x^2+2x^4}}-$$
$$\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\Big[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\big[2^{1/4}\,x\big],\frac{1}{4}\left(2-\sqrt{2}\right)\Big]}{378\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 154 leaves) :

$$-\frac{1}{27x^3\sqrt{1+2x^2+2x^4}}\Big(3+6x^2+6x^4+3\,(1-i)^{3/2}x^3\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\big[i\,\text{ArcSinh}\big[\sqrt{1-i}\,x\big],i\big]-$$
$$5\,(1-i)^{3/2}x^3\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\Big[\frac{1}{3}+\frac{i}{3},\,i\,\text{ArcSinh}\big[\sqrt{1-i}\,x\big],i\Big]\Big)$$

Problem 321: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx$$

Optimal (type 4, 546 leaves, 13 steps) :

$$-\frac{\sqrt{1+2x^2+2x^4}}{15x^5}+\frac{4\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{4\sqrt{1+2x^2+2x^4}}{45x}+\frac{4\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}\,x^2\right)}-$$

$$\frac{2}{27}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{4\times 2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{45\sqrt{1+2x^2+2x^4}}+$$

$$\frac{5\times 2^{1/4}\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{189\sqrt{1+2x^2+2x^4}}-$$

$$\frac{2^{1/4}\left(19-2\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{135\sqrt{1+2x^2+2x^4}}+$$

$$\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{567\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 224 leaves) :

$$-\frac{1}{405x^5\sqrt{1+2x^2+2x^4}}\left(27+42x^2+66x^4+48x^6+72x^8+36\,i\sqrt{1-i}\,x^5\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$\left(12+24\,i\right)\sqrt{1-i}\,x^5\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.50\,(1-i)^{3/2}\,x^5\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 327: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$$

Optimal (type 4, 463 leaves, 19 steps) :

$$-\frac{213}{140}x\sqrt{1+2x^2+2x^4}-\frac{27}{70}x^3\sqrt{1+2x^2+2x^4}-\frac{2211x\sqrt{1+2x^2+2x^4}}{140\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}+$$

$$\frac{17}{16}\sqrt{51}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]+\frac{2211\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{140\times2^{3/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{3\left(514+2717\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{140\times2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12+11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{16\times2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 214 leaves) :

$$\frac{1}{560\sqrt{1+2x^2+2x^4}}\left(-892x-2080x^3-2456x^5-752x^7-160x^9+4422\,i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$\left(9669-5247\,i\right)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.10115\,(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$$

Optimal (type 4, 428 leaves, 12 steps) :

$$-\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}+$$

$$\frac{17}{8}\sqrt{\frac{17}{3}}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]+\frac{103\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{\left(66+383\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12+11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{24\times 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 209 leaves) :

$$\frac{1}{120\sqrt{1+2x^2+2x^4}}\left(-108x-240x^3-264x^5-48x^7+618\,i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$\left(1371-753\,i\right)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.1445\,(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$$

Optimal (type 4, 722 leaves, 13 steps) :

$$-\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+$$

$$\frac{17}{12}\sqrt{\frac{17}{3}}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]+\frac{17\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{3\times 2^{3/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{2^{1/4}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{3\sqrt{1+2x^2+2x^4}}+\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{3\times 2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{84\times 2^{1/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{17\left(5+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{12\times 2^{1/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12+11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{504\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 213 leaves):

$$\frac{1}{36x\sqrt{1+2x^2+2x^4}}\left(-12-36x^2-48x^4-24x^6+90\,i\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$(255-165\,i)\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.289\,(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 330: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx$$

Optimal (type 4, 625 leaves, 13 steps):

$$-\frac{2\sqrt{1+2x^2+2x^4}}{x}-\frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)}+$$

$$\frac{17}{18}\sqrt{\frac{17}{3}}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{9\sqrt{1+2x^2+2x^4}}+$$

$$\frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{126\times 2^{1/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{17\left(5+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{18\times 2^{1/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{2^{1/4}\left(9+5\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{9\sqrt{1+2x^2+2x^4}}-$$

$$\frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12+11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{756\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 219 leaves) :

$$\frac{1}{54x^3\sqrt{1+2x^2+2x^4}}\Bigg(-6-72x^2-132x^4-120x^6-6\,i\sqrt{1-i}\,x^3\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$(195-201\,i)\sqrt{1-i}\,x^3\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$289\,(1-i)^{3/2}x^3\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\Bigg)$$

$$\int\frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx$$

Optimal (type 4, 553 leaves, 15 steps) :

$$\frac{74\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{262\sqrt{1+2x^2+2x^4}}{135x}-\frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5}+\frac{262\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{135\left(1+\sqrt{2}\,x^2\right)}+$$

$$\frac{17}{27}\sqrt{\frac{17}{3}}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{262\times 2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{135\sqrt{1+2x^2+2x^4}}+$$

$$\frac{85\times 2^{3/4}\left(3-\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{189\sqrt{1+2x^2+2x^4}}+$$

$$\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{135\sqrt{1+2x^2+2x^4}}-$$

$$\frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12+11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{1134\times 2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 224 leaves):

$$-\frac{1}{405x^5\sqrt{1+2x^2+2x^4}}\Big(27+192x^2+1116x^4+1848x^6+1572x^8+786\,i\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$(543-1329\,i)\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$1445\,(1-i)^{3/2}x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[-\frac{1}{3}-\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\Big)$$

Problem 337: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

Optimal (type 4, 418 leaves, 4 steps):

$$\frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\mathrm{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]}{4\left(2-3\sqrt{2}\right)}-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{2\times2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(1-3\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{2\times2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}+$$

$$\frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\mathrm{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{8\times2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 127 leaves):

$$-\frac{1}{4\sqrt{1-i}\sqrt{1+2x^2+2x^4}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}$$
$$\left((1+i)\,\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-(1+4i)\,\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+3i\,\mathrm{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\mathrm{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{14\times2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\mathrm{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{56\times2^{1/4}\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 99 leaves):

$$\frac{1}{4\sqrt{1+2x^2+2x^4}}(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\mathrm{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\mathrm{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(3+2 x^2\right) \sqrt{1+2 x^2+2 x^4}} \, dx$$

Optimal (type 4, 245 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right]}{2 \sqrt{15}}+\frac{\left(3+\sqrt{2}\right) \left(1+\sqrt{2} x^2\right) \sqrt{\frac{1+2 x^2+2 x^4}{\left(1+\sqrt{2} x^2\right)^2}} \, \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} \left(2-\sqrt{2}\right)\right]}{14\times 2^{1/4} \sqrt{1+2 x^2+2 x^4}}-\frac{\left(3+\sqrt{2}\right)^2 \left(1+\sqrt{2} x^2\right) \sqrt{\frac{1+2 x^2+2 x^4}{\left(1+\sqrt{2} x^2\right)^2}} \, \text{EllipticPi}\left[\frac{1}{24} \left(12-11 \sqrt{2}\right), 2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} \left(2-\sqrt{2}\right)\right]}{84\times 2^{1/4} \sqrt{1+2 x^2+2 x^4}}$$

Result (type 4, 80 leaves):

$$-\frac{i \sqrt{1+(1-i) x^2} \sqrt{1+(1+i) x^2} \, \text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3}, i \, \text{ArcSinh}\left[\sqrt{1-i} \, x\right], i\right]}{3 \sqrt{1-i} \sqrt{1+2 x^2+2 x^4}}$$

$$\int \frac{1}{x^2 \left(3+2 x^2\right) \sqrt{1+2 x^2+2 x^4}} \, dx$$

Optimal (type 4, 399 leaves, 6 steps):

$$-\frac{\sqrt{1+2 x^2+2 x^4}}{3 x}+\frac{\sqrt{2} x \sqrt{1+2 x^2+2 x^4}}{3 \left(1+\sqrt{2} x^2\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{1+2 x^2+2 x^4}}\right]}{3 \sqrt{15}}-\frac{2^{1/4} \left(1+\sqrt{2} x^2\right) \sqrt{\frac{1+2 x^2+2 x^4}{\left(1+\sqrt{2} x^2\right)^2}} \, \text{EllipticE}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} \left(2-\sqrt{2}\right)\right]}{3 \sqrt{1+2 x^2+2 x^4}}+\frac{\left(5-3 \sqrt{2}\right) \left(1+\sqrt{2} x^2\right) \sqrt{\frac{1+2 x^2+2 x^4}{\left(1+\sqrt{2} x^2\right)^2}} \, \text{EllipticF}\left[2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} \left(2-\sqrt{2}\right)\right]}{21\times 2^{3/4} \sqrt{1+2 x^2+2 x^4}}+\frac{\left(3+\sqrt{2}\right)^2 \left(1+\sqrt{2} x^2\right) \sqrt{\frac{1+2 x^2+2 x^4}{\left(1+\sqrt{2} x^2\right)^2}} \, \text{EllipticPi}\left[\frac{1}{24} \left(12-11 \sqrt{2}\right), 2 \text{ArcTan}\left[2^{1/4} x\right], \frac{1}{4} \left(2-\sqrt{2}\right)\right]}{126\times 2^{1/4} \sqrt{1+2 x^2+2 x^4}}$$

Result (type 4, 147 leaves):

$$-\frac{1}{9\,x\,\sqrt{1+2\,x^2+2\,x^4}}\,\mathrm{i}\,\Big(-3\,\mathrm{i}\,\left(1+2\,x^2+2\,x^4\right)+\sqrt{1-\mathrm{i}}\,x\,\sqrt{1+(1-\mathrm{i})\,x^2}\,\sqrt{1+(1+\mathrm{i})\,x^2}$$
$$\left(3\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]-3\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]-(1+\mathrm{i})\,\text{EllipticPi}\left[\frac{1}{3}+\frac{\mathrm{i}}{3},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]\right)\Big)$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^4\,\left(3+2\,x^2\right)\,\sqrt{1+2\,x^2+2\,x^4}}\,dx$$

Optimal (type 4, 422 leaves, 7 steps):

$$-\frac{\sqrt{1+2\,x^2+2\,x^4}}{9\,x^3}+\frac{2\,\sqrt{1+2\,x^2+2\,x^4}}{3\,x}-\frac{2\,\sqrt{2}\,x\,\sqrt{1+2\,x^2+2\,x^4}}{3\,\left(1+\sqrt{2}\,x^2\right)}+$$
$$\frac{2\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2\,x^2+2\,x^4}}\right]}{9\,\sqrt{15}}+\frac{2\times 2^{1/4}\,\left(1+\sqrt{2}\,x^2\right)\,\sqrt{\frac{1+2\,x^2+2\,x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\,\left(2-\sqrt{2}\right)\right]}{3\,\sqrt{1+2\,x^2+2\,x^4}}-$$
$$\frac{\left(1+19\,\sqrt{2}\right)\,\left(1+\sqrt{2}\,x^2\right)\,\sqrt{\frac{1+2\,x^2+2\,x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\,\left(2-\sqrt{2}\right)\right]}{63\times 2^{1/4}\,\sqrt{1+2\,x^2+2\,x^4}}-$$
$$\frac{\left(3+\sqrt{2}\right)^2\,\left(1+\sqrt{2}\,x^2\right)\,\sqrt{\frac{1+2\,x^2+2\,x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\,\left(12-11\,\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\,\left(2-\sqrt{2}\right)\right]}{189\times 2^{1/4}\,\sqrt{1+2\,x^2+2\,x^4}}$$

Result (type 4, 219 leaves):

$$\frac{1}{27\,x^3\,\sqrt{1+2\,x^2+2\,x^4}}\Big(-3+12\,x^2+30\,x^4+36\,x^6+18\,\mathrm{i}\,\sqrt{1-\mathrm{i}}\,x^3\,\sqrt{1+(1-\mathrm{i})\,x^2}\,\sqrt{1+(1+\mathrm{i})\,x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]-$$
$$(3+15\,\mathrm{i})\,\sqrt{1-\mathrm{i}}\,x^3\,\sqrt{1+(1-\mathrm{i})\,x^2}\,\sqrt{1+(1+\mathrm{i})\,x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]+$$
$$2\,(1-\mathrm{i})^{3/2}\,x^3\,\sqrt{1+(1-\mathrm{i})\,x^2}\,\sqrt{1+(1+\mathrm{i})\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{\mathrm{i}}{3},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{1-\mathrm{i}}\,x\right],\mathrm{i}\right]\Big)$$

Problem 348: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^8}{\left(3+2\,x^2\right)\,\left(1+2\,x^2+2\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 449 leaves, 10 steps):

$$\frac{x^3\left(1-2x^2\right)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}+$$

$$\frac{27}{80}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(-2+7\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{8\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}+$$

$$\frac{27\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{80\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 199 leaves):

$$\frac{1}{80\sqrt{1+2x^2+2x^4}}\left(4x+12x^3-4\,i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$(29-33\,i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.27(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 349: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 423 leaves, 8 steps):

$$\frac{x\left(1-2x^2\right)}{20\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{9}{40}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}}-$$

$$\frac{\left(2^{1/4}+2^{3/4}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{8\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{40\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 199 leaves) :

$$\frac{1}{40\sqrt{1+2x^2+2x^4}}\left(2x-4x^3-2\,i\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+\right.$$
$$(8-6\,i)\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$\left.9\,(1-i)^{3/2}\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 422 leaves, 8 steps) :

$$-\frac{x\left(2+x^2\right)}{10\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{3}{20}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]-$$

$$\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{4\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}+$$

$$\frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{20\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 199 leaves):

$$-\frac{1}{20\sqrt{1+2x^2+2x^4}}\left(4x+2x^3+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+(1-2i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\right.$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 423 leaves, 8 steps):

$$\frac{x\left(3+4x^2\right)}{10\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{10}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]+$$

$$\frac{2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{5\sqrt{1+2x^2+2x^4}}-$$

$$\frac{\left(2^{1/4}+2^{3/4}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{4\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{10\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 199 leaves) :

$$\frac{1}{20\sqrt{1+2x^2+2x^4}}\left(6x+8x^3+4\,i\,\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$(1+3\,i)\,\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$\left.2\,(1-i)^{3/2}\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},\,i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 422 leaves, 8 steps) :

$$-\frac{x\left(1+3x^2\right)}{5\sqrt{1+2x^2+2x^4}}+\frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]}{5\sqrt{15}}-\frac{3\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{5\times 2^{3/4}\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{2\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{15\times 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 199 leaves):

$$\frac{1}{30\sqrt{1+2x^2+2x^4}}\left(-6x-18x^3-9\,i\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+\right.$$
$$\left(6+3\,i\right)\sqrt{1-i}\,\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]+$$
$$\left.2\,(1-i)^{3/2}\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 468 leaves, 15 steps):

$$-\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{15\left(1+\sqrt{2}\,x^2\right)}-$$

$$\frac{2\,\text{ArcTan}\left[\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right]}{15\sqrt{15}}-\frac{2\times 2^{1/4}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{15\sqrt{1+2x^2+2x^4}}+$$

$$\frac{\left(-7+3\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{3\times 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}-$$

$$\frac{2^{1/4}\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{1}{24}\left(12-11\sqrt{2}\right),2\,\text{ArcTan}\left[2^{1/4}\,x\right],\frac{1}{4}\left(2-\sqrt{2}\right)\right]}{45\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Result (type 4, 211 leaves):

$$\frac{1}{90\,x\sqrt{1+2x^2+2x^4}}\left(-12\,i\sqrt{1-i}\,x\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-\right.$$
$$\left(27-39\,i\right)\sqrt{1-i}\,x\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]-$$
$$\left.2\left(15+39x^2+12x^4+2\,(1-i)^{3/2}\,x\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticPi}\left[\frac{1}{3}+\frac{i}{3},i\,\text{ArcSinh}\left[\sqrt{1-i}\,x\right],i\right]\right)\right)$$

Problem 361: Unable to integrate problem.

$$\int\frac{x^4\sqrt{d+e\,x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 390 leaves, 10 steps):

$$\frac{x\sqrt{d+e x^2}}{2 c}-\frac{\left(b c d-b^2 e+a c e-\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{c^2 \sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-$$

$$\frac{\left(b c d-b^2 e+a c e+\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{c^2 \sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}+\frac{(c d-2 b e) \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{2 c^2 \sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4 \sqrt{d+e x^2}}{a+b x^2+c x^4}\, dx$$

Problem 362: Unable to integrate problem.

$$\int \frac{x^2 \sqrt{d+e x^2}}{a+b x^2+c x^4}\, dx$$

Optimal (type 3, 324 leaves, 9 steps):

$$\frac{\left(c d-b e-\frac{b c d-b^2 e+2 a c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{c \sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}+\frac{\left(c d-b e+\frac{b c d-b^2 e+2 a c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{c \sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}+\frac{\sqrt{e}\ \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{c}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2 \sqrt{d+e x^2}}{a+b x^2+c x^4}\, dx$$

Problem 363: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{a+b x^2+c x^4} \, dx$$

Optimal (type 3, 240 leaves, 11 steps):

$$\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \sqrt{b-\sqrt{b^2-4 a c}}}-\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \sqrt{b+\sqrt{b^2-4 a c}}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{d+e x^2}}{a+b x^2+c x^4} \, dx$$

Problem 364: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 291 leaves, 8 steps):

$$-\frac{\sqrt{d+e x^2}}{a x}-\frac{c \left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a \sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-\frac{c \left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a \sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+e x^2}}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

Problem 365: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{x^4 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 373 leaves, 12 steps):

$$-\frac{\sqrt{d+e x^2}}{3 a x^3}+\frac{2 e \sqrt{d+e x^2}}{3 a d x}+\frac{(b d-a e) \sqrt{d+e x^2}}{a^2 d x}+$$

$$\frac{c \left(b d-a e+\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}+\frac{c \left(b d-a e-\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+e x^2}}{x^4 \left(a+b x^2+c x^4\right)} \, dx$$

Problem 366: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}}{x^6 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 512 leaves, 15 steps):

$$-\frac{\sqrt{d+e x^2}}{5 a x^5}+\frac{4 e \sqrt{d+e x^2}}{15 a d x^3}+\frac{(b d-a e) \sqrt{d+e x^2}}{3 a^2 d x^3}-\frac{8 e^2 \sqrt{d+e x^2}}{15 a d^2 x}-\frac{2 e (b d-a e) \sqrt{d+e x^2}}{3 a^2 d^2 x}-$$

$$\frac{\left(b^2 d-a c d-a b e\right) \sqrt{d+e x^2}}{a^3 d x}-\frac{c \left(b^2 d-a c d-a b e+\frac{b^3 d-3 a b c d-a b^2 e+2 a^2 c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{a^3 \sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-$$

$$\frac{c \left(b^2 d-a c d-a b e-\frac{b^3 d-3 a b c d-a b^2 e+2 a^2 c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{a^3 \sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{\sqrt{d+e x^2}}{x^6 \left(a+b x^2+c x^4\right)} \, dx$$

Problem 371: Unable to integrate problem.

$$\int \frac{x^4 \left(d+e x^2\right)^{3/2}}{a+b x^2+c x^4} \, dx$$

Optimal (type 3, 595 leaves, 17 steps):

$$\frac{(3 c d-4 b e) x \sqrt{d+e x^2}}{8 c^2}+\frac{x \left(d+e x^2\right)^{3/2}}{4 c}-\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e} \left(b c d-b^2 e+a c e-\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e} \, x}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{2 c^3 \sqrt{b-\sqrt{b^2-4 a c}}}-$$

$$\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e} \left(b c d-b^2 e+a c e+\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e} \, x}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{2 c^3 \sqrt{b+\sqrt{b^2-4 a c}}}+\frac{d (3 c d-4 b e) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{8 c^2 \sqrt{e}}-$$

$$\frac{\sqrt{e} \left(b c d-b^2 e+a c e-\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{2 c^3}-\frac{\sqrt{e} \left(b c d-b^2 e+a c e+\frac{b^2 c d-2 a c^2 d-b^3 e+3 a b c e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{2 c^3}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4 \left(d+e x^2\right)^{3/2}}{a+b x^2+c x^4} \, dx$$

Problem 372: Unable to integrate problem.

$$\int \frac{x^2 \left(d+e x^2\right)^{3/2}}{a+b x^2+c x^4} \, dx$$

Optimal (type 3, 491 leaves, 16 steps):

$$\frac{e\,x\,\sqrt{d+e\,x^2}}{2\,c}+\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}\,\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}\,\,x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\,\,\sqrt{d+e\,x^2}}\right]}{2\,c^2\,\sqrt{b-\sqrt{b^2-4\,a\,c}}}+$$

$$\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\,\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\,\,x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\,\,\sqrt{d+e\,x^2}}\right]}{2\,c^2\,\sqrt{b+\sqrt{b^2-4\,a\,c}}}+\frac{d\,\sqrt{e}\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\,x}{\sqrt{d+e\,x^2}}\right]}{2\,c}+$$

$$\frac{\sqrt{e}\,\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\,x}{\sqrt{d+e\,x^2}}\right]}{2\,c^2}+\frac{\sqrt{e}\,\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\,x}{\sqrt{d+e\,x^2}}\right]}{2\,c^2}$$

Result (type 8, 31 leaves) :

$$\int\frac{x^2\,\left(d+e\,x^2\right)^{3/2}}{a+b\,x^2+c\,x^4}\,dx$$

Problem 373: Unable to integrate problem.

$$\int\frac{\left(d+e\,x^2\right)^{3/2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 487 leaves, 13 steps) :

$$\frac{\left(2 c^2 d^2+b \left(b-\sqrt{b^2-4 a c}\right) e^2-2 c e \left(b d-\sqrt{b^2-4 a c}\, d+a e\right)\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{c \sqrt{b^2-4 a c}\, \sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-$$

$$\frac{\left(2 c^2 d^2+b \left(b+\sqrt{b^2-4 a c}\right) e^2-2 c e \left(b d+\sqrt{b^2-4 a c}\, d+a e\right)\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{c \sqrt{b^2-4 a c}\, \sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}+$$

$$\frac{\sqrt{e} \left(3 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{2 c \sqrt{b^2-4 a c}}-\frac{\sqrt{e} \left(3 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{2 c \sqrt{b^2-4 a c}}$$

Result (type 8, 28 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2}}{a+b x^2+c x^4} \, dx$$

Problem 374: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2}}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d \sqrt{d+e x^2}}{a x}-\frac{\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \left(b-\sqrt{b^2-4 a c}\right)^{3/2}}+\frac{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \left(b+\sqrt{b^2-4 a c}\right)^{3/2}}$$

Result (type 8, 31 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2}}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

Problem 375: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2}}{x^4 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 523 leaves, 19 steps):

$$\frac{(b d-a e) \sqrt{d+e x^2}}{a^2 x}-\frac{\left(d+e x^2\right)^{3/2}}{3 a x^3}+\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e} \left(b d-a e+\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e} \, x}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{2 a^2 \sqrt{b-\sqrt{b^2-4 a c}}}+$$

$$\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e} \left(b d-a e-\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e} \, x}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{2 a^2 \sqrt{b+\sqrt{b^2-4 a c}}}-\frac{\sqrt{e} (b d-a e) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{a^2}+$$

$$\frac{\sqrt{e} \left(b d-a e-\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{2 a^2}+\frac{\sqrt{e} \left(b d-a e+\frac{b^2 d-2 a c d-a b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{2 a^2}$$

Result (type 8, 31 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2}}{x^4 \left(a+b x^2+c x^4\right)} \, dx$$

Problem 381: Unable to integrate problem.

$$\int \frac{x^4 \sqrt{1-x^2}}{a+b x^2+c x^4} \, dx$$

Optimal (type 3, 325 leaves, 9 steps):

$$\frac{x\sqrt{1-x^2}}{2c}+\frac{(2b+c)\,\text{ArcSin}[x]}{2c^2}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{1-x^2}}\right]}{c^2\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{b+2c-\sqrt{b^2-4ac}}}-$$

$$\frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\;x}{\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{1-x^2}}\right]}{c^2\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves):

$$\int\frac{x^4\sqrt{1-x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Problem 382: Unable to integrate problem.

$$\int\frac{x^2\sqrt{1-x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 263 leaves, 8 steps):

$$-\frac{\text{ArcSin}[x]}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{1-x^2}}\right]}{c\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\;x}{\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{1-x^2}}\right]}{c\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves):

$$\int\frac{x^2\sqrt{1-x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Problem 383: Unable to integrate problem.

$$\int\frac{\sqrt{1-x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 3, 220 leaves, 9 steps):

$$\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\ \mathrm{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\ x}{\sqrt{b-\sqrt{b^2-4ac}}\ \sqrt{1-x^2}}\right]}{\sqrt{b^2-4ac}\ \sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\ \mathrm{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\ x}{\sqrt{b+\sqrt{b^2-4ac}}\ \sqrt{1-x^2}}\right]}{\sqrt{b^2-4ac}\ \sqrt{b+\sqrt{b^2-4ac}}}$$

Result (type 8, 28 leaves) :

$$\int\frac{\sqrt{1-x^2}}{a+b\,x^2+c\,x^4}\,dx$$

Problem 384: Unable to integrate problem.

$$\int\frac{\sqrt{1-x^2}}{x^2\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 265 leaves, 8 steps) :

$$-\frac{\sqrt{1-x^2}}{a\,x}-\frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\ x}{\sqrt{b-\sqrt{b^2-4ac}}\ \sqrt{1-x^2}}\right]}{a\sqrt{b-\sqrt{b^2-4ac}}\ \sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\ x}{\sqrt{b+\sqrt{b^2-4ac}}\ \sqrt{1-x^2}}\right]}{a\sqrt{b+\sqrt{b^2-4ac}}\ \sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Result (type 8, 31 leaves) :

$$\int\frac{\sqrt{1-x^2}}{x^2\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Problem 385: Unable to integrate problem.

$$\int\frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$$

Optimal (type 3, 96 leaves, 8 steps) :

$$-\mathrm{ArcSin}[x]+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \mathrm{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]-\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \mathrm{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]$$

Result (type 8, 27 leaves) :

$$\int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} \, dx$$

Problem 386: Unable to integrate problem.

$$\int \frac{x^8}{\sqrt{d+e x^2} \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 479 leaves, 17 steps):

$$-\frac{3 d x \sqrt{d+e x^2}}{8 c e^2}-\frac{b x \sqrt{d+e x^2}}{2 c^2 e}+\frac{x^3 \sqrt{d+e x^2}}{4 c e}-\frac{\left(b^3-2 a b c-\frac{b^4-4 a b^2 c+2 a^2 c^2}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c^3 \sqrt{b-\sqrt{b^2-4 a c}} \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-$$

$$\frac{\left(b^3-2 a b c+\frac{b^4-4 a b^2 c+2 a^2 c^2}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}} \sqrt{d+e x^2}}\right]}{c^3 \sqrt{b+\sqrt{b^2-4 a c}} \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}+\frac{3 d^2 \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{8 c e^{5/2}}+\frac{b d \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{2 c^2 e^{3/2}}+\frac{\left(b^2-a c\right) \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{c^3 \sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^8}{\sqrt{d+e x^2} \left(a+b x^2+c x^4\right)} \, dx$$

Problem 387: Unable to integrate problem.

$$\int \frac{x^6}{\sqrt{d+e x^2} \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 366 leaves, 13 steps):

$$\frac{x\sqrt{d+e x^2}}{2 c e}+\frac{\left(b^2-a c-\frac{b\left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{c^2\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}+$$

$$\frac{\left(b^2-a c+\frac{b\left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{c^2\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{e}\; x}{\sqrt{d+e x^2}}\right]}{2 c e^{3/2}}-\frac{b\,\text{ArcTanh}\left[\frac{\sqrt{e}\; x}{\sqrt{d+e x^2}}\right]}{c^2\sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int\frac{x^6}{\sqrt{d+e x^2}\;\left(a+b x^2+c x^4\right)}\,dx$$

Problem 388: Unable to integrate problem.

$$\int\frac{x^4}{\sqrt{d+e x^2}\;\left(a+b x^2+c x^4\right)}\,dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$-\frac{\left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{c\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-\frac{\left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{c\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{e}\; x}{\sqrt{d+e x^2}}\right]}{c\sqrt{e}}$$

Result (type 8, 31 leaves):

$$\int\frac{x^4}{\sqrt{d+e x^2}\;\left(a+b x^2+c x^4\right)}\,dx$$

Problem 389: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{d+e x^2}\,\left(a+b x^2+c x^4\right)}\,dx$$

Optimal (type 3, 240 leaves, 6 steps):

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\;\text{ArcTan}\left[\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{d+e x^2}}\right]}{\sqrt{b^2-4ac}\;\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\;\text{ArcTan}\left[\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{d+e x^2}}\right]}{\sqrt{b^2-4ac}\;\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^2}{\sqrt{d+e x^2}\,\left(a+b x^2+c x^4\right)}\,dx$$

Problem 390: Unable to integrate problem.

$$\int \frac{1}{\sqrt{d+e x^2}\,\left(a+b x^2+c x^4\right)}\,dx$$

Optimal (type 3, 243 leaves, 5 steps):

$$\frac{2c\;\text{ArcTan}\left[\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{d+e x^2}}\right]}{\sqrt{b^2-4ac}\;\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{2c\;\text{ArcTan}\left[\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{d+e x^2}}\right]}{\sqrt{b^2-4ac}\;\sqrt{b+\sqrt{b^2-4ac}}\;\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{d+e x^2}\,\left(a+b x^2+c x^4\right)}\,dx$$

Problem 391: Unable to integrate problem.

$$\int \frac{1}{x^2 \sqrt{d+e x^2}\left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 280 leaves, 9 steps):

$$-\frac{\sqrt{d+e x^2}}{a d x}-\frac{c\left(1+\frac{b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a \sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a \sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^2 \sqrt{d+e x^2}\left(a+b x^2+c x^4\right)} \, dx$$

Problem 392: Unable to integrate problem.

$$\int \frac{1}{x^4 \sqrt{d+e x^2}\left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 341 leaves, 11 steps):

$$-\frac{\sqrt{d+e x^2}}{3 a d x^3}+\frac{b \sqrt{d+e x^2}}{a^2 d x}+\frac{2 e \sqrt{d+e x^2}}{3 a d^2 x}+\frac{c\left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}+\frac{c\left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a^2 \sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^4 \sqrt{d+e x^2}\left(a+b x^2+c x^4\right)} \, dx$$

Problem 393: Unable to integrate problem.

$$\int \frac{1}{x^6 \sqrt{d+e x^2}\, \left(a+b x^2+c x^4\right)}\, dx$$

Optimal (type 3, 443 leaves, 14 steps):

$$-\frac{\sqrt{d+e x^2}}{5 a d x^5}+\frac{b \sqrt{d+e x^2}}{3 a^2 d x^3}+\frac{4 e \sqrt{d+e x^2}}{15 a d^2 x^3}-\frac{\left(b^2-a c\right) \sqrt{d+e x^2}}{a^3 d x}-\frac{2 b e \sqrt{d+e x^2}}{3 a^2 d^2 x}-\frac{8 e^2 \sqrt{d+e x^2}}{15 a d^3 x}-$$

$$\frac{c \left(b^2-a c+\frac{b \left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a^3 \sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}-\frac{c \left(b^2-a c-\frac{b \left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{a^3 \sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^6 \sqrt{d+e x^2}\, \left(a+b x^2+c x^4\right)}\, dx$$

Problem 394: Unable to integrate problem.

$$\int \frac{x^6}{\left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)}\, dx$$

Optimal (type 3, 350 leaves, 14 steps):

$$-\frac{d^2 x}{e \left(c d^2-b d e+a e^2\right) \sqrt{d+e x^2}}+\frac{2 \left(b^2-a c-\frac{b \left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{c \sqrt{b-\sqrt{b^2-4 a c}}\, \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2}}+$$

$$\frac{2 \left(b^2-a c+\frac{b \left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{c \sqrt{b+\sqrt{b^2-4 a c}}\, \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{c\, e^{3/2}}$$

Result (type 8, 31 leaves):

$$\int \frac{x^6}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Problem 395: Unable to integrate problem.

$$\int \frac{x^4}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 360 leaves, 8 steps):

$$\frac{d\,x}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{d+e\,x^2}}-\frac{\left(b\,d-a\,e-\frac{b^2\,d-2\,a\,c\,d-a\,b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}-$$

$$\frac{\left(b\,d-a\,e+\frac{b^2\,d-2\,a\,c\,d-a\,b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}$$

Result (type 8, 31 leaves):

$$\int \frac{x^4}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Problem 396: Unable to integrate problem.

$$\int \frac{x^2}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 333 leaves, 8 steps):

$$-\frac{e\,x}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{d+e\,x^2}}+\frac{c\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}+$$

$$\frac{c\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}$$

Result (type 8, 31 leaves):

$$\int\frac{x^2}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Problem 397: Unable to integrate problem.

$$\int\frac{1}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 341 leaves, 8 steps):

$$\frac{e^2\,x}{d\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{d+e\,x^2}}-\frac{c\left(e-\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}-$$

$$\frac{c\left(e+\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx$$

Problem 398: Unable to integrate problem.

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 339 leaves, 12 steps):

$$\frac{e\left(c d-b e\right) x}{a d\left(c d^2+e\left(-b d+a e\right)\right)\sqrt{d+e x^2}}+\frac{-d-2 e x^2}{a d^2 x\sqrt{d+e x^2}}-\frac{2 c^2\left(1+\frac{b}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{a\sqrt{b-\sqrt{b^2-4 a c}}\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2}}-\frac{2 c^2\left(1-\frac{b}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{a\sqrt{b+\sqrt{b^2-4 a c}}\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx$$

Problem 399: Unable to integrate problem.

$$\int \frac{1}{x^4 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 3, 419 leaves, 15 steps):

$$-\frac{1}{3 a d x^3\sqrt{d+e x^2}}+\frac{3 b d+4 a e}{3 a^2 d^2 x\sqrt{d+e x^2}}+\frac{2 e\left(3 b d+4 a e\right) x}{3 a^2 d^3\sqrt{d+e x^2}}-\frac{e\left(b c d-b^2 e+a c e\right) x}{a^2 d\left(c d^2+e\left(-b d+a e\right)\right)\sqrt{d+e x^2}}+\frac{2 c^2\left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b-\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{a^2\sqrt{b-\sqrt{b^2-4 a c}}\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2}}+\frac{2 c^2\left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\; x}{\sqrt{b+\sqrt{b^2-4 a c}}\;\sqrt{d+e x^2}}\right]}{a^2\sqrt{b+\sqrt{b^2-4 a c}}\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2}}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x^4 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)}\,dx$$

Problem 400: Unable to integrate problem.

$$\int \frac{(f x)^m \left(d+e x^2\right)^q}{a+b x^2+c x^4}\,dx$$

Optimal (type 6, 243 leaves, 6 steps):

$$\frac{2 c (f x)^{1+m} \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{\sqrt{b^2-4 a c} \left(b-\sqrt{b^2-4 a c}\right) f (1+m)}-\frac{2 c (f x)^{1+m} \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{\sqrt{b^2-4 a c} \left(b+\sqrt{b^2-4 a c}\right) f (1+m)}$$

Result (type 8, 31 leaves):

$$\int \frac{(f x)^m \left(d+e x^2\right)^q}{a+b x^2+c x^4}\,dx$$

Problem 405: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^q}{x \left(a+b x^2+c x^4\right)}\,dx$$

Optimal (type 5, 262 leaves, 8 steps):

$$\frac{c \left(1+\frac{b}{\sqrt{b^2-4 a c}}\right) \left(d+e x^2\right)^{1+q} \text{Hypergeometric2F1}\left[1,1+q,2+q,\frac{2 c \left(d+e x^2\right)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 a \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) (1+q)}+\frac{c \left(1-\frac{b}{\sqrt{b^2-4 a c}}\right) \left(d+e x^2\right)^{1+q} \text{Hypergeometric2F1}\left[1,1+q,2+q,\frac{2 c \left(d+e x^2\right)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 a \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) (1+q)}-\frac{\left(d+e x^2\right)^{1+q} \text{Hypergeometric2F1}\left[1,1+q,2+q,1+\frac{e x^2}{d}\right]}{2 a d (1+q)}$$

Result (type 8, 29 leaves):

$$\int \frac{(d+e x^2)^q}{x\,(a+b x^2+c x^4)}\,dx$$

Problem 407: Unable to integrate problem.

$$\int \frac{x^6\,(d+e x^2)^q}{a+b x^2+c x^4}\,dx$$

Optimal (type 6, 339 leaves, 12 steps):

$$\frac{\left(b^2-a c-\frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}}\right) x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{c^2\left(b-\sqrt{b^2-4 a c}\right)}+$$
$$\frac{\left(b^2-a c+\frac{b(b^2-3 a c)}{\sqrt{b^2-4 a c}}\right) x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{c^2\left(b+\sqrt{b^2-4 a c}\right)}-$$
$$\frac{b x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[\frac{1}{2},-q,\frac{3}{2},-\frac{e x^2}{d}\right]}{c^2}+\frac{x^3\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[\frac{3}{2},-q,\frac{5}{2},-\frac{e x^2}{d}\right]}{3 c}$$

Result (type 8, 29 leaves):

$$\int \frac{x^6\,(d+e x^2)^q}{a+b x^2+c x^4}\,dx$$

Problem 408: Unable to integrate problem.

$$\int \frac{x^4\,(d+e x^2)^q}{a+b x^2+c x^4}\,dx$$

Optimal (type 6, 273 leaves, 10 steps):

$$-\frac{\left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{c\left(b-\sqrt{b^2-4 a c}\right)}-$$
$$\frac{\left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right) x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right]}{c\left(b+\sqrt{b^2-4 a c}\right)}+\frac{x\,(d+e x^2)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[\frac{1}{2},-q,\frac{3}{2},-\frac{e x^2}{d}\right]}{c}$$

Result (type 8, 29 leaves):

$$\int \frac{x^4 \left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Problem 409: Unable to integrate problem.

$$\int \frac{x^2 \left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Optimal (type 6, 162 leaves, 6 steps):

$$-\frac{x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]}{\sqrt{b^2-4 a c}}+\frac{x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]}{\sqrt{b^2-4 a c}}$$

Result (type 8, 29 leaves):

$$\int \frac{x^2 \left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Problem 410: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Optimal (type 6, 190 leaves, 5 steps):

$$-\frac{2 c x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]}{b^2-4 a c-b \sqrt{b^2-4 a c}}-\frac{2 c x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2}, 1, -q, \frac{3}{2}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, -\frac{e x^2}{d}\right]}{b^2-4 a c+b \sqrt{b^2-4 a c}}$$

Result (type 8, 26 leaves):

$$\int \frac{\left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Problem 411: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^q}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

Optimal (type 6, 264 leaves, 10 steps):

$$-\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]}{a\left(b-\sqrt{b^2-4ac}\right)}-$$

$$\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)x\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]}{a\left(b+\sqrt{b^2-4ac}\right)}-\frac{\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[-\frac{1}{2},-q,\frac{1}{2},-\frac{e x^2}{d}\right]}{a x}$$

Result (type 8, 29 leaves) :

$$\int\frac{\left(d+e x^2\right)^q}{x^2\left(a+b x^2+c x^4\right)}\,dx$$

Problem 412: Unable to integrate problem.

$$\int\frac{\left(d+e x^2\right)^q}{x^4\left(a+b x^2+c x^4\right)}\,dx$$

Optimal (type 6, 328 leaves, 12 steps) :

$$\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]}{a^2\left(b-\sqrt{b^2-4ac}\right)}+$$

$$\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{e x^2}{d}\right]}{a^2\left(b+\sqrt{b^2-4ac}\right)}-$$

$$\frac{\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[-\frac{3}{2},-q,-\frac{1}{2},-\frac{e x^2}{d}\right]}{3 a x^3}+\frac{b\left(d+e x^2\right)^q\left(1+\frac{e x^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left[-\frac{1}{2},-q,\frac{1}{2},-\frac{e x^2}{d}\right]}{a^2 x}$$

Result (type 8, 29 leaves) :

$$\int\frac{\left(d+e x^2\right)^q}{x^4\left(a+b x^2+c x^4\right)}\,dx$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x}{1+x^2+x^4}\,dx$$

Optimal (type 3, 92 leaves, 15 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{d\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{4}\,d\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}\,d\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 98 leaves):

$$\frac{1}{6}\,i\left(\sqrt{6-6\,i\,\sqrt{3}}\;d\,\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\sqrt{6+6\,i\,\sqrt{3}}\;d\,\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]+2\,i\,\sqrt{3}\;e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2}{1+x^2+x^4}\,dx$$

Optimal (type 3, 104 leaves, 14 steps):

$$-\frac{(d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 121 leaves):

$$\frac{\left(2\,i\,d+\left(-i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{6+6\,i\,\sqrt{3}}}+\frac{\left(-2\,i\,d+\left(i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{6-6\,i\,\sqrt{3}}}-\frac{e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]}{\sqrt{3}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3}{1+x^2+x^4}\,dx$$

Optimal (type 3, 127 leaves, 15 steps):

$$-\frac{(d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]+\frac{1}{4}g\,\text{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 150 leaves):

$$\frac{1}{8\sqrt{3}}\left(2\sqrt{2-2\,i\sqrt{3}}\,\left(2\,i\,d+\left(-i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]+\right.$$
$$\left.2\left(\sqrt{2+2\,i\sqrt{3}}\,\left(-2\,i\,d+\left(i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]+(-4e+2g)\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+\sqrt{3}\,g\,\text{Log}\left[1+x^2+x^4\right]\right)\right)$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4}{1+x^2+x^4}\,dx$$

Optimal (type 3, 136 leaves, 17 steps):

$$h\,x-\frac{(d+f-2h)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f-2h)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+$$
$$\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]+\frac{1}{4}g\,\text{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 165 leaves):

$$\frac{1}{24}\left(24\,h\,x+4\left(\left(3\,i+\sqrt{3}\right)d+\left(-3\,i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]+\right.$$
$$\left.4\left(\left(-3\,i+\sqrt{3}\right)d+\left(3\,i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]-8\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+4\sqrt{3}\,g\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+6\,g\,\text{Log}\left[1+x^2+x^4\right]\right)$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4+i\,x^5}{1+x^2+x^4}\,dx$$

Optimal (type 3, 151 leaves, 19 steps):

$$h x+\frac{i x^2}{2}-\frac{(d+f-2 h) \operatorname{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}+\frac{(d+f-2 h) \operatorname{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}+$$
$$\frac{(2 e-g-i) \operatorname{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{2 \sqrt{3}}-\frac{1}{4}(d-f) \operatorname{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f) \operatorname{Log}\left[1+x+x^2\right]+\frac{1}{4}(g-i) \operatorname{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 187 leaves):

$$\frac{1}{12}\left(6 x(2 h+i x)+\left(1+\mathrm{i} \sqrt{3}\right)\left(2 \sqrt{3} d-\left(3 \mathrm{i}+\sqrt{3}\right) f-\left(-3 \mathrm{i}+\sqrt{3}\right) h\right) \operatorname{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right) x\right]+\right.$$
$$\left.\left(\mathrm{i}+\sqrt{3}\right)\left(-2 \mathrm{i} \sqrt{3} d+\left(3+\mathrm{i} \sqrt{3}\right) f+\mathrm{i}\left(3 \mathrm{i}+\sqrt{3}\right) h\right) \operatorname{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right) x\right]-2 \sqrt{3}(2 e-g-i) \operatorname{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]+3(g-i) \operatorname{Log}\left[1+x^2+x^4\right]\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x}{\left(1+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 140 leaves, 17 steps):

$$\frac{d x\left(1-x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{e\left(1+2 x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{d \operatorname{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{d \operatorname{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{2 e \operatorname{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{1}{4} d \operatorname{Log}\left[1-x+x^2\right]+\frac{1}{4} d \operatorname{Log}\left[1+x+x^2\right]$$

Result (type 3, 146 leaves):

$$\frac{e+2 e x^2+d\left(x-x^3\right)}{6\left(1+x^2+x^4\right)}-\frac{\left(-11 \mathrm{i}+\sqrt{3}\right) d \operatorname{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right) x\right]}{6 \sqrt{6+6 \mathrm{i} \sqrt{3}}}-\frac{\left(11 \mathrm{i}+\sqrt{3}\right) d \operatorname{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right) x\right]}{6 \sqrt{6-6 \mathrm{i} \sqrt{3}}}-\frac{2 e \operatorname{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]}{3 \sqrt{3}}$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x+f x^2}{\left(1+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 165 leaves, 16 steps):

$$\frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{x\left(d+f-\left(d-2f\right)x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{\left(4d+f\right)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(4d+f\right)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{2e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{8}\left(2d-f\right)\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\left(2d-f\right)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 186 leaves):

$$\frac{1}{36}\left(\frac{6\left(e+2ex^2+x\left(d+f-dx^2+2fx^2\right)\right)}{1+x^2+x^4}-\frac{\left(\left(-11\,i+\sqrt{3}\right)d-2\left(-2\,i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(11\,i+\sqrt{3}\right)d-2\left(2\,i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-8\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\left(1+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 179 leaves, 15 steps):

$$\frac{x\left(d+f-\left(d-2f\right)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{e-2g+\left(2e-g\right)x^2}{6\left(1+x^2+x^4\right)}-\frac{\left(4d+f\right)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(4d+f\right)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(2e-g\right)\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{8}\left(2d-f\right)\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\left(2d-f\right)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 200 leaves):

$$\frac{1}{36}\left(\frac{6\left(e+2 e x^2-g\left(2+x^2\right)+x\left(d+f-d x^2+2 f x^2\right)\right)}{1+x^2+x^4}-\frac{\left(\left(-11 i+\sqrt{3}\right) d-2\left(-2 i+\sqrt{3}\right) f\right) \text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1+i \sqrt{3}\right)}}-\right.$$

$$\left.\frac{\left(\left(11 i+\sqrt{3}\right) d-2\left(2 i+\sqrt{3}\right) f\right) \text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1-i \sqrt{3}\right)}}-4 \sqrt{3}\left(2 e-g\right) \text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]\right)$$

$$\int \frac{d+e x+f x^2+g x^3+h x^4}{\left(1+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 187 leaves, 15 steps):

$$\frac{e-2 g+\left(2 e-g\right) x^2}{6\left(1+x^2+x^4\right)}+\frac{x\left(d+f-2 h-\left(d-2 f+h\right) x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{\left(4 d+f+h\right) \text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{12 \sqrt{3}}+$$

$$\frac{\left(4 d+f+h\right) \text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{12 \sqrt{3}}+\frac{\left(2 e-g\right) \text{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{1}{8}\left(2 d-f+h\right) \text{Log}\left[1-x+x^2\right]+\frac{1}{8}\left(2 d-f+h\right) \text{Log}\left[1+x+x^2\right]$$

Result (type 3, 234 leaves):

$$\frac{1}{36}\left(-\frac{6\left(g\left(2+x^2\right)-e\left(1+2 x^2\right)+x\left(d\left(-1+x^2\right)+h\left(2+x^2\right)-f\left(1+2 x^2\right)\right)\right)}{1+x^2+x^4}-\right.$$

$$\frac{\left(\left(-11 i+\sqrt{3}\right) d-2\left(-2 i+\sqrt{3}\right) f+\left(-5 i+\sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1+i \sqrt{3}\right)}}-$$

$$\left.\frac{\left(\left(11 i+\sqrt{3}\right) d-2\left(2 i+\sqrt{3}\right) f+\left(5 i+\sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1-i \sqrt{3}\right)}}-4 \sqrt{3}\left(2 e-g\right) \text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]\right)$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x+f x^2+g x^3+h x^4+i x^5}{\left(1+x^2+x^4\right)^2}\, dx$$

Optimal (type 3, 194 leaves, 16 steps):

$$\frac{x\left(d+f-2h-\left(d-2f+h\right)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{e-2g+i+\left(2e-g-i\right)x^2}{6\left(1+x^2+x^4\right)}-\frac{\left(4d+f+h\right)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(4d+f+h\right)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(2e-g+2i\right)\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{8}\left(2d-f+h\right)\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\left(2d-f+h\right)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 243 leaves):

$$\frac{1}{36}\left(\frac{6\left(e+i+d x+f x-2h x+2e x^2-i x^2-d x^3+2f x^3-h x^3-g\left(2+x^2\right)\right)}{1+x^2+x^4}-\frac{\left(\left(-11\,\mathrm{i}+\sqrt{3}\right)d-2\left(-2\,\mathrm{i}+\sqrt{3}\right)f+\left(-5\,\mathrm{i}+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+\mathrm{i}\sqrt{3}\right)}}-\frac{\left(\left(11\,\mathrm{i}+\sqrt{3}\right)d-2\left(2\,\mathrm{i}+\sqrt{3}\right)f+\left(5\,\mathrm{i}+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-\mathrm{i}\sqrt{3}\right)}}-4\sqrt{3}\left(2e-g+2i\right)\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x}{\left(1+x^2+x^4\right)^3}\, dx$$

Optimal (type 3, 185 leaves, 19 steps):

$$\frac{d\,x\,\left(1-x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{e\,\left(1+2\,x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{d\,x\,\left(2-7\,x^2\right)}{24\,\left(1+x^2+x^4\right)}+\frac{e\,\left(1+2\,x^2\right)}{6\,\left(1+x^2+x^4\right)}-$$
$$\frac{13\,d\,\text{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{13\,d\,\text{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{2\,e\,\text{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\,\sqrt{3}}-\frac{9}{32}\,d\,\text{Log}\left[1-x+x^2\right]+\frac{9}{32}\,d\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 186 leaves) :

$$\frac{1}{144}\left(\frac{6\,\left(d\,x\,\left(2-7\,x^2\right)+e\,\left(4+8\,x^2\right)\right)}{1+x^2+x^4}+\frac{12\,\left(e+2\,e\,x^2+d\,\left(x-x^3\right)\right)}{\left(1+x^2+x^4\right)^2}-\right.$$
$$\left.\frac{\left(-47\,i+7\,\sqrt{3}\right)\,d\,\text{ArcTan}\left[\frac{1}{2}\,\left(-i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1+i\,\sqrt{3}\right)}}-\frac{\left(47\,i+7\,\sqrt{3}\right)\,d\,\text{ArcTan}\left[\frac{1}{2}\,\left(i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1-i\,\sqrt{3}\right)}}-32\,\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 18 steps) :

$$\frac{e\,\left(1+2\,x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{x\,\left(d+f-\left(d-2\,f\right)\,x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{e\,\left(1+2\,x^2\right)}{6\,\left(1+x^2+x^4\right)}+\frac{x\,\left(2\,d+3\,f-7\,\left(d-f\right)\,x^2\right)}{24\,\left(1+x^2+x^4\right)}-\frac{\left(13\,d+2\,f\right)\,\text{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+$$
$$\frac{\left(13\,d+2\,f\right)\,\text{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{2\,e\,\text{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\,\sqrt{3}}-\frac{1}{32}\,\left(9\,d-4\,f\right)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{32}\,\left(9\,d-4\,f\right)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 235 leaves) :

$$\frac{1}{144}\left(\frac{6\left(2\,d\,x+3\,f\,x-7\,d\,x^3+7\,f\,x^3+e\left(4+8\,x^2\right)\right)}{1+x^2+x^4}+\right.$$

$$\frac{12\left(e+2\,e\,x^2+x\left(d+f-d\,x^2+2\,f\,x^2\right)\right)}{\left(1+x^2+x^4\right)^2}-\frac{\left(\left(-47\,i+7\sqrt{3}\right)d+\left(17\,i-7\sqrt{3}\right)f\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-$$

$$\left.\frac{\left(\left(47\,i+7\sqrt{3}\right)d-\left(17\,i+7\sqrt{3}\right)f\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-32\sqrt{3}\,e\,\mathrm{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 243 leaves, 17 steps):

$$\frac{x\left(d+f-\left(d-2\,f\right)x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{e-2\,g+\left(2\,e-g\right)x^2}{12\left(1+x^2+x^4\right)^2}+\frac{\left(2\,e-g\right)\left(1+2\,x^2\right)}{12\left(1+x^2+x^4\right)}+\frac{x\left(2\,d+3\,f-7\left(d-f\right)x^2\right)}{24\left(1+x^2+x^4\right)}-\frac{\left(13\,d+2\,f\right)\mathrm{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{48\sqrt{3}}+$$

$$\frac{\left(13\,d+2\,f\right)\mathrm{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{\left(2\,e-g\right)\mathrm{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{32}\left(9\,d-4\,f\right)\mathrm{Log}\left[1-x+x^2\right]+\frac{1}{32}\left(9\,d-4\,f\right)\mathrm{Log}\left[1+x+x^2\right]$$

Result (type 3, 259 leaves):

$$\frac{1}{144}\left(\frac{6\left(2\,d\,x+3\,f\,x-7\,d\,x^3+7\,f\,x^3-2\,g\left(1+2\,x^2\right)+e\left(4+8\,x^2\right)\right)}{1+x^2+x^4}+\right.$$

$$\frac{12\left(e+2\,e\,x^2-g\left(2+x^2\right)+x\left(d+f-d\,x^2+2\,f\,x^2\right)\right)}{\left(1+x^2+x^4\right)^2}-\frac{\left(\left(-47\,\mathrm{i}+7\,\sqrt{3}\right)d+\left(17\,\mathrm{i}-7\,\sqrt{3}\right)f\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+\mathrm{i}\,\sqrt{3}\right)}}-$$

$$\left.\frac{\left(\left(47\,\mathrm{i}+7\,\sqrt{3}\right)d-\left(17\,\mathrm{i}+7\,\sqrt{3}\right)f\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-\mathrm{i}\,\sqrt{3}\right)}}-16\,\sqrt{3}\,\left(2\,e-g\right)\mathrm{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 263 leaves, 17 steps):

$$\frac{e-2\,g+\left(2\,e-g\right)x^2}{12\left(1+x^2+x^4\right)^2}+\frac{x\left(d+f-2\,h-\left(d-2\,f+h\right)x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{\left(2\,e-g\right)\left(1+2\,x^2\right)}{12\left(1+x^2+x^4\right)}+\frac{x\left(2\,d+3\,f-h-\left(7\,d-7\,f+4\,h\right)x^2\right)}{24\left(1+x^2+x^4\right)}-\frac{\left(13\,d+2\,f+h\right)\mathrm{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+$$

$$\frac{\left(13\,d+2\,f+h\right)\mathrm{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{\left(2\,e-g\right)\mathrm{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\,\sqrt{3}}-\frac{1}{32}\left(9\,d-4\,f+3\,h\right)\mathrm{Log}\left[1-x+x^2\right]+\frac{1}{32}\left(9\,d-4\,f+3\,h\right)\mathrm{Log}\left[1+x+x^2\right]$$

Result (type 3, 303 leaves):

$$\frac{1}{144}\left(-\frac{6\left(-4 e\left(1+2 x^2\right)+g\left(2+4 x^2\right)+x\left(-2 d-3 f+h+7 d x^2-7 f x^2+4 h x^2\right)\right)}{1+x^2+x^4}+\frac{12\left(e+2 e x^2-g\left(2+x^2\right)+x\left(d+f-d x^2+2 f x^2-h\left(2+x^2\right)\right)\right)}{\left(1+x^2+x^4\right)^2}-\right.$$

$$\frac{\left(\left(-47 i+7 \sqrt{3}\right) d+\left(17 i-7 \sqrt{3}\right) f+2\left(-7 i+2 \sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1+i \sqrt{3}\right)}}-$$

$$\left.\frac{\left(\left(47 i+7 \sqrt{3}\right) d-\left(17 i+7 \sqrt{3}\right) f+2\left(7 i+2 \sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right) x\right]}{\sqrt{\frac{1}{6}\left(1-i \sqrt{3}\right)}}-16 \sqrt{3}\,(2 e-g) \text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x+f x^2+g x^3+h x^4+i x^5}{\left(1+x^2+x^4\right)^3} \, dx$$

Optimal (type 3, 269 leaves, 18 steps):

$$\frac{x\left(d+f-2 h-(d-2 f+h) x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{e-2 g+i+(2 e-g-i) x^2}{12\left(1+x^2+x^4\right)^2}+\frac{(2 e-g+i)\left(1+2 x^2\right)}{12\left(1+x^2+x^4\right)}+$$

$$\frac{x\left(2 d+3 f-h-(7 d-7 f+4 h) x^2\right)}{24\left(1+x^2+x^4\right)}-\frac{(13 d+2 f+h) \text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{48 \sqrt{3}}+\frac{(13 d+2 f+h) \text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{48 \sqrt{3}}+$$

$$\frac{(2 e-g+i) \text{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{1}{32}(9 d-4 f+3 h) \text{Log}\left[1-x+x^2\right]+\frac{1}{32}(9 d-4 f+3 h) \text{Log}\left[1+x+x^2\right]$$

Result (type 3, 325 leaves):

$$\frac{1}{144}\left(\frac{12\,(e+i+d\,x+f\,x-2\,h\,x+2\,e\,x^2-i\,x^2-d\,x^3+2\,f\,x^3-h\,x^3-g\,(2+x^2))}{(1+x^2+x^4)^2}+\right.$$

$$\frac{6\,(2\,i+2\,d\,x+3\,f\,x-h\,x+4\,i\,x^2-7\,d\,x^3+7\,f\,x^3-4\,h\,x^3-2\,g\,(1+2\,x^2)+e\,(4+8\,x^2))}{1+x^2+x^4}-$$

$$\frac{\left(\left(-47\,\mathrm{i}+7\,\sqrt{3}\right)d+\left(17\,\mathrm{i}-7\,\sqrt{3}\right)f+2\left(-7\,\mathrm{i}+2\,\sqrt{3}\right)h\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+\mathrm{i}\,\sqrt{3}\right)}}-$$

$$\left.\frac{\left(\left(47\,\mathrm{i}+7\,\sqrt{3}\right)d-\left(17\,\mathrm{i}+7\,\sqrt{3}\right)f+2\left(7\,\mathrm{i}+2\,\sqrt{3}\right)h\right)\mathrm{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-\mathrm{i}\,\sqrt{3}\right)}}-16\,\sqrt{3}\,(2\,e-g+i)\,\mathrm{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (d+e\,x+f\,x^2+g\,x^3)\,(a+b\,x^2+c\,x^4)^{3/2}\,dx$$

Optimal (type 4, 717 leaves, 12 steps) :

$$-\frac{\left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f\right) x \sqrt{a+b x^2+c x^4}}{315 c^{5/2} \left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{3 \left(b^2-4 a c\right) (2 c e-b g) \left(b+2 c x^2\right) \sqrt{a+b x^2+c x^4}}{256 c^3}+$$

$$\frac{x \left(9 b^2 c d+90 a c^2 d-4 b^3 f+9 a b c f+3 c (9 b c d-4 b^2 f+14 a c f) x^2\right) \sqrt{a+b x^2+c x^4}}{315 c^2}+$$

$$\frac{(2 c e-b g) \left(b+2 c x^2\right) \left(a+b x^2+c x^4\right)^{3/2}}{32 c^2}+\frac{x \left(3 (3 c d+b f)+7 c f x^2\right) \left(a+b x^2+c x^4\right)^{3/2}}{63 c}+$$

$$\frac{g \left(a+b x^2+c x^4\right)^{5/2}}{10 c}+\frac{3 \left(b^2-4 a c\right)^2 (2 c e-b g) \operatorname{ArcTanh}\left[\frac{b+2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2+c x^4}}\right]}{512 c^{7/2}}+\frac{1}{315 c^{11/4} \sqrt{a+b x^2+c x^4}}$$

$$a^{1/4} \left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]-$$

$$\frac{1}{630 c^{11/4} \sqrt{a+b x^2+c x^4}} a^{1/4} \left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f+\sqrt{a} \sqrt{c} \left(9 b^2 c d-180 a c^2 d-4 b^3 f+24 a b c f\right)\right)$$

$$\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]$$

Result (type 4, 2588 leaves) :

$$\frac{1}{161280 c^{7/2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}} \left(-2 \sqrt{c} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \left(a+b x^2+c x^4\right) \left(-945 b^4 g+2 b^3 c \left(945 e+x (512 f+315 g x)\right)-\right.\right.$$

$$12 b^2 c \left(-525 a g+c x \left(192 d+105 e x+64 f x^2+42 g x^3\right)\right)-8 b c^2 \left(3 a \left(525 e+256 f x+147 g x^2\right)+2 c x^3 \left(1152 d+945 e x+800 f x^2+693 g x^3\right)\right)-$$

$$\left.16 c^2 \left(504 a^2 g+2 c^2 x^5 \left(360 d+7 x \left(45 e+40 f x+36 g x^2\right)\right)+a c x \left(2160 d+7 x \left(225 e+16 x (11 f+9 g x)\right)\right)\right)\right)+$$

$$2304 i \sqrt{2} b^3 c^{3/2} \left(b-\sqrt{b^2-4 a c}\right) d \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)+$$

$$18432 i \sqrt{2} a b c^{5/2} \left(-b+\sqrt{b^2-4 a c}\right) d \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$7296\,i\,\sqrt{2}\,a\,b^2\,c^{3/2}\left(b-\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$1024\,i\,\sqrt{2}\,b^4\,\sqrt{c}\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$10\,752\,i\,\sqrt{2}\,a^2\,c^{5/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$2304\,i\,\sqrt{2}\,a\,b^2\,c^{5/2}\,d\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$46\,080\,i\,\sqrt{2}\,a^2\,c^{7/2}\,d\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$1024\,i\,\sqrt{2}\,a\,b^3\,c^{3/2}\,f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$6144\,i\,\sqrt{2}\,a^2\,b\,c^{5/2}\,f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$1890\, b^4\, c \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; e \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] -$$

$$15\,120\, a\, b^2\, c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; e \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] +$$

$$30\,240\, a^2\, c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; e \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] -$$

$$945\, b^5 \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; g \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] +$$

$$7560\, a\, b^3\, c \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; g \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] -$$

$$15\,120\, a^2\, b\, c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\; g \sqrt{a+b\,x^2+c\,x^4}\; \text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right] \Bigg)$$

Problem 104: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(d+e\,x+f\,x^2+g\,x^3\right) \sqrt{a+b\,x^2+c\,x^4}\; dx$$

Optimal (type 4, 505 leaves, 10 steps):

$$\frac{\left(5 b c d-2 b^2 f+6 a c f\right) x \sqrt{a+b x^2+c x^4}}{15 c^{3/2} \left(\sqrt{a}+\sqrt{c} x^2\right)}+\frac{(2 c e-b g) \left(b+2 c x^2\right) \sqrt{a+b x^2+c x^4}}{16 c^2}+$$

$$\frac{x \left(5 c d+b f+3 c f x^2\right) \sqrt{a+b x^2+c x^4}}{15 c}+\frac{g \left(a+b x^2+c x^4\right)^{3/2}}{6 c}-\frac{\left(b^2-4 a c\right) (2 c e-b g) \text{ArcTanh}\left[\frac{b+2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2+c x^4}}\right]}{32 c^{5/2}}-$$

$$\frac{a^{1/4} \left(5 b c d-2 b^2 f+6 a c f\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{15 c^{7/4} \sqrt{a+b x^2+c x^4}}+\frac{1}{30 c^{7/4} \sqrt{a+b x^2+c x^4}}$$

$$a^{1/4} \left(b+2 \sqrt{a} \sqrt{c}\right) \left(5 c d-2 b f+3 \sqrt{a} \sqrt{c} f\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]$$

Result (type 4, 1534 leaves):

$$\sqrt{a+b x^2+c x^4} \left(\frac{6 b c e-3 b^2 g+8 a c g}{48 c^2}+\frac{(5 c d+b f) x}{15 c}+\frac{(6 c e+b g) x^2}{24 c}+\frac{f x^3}{5}+\frac{g x^4}{6}\right)+$$

$$\frac{1}{240 c^2}\left(\left(20 i \sqrt{2} b c \left(-b+\sqrt{b^2-4 a c}\right) d \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right],\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(8 i \sqrt{2} b^2 \left(-b+\sqrt{b^2-4 a c}\right) f \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right],\right.\right.\right.$$

$$\left.\left.\left.\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(24 i \sqrt{2} a c \left(-b+\sqrt{b^2-4 a c}\right) f \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right],\right.\right.\right.$$

$$\left.\left.\left.\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(80\,i\sqrt{2}\,a\,c^2\,d\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(8\,i\sqrt{2}\,a\,b\,c\,f\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-15\,b^2\sqrt{c}\,e\,\text{Log}\left[b+2\,c\,x^2+2\sqrt{c}\sqrt{a+b\,x^2+c\,x^4}\right]+$$
$$60\,a\,c^{3/2}\,e\,\text{Log}\left[b+2\,c\,x^2+2\sqrt{c}\sqrt{a+b\,x^2+c\,x^4}\right]+\frac{15\,b^3\,g\,\text{Log}\left[b+2\,c\,x^2+2\sqrt{c}\sqrt{a+b\,x^2+c\,x^4}\right]}{2\sqrt{c}}-$$
$$\left.30\,a\,b\sqrt{c}\,g\,\text{Log}\left[b+2\,c\,x^2+2\sqrt{c}\sqrt{a+b\,x^2+c\,x^4}\right]\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 359 leaves, 8 steps):

$$\frac{g\sqrt{a+b\,x^2+c\,x^4}}{2\,c}+\frac{f\,x\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{(2\,c\,e-b\,g)\,\text{ArcTanh}\left[\frac{b+2\,c\,x^2}{2\sqrt{c}\sqrt{a+b\,x^2+c\,x^4}}\right]}{4\,c^{3/2}}-$$
$$\frac{a^{1/4}\,f\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{c^{3/4}\sqrt{a+b\,x^2+c\,x^4}}+$$
$$\frac{a^{1/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+f\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2\,c^{3/4}\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 526 leaves):

$$\frac{1}{4\,c^{3/2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}\left(i\,\sqrt{2}\,\sqrt{c}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,f\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-i\,\sqrt{2}\,\sqrt{c}\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,f\right)$$

$$\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(2\,\sqrt{c}\,g\,\left(a+b\,x^2+c\,x^4\right)+\left(2\,c\,e-b\,g\right)\,\sqrt{a+b\,x^2+c\,x^4}\,\text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\right]\right)\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 447 leaves, 7 steps):

$$\frac{x\,\left(b^2\,d-2\,a\,c\,d-a\,b\,f+c\,\left(b\,d-2\,a\,f\right)\,x^2\right)}{a\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{b\,e-2\,a\,g+\left(2\,c\,e-b\,g\right)\,x^2}{\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{\sqrt{c}\,\left(b\,d-2\,a\,f\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{a\,\left(b^2-4\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\frac{c^{1/4}\,\left(b\,d-2\,a\,f\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{a^{3/4}\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}-$$

$$\frac{\left(\sqrt{c}\,d-\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,a^{3/4}\,\left(b-2\,\sqrt{a}\,\sqrt{c}\right)\,c^{1/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 513 leaves):

$$-\frac{1}{4\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(-2\,a^2\,g-b\,d\,x\,\left(b+c\,x^2\right)+2\,a\,c\,x\,\left(d+x\,\left(e+f\,x\right)\right)+a\,b\,\left(e+x\,\left(f-g\,x\right)\right)\right)+\mathrm{i}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b\,d-2\,a\,f\right)\right.$$

$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$\mathrm{i}\,\left(-b^2\,d+4\,a\,c\,d+b\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,\sqrt{b^2-4\,a\,c}\,f\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\left(a+b\,x^2+c\,x^4\right)^{5/2}}\,dx$$

Optimal (type 4, 680 leaves, 9 steps):

$$\frac{x\left(b^2 d-2 a c d-a b f+c\left(b d-2 a f\right) x^2\right)}{3 a\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^{3/2}}-\frac{b e-2 a g+\left(2 c e-b g\right) x^2}{3\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^{3/2}}+\frac{4\left(2 c e-b g\right)\left(b+2 c x^2\right)}{3\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}}+$$
$$\frac{x\left(2 b^4 d-17 a b^2 c d+20 a^2 c^2 d+a b^3 f+4 a^2 b c f+c\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right) x^2\right)}{3 a^2\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}}-$$
$$\frac{\sqrt{c}\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right) x\sqrt{a+b x^2+c x^4}}{3 a^2\left(b^2-4 a c\right)^2\left(\sqrt{a}+\sqrt{c} x^2\right)}+$$
$$\left(c^{1/4}\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$
$$\left(3 a^{7/4}\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}\right)-$$
$$\left(c^{1/4}\left(2 b^2 d-3\sqrt{a}\,b\sqrt{c}\,d-10 a c d+a b f+6 a^{3/2}\sqrt{c}\,f\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$
$$\left(6 a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 598 leaves) :

$$\frac{1}{12\,a^2\,(b^2-4\,a\,c)^2\,(a+b\,x^2+c\,x^4)^{3/2}}\Bigg(-4\,a\,(b^2-4\,a\,c)\,\left(-2\,a^2\,g-b\,d\,x\,(b+c\,x^2)+2\,a\,c\,x\,(d+x\,(e+f\,x))+a\,b\,(e+x\,(f-g\,x))\right)+4\,(a+b\,x^2+c\,x^4)$$
$$\left(2\,b^3\,d\,x\,(b+c\,x^2)+a\,b\,x\,(-17\,b\,c\,d+b^2\,f-16\,c^2\,d\,x^2+b\,c\,f\,x^2)+4\,a^2\,\left(-b^2\,g+c^2\,x\,(5\,d+x\,(4\,e+3\,f\,x))+b\,c\,(2\,e+x\,(f-2\,g\,x))\right)\right)+$$
$$\frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}}\,i\,\sqrt{2}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,(a+b\,x^2+c\,x^4)$$
$$\Bigg(-\left(-b+\sqrt{b^2-4\,a\,c}\right)\,(2\,b^3\,d-16\,a\,b\,c\,d+a\,b^2\,f+12\,a^2\,c\,f)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$
$$\left(-2\,b^4\,d+b^3\,\left(2\,\sqrt{b^2-4\,a\,c}\,d-a\,f\right)+4\,a\,b\,c\,\left(-4\,\sqrt{b^2-4\,a\,c}\,d+a\,f\right)+a\,b^2\,\left(18\,c\,d+\sqrt{b^2-4\,a\,c}\,f\right)+4\,a^2\,c\,\left(-10\,c\,d+3\,\sqrt{b^2-4\,a\,c}\,f\right)\right)$$
$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)$$

Test results for the 145 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Problem 40: Result is not expressed in closed-form.

$$\int \frac{(d\,x)^m\,(A+B\,x+C\,x^2)}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 5, 368 leaves, 8 steps):

$$\frac{\left(C+\frac{2\,A\,c-b\,C}{\sqrt{b^2-4\,a\,c}}\right)\,(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{1+m}{2},\,\frac{3+m}{2},\,-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]}{\left(b-\sqrt{b^2-4\,a\,c}\right)\,d\,(1+m)}+\frac{\left(C-\frac{2\,A\,c-b\,C}{\sqrt{b^2-4\,a\,c}}\right)\,(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{1+m}{2},\,\frac{3+m}{2},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,d\,(1+m)}+$$
$$\frac{2\,B\,c\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{2+m}{2},\,\frac{4+m}{2},\,-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,d^2\,(2+m)}-\frac{2\,B\,c\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{2+m}{2},\,\frac{4+m}{2},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,(2+m)}$$

Result (type 7, 438 leaves) :

$$\frac{1}{2\,m\,(1+m)\,(2+m)}(d\,x)^m\left(A\,(2+3\,m+m^2)\,\text{RootSum}\left[a+b\,\sharp1^2+c\,\sharp1^4\,\&,\,\frac{\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}}{b\,\sharp1+2\,c\,\sharp1^3}\,\&\right]+\right.$$

$$B\,(2+m)\,\text{RootSum}\left[a+b\,\sharp1^2+c\,\sharp1^4\,\&,\,\frac{1}{b\,\sharp1+2\,c\,\sharp1^3}\right.$$

$$\left.\left(m\,x+\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1+m\,\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1\right)\,\&\right]+$$

$$C\,\text{RootSum}\left[a+b\,\sharp1^2+c\,\sharp1^4\,\&,\,\frac{1}{b\,\sharp1+2\,c\,\sharp1^3}\left(m\,x^2+m^2\,x^2+2\,m\,x\,\sharp1+m^2\,x\,\sharp1+2\,\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+\right.\right.$$

$$3\,m\,\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+$$

$$\left.\left.\left.m^2\,\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,-\frac{\sharp1}{x-\sharp1}\right]\left(\frac{x}{x-\sharp1}\right)^{-m}\sharp1^2+m\left(\frac{x}{\sharp1}\right)^{-m}\sharp1^2\right)\,\&\right]\right)$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int\frac{(d\,x)^m\,(A+B\,x+C\,x^2)}{(a+b\,x^2+c\,x^4)^2}\,dx$$

Optimal (type 5, 685 leaves, 10 steps) :

$$\frac{B\,(d\,x)^{2+m}\,\left(b^2-2\,a\,c+b\,c\,x^2\right)}{2\,a\,\left(b^2-4\,a\,c\right)\,d^2\,\left(a+b\,x^2+c\,x^4\right)}+\frac{(d\,x)^{1+m}\,\left(A\,\left(b^2-2\,a\,c\right)-a\,b\,C+c\,\left(A\,b-2\,a\,C\right)\,x^2\right)}{2\,a\,\left(b^2-4\,a\,c\right)\,d\,\left(a+b\,x^2+c\,x^4\right)}+$$

$$\left(c\,\left(2\,a\,C\,\left(2\,b-\sqrt{b^2-4\,a\,c}\,\left(1-m\right)\right)+A\,\left(b^2\,\left(1-m\right)+b\,\sqrt{b^2-4\,a\,c}\,\left(1-m\right)-4\,a\,c\,\left(3-m\right)\right)\right)\,(d\,x)^{1+m}\right.$$

$$\left.\text{Hypergeometric2F1}\left[1,\,\frac{1+m}{2},\,\frac{3+m}{2},\,-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,d\,\left(1+m\right)\right)-$$

$$\left(c\,\left(2\,a\,C\,\left(2\,b+\sqrt{b^2-4\,a\,c}\,\left(1-m\right)\right)+A\,\left(b^2\,\left(1-m\right)-b\,\sqrt{b^2-4\,a\,c}\,\left(1-m\right)-4\,a\,c\,\left(3-m\right)\right)\right)\,(d\,x)^{1+m}\right.$$

$$\left.\text{Hypergeometric2F1}\left[1,\,\frac{1+m}{2},\,\frac{3+m}{2},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d\,\left(1+m\right)\right)-$$

$$\frac{B\,c\,\left(4\,a\,c\,\left(2-m\right)+b\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,m\right)\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{2+m}{2},\,\frac{4+m}{2},\,-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]}{2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,d^2\,\left(2+m\right)}+$$

$$\frac{B\,c\,\left(4\,a\,c\,\left(2-m\right)+b\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,m\right)\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\,\frac{2+m}{2},\,\frac{4+m}{2},\,-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,\left(2+m\right)}$$

Result (type 6, 999 leaves) :

$$\frac{1}{4\,c\,\left(a+b\,x^2+c\,x^4\right)^3}$$

$$a\,x\,(d\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(\left(\left(A\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/\right.$$

$$\left((1+m)\,\left(a\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{3+m}{2},2,3,\frac{5+m}{2},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$x\,\left(\left(B\,(4+m)\,\text{AppellF1}\left[\frac{2+m}{2},2,2,\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left((2+m)\,\left(a\,(4+m)\,\text{AppellF1}\left[\frac{2+m}{2},2,2,\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{4+m}{2},2,3,\frac{6+m}{2},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{4+m}{2},3,2,\frac{6+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(C\,(5+m)\,x\,\text{AppellF1}\left[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left((3+m)\,\left(a\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]-2\,x^2\,\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5+m}{2},2,3,\frac{7+m}{2},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[\frac{5+m}{2},3,2,\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x\,\left(3+2\,x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 66 leaves, 8 steps):

$$\frac{25\,\left(1-x^2\right)}{24\,\left(3+2\,x^2+x^4\right)}+\frac{89\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{72\,\sqrt{2}}+\frac{4\,\text{Log}[x]}{9}-\frac{1}{9}\,\text{Log}\left[3+2\,x^2+x^4\right]$$

Result (type 3, 93 leaves):

$$\frac{1}{288}\,\left(-\frac{300\,\left(-1+x^2\right)}{3+2\,x^2+x^4}+128\,\text{Log}[x]-\sqrt{2}\,\left(89\,i+16\,\sqrt{2}\right)\,\text{Log}\left[1-i\,\sqrt{2}+x^2\right]+\sqrt{2}\,\left(89\,i-16\,\sqrt{2}\right)\,\text{Log}\left[1+i\,\sqrt{2}+x^2\right]\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 71 leaves, 8 steps):

$$-\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{71\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{216\sqrt{2}}-\frac{13\,\text{Log}[x]}{27}+\frac{13}{108}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 97 leaves):

$$\frac{1}{864}\left(-\frac{192}{x^2}-\frac{300\left(5+x^2\right)}{3+2x^2+x^4}-416\,\text{Log}[x]+\sqrt{2}\left(71\,i+52\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]+\sqrt{2}\left(-71\,i+52\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 80 leaves, 8 steps):

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{125\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{216\sqrt{2}}+\frac{13\,\text{Log}[x]}{27}-\frac{13}{108}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 105 leaves):

$$\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{100\left(7+5x^2\right)}{3+2x^2+x^4}+416\,\text{Log}[x]-\sqrt{2}\left(125\,i+52\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]+\sqrt{2}\left(125\,i-52\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 87 leaves, 8 steps):

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1237\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{1944\sqrt{2}}+\frac{61\,\text{Log}[x]}{243}-\frac{61}{972}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 110 leaves):

$$\frac{1}{7776}\left(-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300\left(-1+7x^2\right)}{3+2x^2+x^4}+1952\,\text{Log}[x]+\sqrt{2}\left(1237\,i-244\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]-\sqrt{2}\left(1237\,i+244\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 248 leaves, 12 steps):

$$38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]+$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 145 leaves):

$$38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{\left(352\,i+1339\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}-\frac{\left(-352\,i+1339\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 237 leaves, 12 steps):

$$19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\,\mathrm{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\,\mathrm{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 132 leaves):

$$19x-\frac{17x^3}{3}+x^5-\frac{25x\left(-3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{9\left(90\,i+31\sqrt{2}\right)\mathrm{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}+\frac{9\left(-90\,i+31\sqrt{2}\right)\mathrm{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 232 leaves, 12 steps):

$$-17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\ \mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\ \mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\ \mathrm{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]+$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\ \mathrm{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 129 leaves):

$$-17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{\left(-356\,i+127\sqrt{2}\right)\mathrm{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}+\frac{\left(356\,i+127\sqrt{2}\right)\mathrm{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 225 leaves, 12 steps):

$$5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\ \mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\ \mathrm{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\ \mathrm{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\ \mathrm{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 121 leaves):

$$5x+\frac{25\left(x+x^3\right)}{8\left(3+2x^2+x^4\right)}-\frac{\left(-34\,i+111\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}-\frac{\left(34\,i+111\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 224 leaves, 10 steps):

$$\frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 115 leaves):

$$\frac{1}{48}\left(-\frac{50x\left(-1+x^2\right)}{3+2x^2+x^4}+\frac{\left(95+44\,i\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\sqrt{2}}}+\frac{\left(95-44\,i\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\sqrt{2}}}\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 229 leaves, 12 steps):

$$-\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$
$$\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 126 leaves):

$$-\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{\left(26\,i+19\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{48\sqrt{2-2\,i\sqrt{2}}}-\frac{\left(-26\,i+19\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{48\sqrt{2+2\,i\sqrt{2}}}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 238 leaves, 12 steps):

$$-\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$
$$\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]-$$
$$\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 131 leaves):

$$\frac{1}{864}\left(\frac{4\left(-96+248 x^2+351 x^4+229 x^6\right)}{x^3\left(3+2 x^2+x^4\right)}+\frac{2\left(229+46\, i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{2\left(229-46\, i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3 x^4+5 x^6}{x^6\left(3+2 x^2+x^4\right)^2}\, dx$$

Optimal (type 3, 245 leaves, 12 steps):

$$-\frac{4}{45 x^5}+\frac{13}{81 x^3}-\frac{13}{27 x}+\frac{25 x\left(1-7 x^2\right)}{648\left(3+2 x^2+x^4\right)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419 \sqrt{3}\right)}\, \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{1296}-$$
$$\frac{\sqrt{\frac{1}{6}\left(-1139381+688419 \sqrt{3}\right)}\, \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{1296}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419 \sqrt{3}\right)}\, \text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\, x+x^2\right]}{2592}+$$
$$\frac{\sqrt{\frac{1}{6}\left(1139381+688419 \sqrt{3}\right)}\, \text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\, x+x^2\right]}{2592}$$

Result (type 3, 140 leaves):

$$\frac{-\frac{4\left(864-984 x^2+3928 x^4+2475 x^6+2435 x^8\right)}{x^5\left(3+2 x^2+x^4\right)}-\frac{10\, i\left(-487\, i+475 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{10\, i\left(487\, i+475 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}}{12960}$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}\left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3}\, dx$$

Optimal (type 3, 243 leaves, 13 steps):

$$58 x-9 x^3+x^5-\frac{25 x \left(15+7 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}+\frac{x \left(3305+252 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+$$
$$\frac{3}{256} \sqrt{-8595619+7678611 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-\frac{3}{256} \sqrt{-8595619+7678611 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+$$
$$\frac{3}{512} \sqrt{8595619+7678611 \sqrt{3}}\ \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]-\frac{3}{512} \sqrt{8595619+7678611 \sqrt{3}}\ \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 156 leaves):

$$58 x-9 x^3+x^5-\frac{25 x \left(15+7 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}+\frac{x \left(3305+252 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+\frac{3 \left(4795 i+148 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{128 \sqrt{2-2 i \sqrt{2}}}+\frac{3 \left(-4795 i+148 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{128 \sqrt{2+2 i \sqrt{2}}}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3}\, dx$$

Optimal (type 3, 242 leaves, 13 steps):

$$-27 x+\frac{5 x^3}{3}+\frac{25 x \left(3+5 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}-\frac{x \left(1468+835 x^2\right)}{64 \left(3+2 x^2+x^4\right)}-$$
$$\frac{21}{256} \sqrt{34271+22721 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+\frac{21}{256} \sqrt{34271+22721 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-$$
$$\frac{21}{512} \sqrt{-34271+22721 \sqrt{3}}\ \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]+\frac{21}{512} \sqrt{-34271+22721 \sqrt{3}}\ \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 155 leaves):

$$-27 x+\frac{5 x^3}{3}+\frac{25 x \left(3+5 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}-\frac{x \left(1468+835 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+\frac{21 \left(-175 i+137 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{128 \sqrt{2-2 i \sqrt{2}}}+\frac{21 \left(175 i+137 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{128 \sqrt{2+2 i \sqrt{2}}}$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3} \, dx$$

Optimal (type 3, 235 leaves, 13 steps):

$$5 x+\frac{25 x \left(3-x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}+\frac{7 x \left(11+58 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+\frac{1}{256} \sqrt{827621+1176531 \sqrt{3}} \operatorname{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-$$
$$\frac{1}{256} \sqrt{827621+1176531 \sqrt{3}} \operatorname{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-\frac{1}{512} \sqrt{-827621+1176531 \sqrt{3}} \operatorname{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]+$$
$$\frac{1}{512} \sqrt{-827621+1176531 \sqrt{3}} \operatorname{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]$$

Result (type 3, 138 leaves):

$$\frac{1}{256} \left(\frac{4 x \left(3411+5112 x^2+4089 x^4+1686 x^6+320 x^8\right)}{\left(3+2 x^2+x^4\right)^2}-\frac{i \left(-2644 i+185 \sqrt{2}\right) \operatorname{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{i \left(2644 i+185 \sqrt{2}\right) \operatorname{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}\right)$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3} \, dx$$

Optimal (type 3, 238 leaves, 11 steps):

$$-\frac{25\,x\,(3+x^2)}{16\,(3+2\,x^2+x^4)^2}+\frac{x\,(238-59\,x^2)}{64\,(3+2\,x^2+x^4)}-\frac{1}{256}\sqrt{3\,\left(-48835+32827\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{256}\sqrt{3\,\left(-48835+32827\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{512}\sqrt{3\,\left(48835+32827\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}-\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]-\frac{1}{512}\sqrt{3\,\left(48835+32827\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}+\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]$$

Result (type 3, 129 leaves) :

$$\frac{1}{256}\left(\frac{4\,x\,(414+199\,x^2+120\,x^4-59\,x^6)}{(3+2\,x^2+x^4)^2}+\frac{3\,\left(174+133\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\,\sqrt{2}}}\right]}{\sqrt{1-i\,\sqrt{2}}}+\frac{3\,\left(174-133\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\,\sqrt{2}}}\right]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,(4+x^2+3\,x^4+5\,x^6)}{(3+2\,x^2+x^4)^3}\,dx$$

Optimal (type 3, 246 leaves, 11 steps) :

$$\frac{25\,x\,(1+x^2)}{16\,(3+2\,x^2+x^4)^2}-\frac{x\,(353+88\,x^2)}{192\,(3+2\,x^2+x^4)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\,\sqrt{3}\right)}\,\text{ArcTan}\Big[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\Big]+$$

$$\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\,\sqrt{3}\right)}\,\text{ArcTan}\Big[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\Big]-$$

$$\frac{11\sqrt{\frac{1}{3}\left(1825+1089\,\sqrt{3}\right)}\,\text{Log}\Big[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\Big]}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1089\,\sqrt{3}\right)}\,\text{Log}\Big[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\Big]}{1536}$$

Result (type 3, 133 leaves) :

$$\frac{1}{768}\left(-\frac{4\,x\,(759+670\,x^2+529\,x^4+88\,x^6)}{(3+2\,x^2+x^4)^2}-\frac{11\,i\left(-16\,i+31\,\sqrt{2}\right)\text{ArcTan}\Big[\frac{x}{\sqrt{1-i\,\sqrt{2}}}\Big]}{\sqrt{1-i\,\sqrt{2}}}+\frac{11\,i\left(16\,i+31\,\sqrt{2}\right)\text{ArcTan}\Big[\frac{x}{\sqrt{1+i\,\sqrt{2}}}\Big]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{(3+2\,x^2+x^4)^3}\,dx$$

Optimal (type 3, 248 leaves, 11 steps) :

$$\frac{25\,x\,(1-x^2)}{48\,(3+2\,x^2+x^4)^2}+\frac{x\,(64+51\,x^2)}{192\,(3+2\,x^2+x^4)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 129 leaves):

$$\frac{1}{768}\left(\frac{4\,x\,(292+181\,x^2+166\,x^4+51\,x^6)}{(3+2\,x^2+x^4)^2}+\frac{3\left(34+21\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\,\sqrt{2}}}\right]}{\sqrt{1-i\,\sqrt{2}}}+\frac{3\left(34-21\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\,\sqrt{2}}}\right]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x^2\,(3+2\,x^2+x^4)^3}\,dx$$

Optimal (type 3, 253 leaves, 13 steps):

$$-\frac{4}{27\,x}-\frac{25\,x\,(5+x^2)}{144\,(3+2\,x^2+x^4)^2}-\frac{x\,(325+242\,x^2)}{1728\,(3+2\,x^2+x^4)}+$$

$$\frac{\sqrt{\frac{1}{3}\left(59\,711+55\,161\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{2304}-\frac{\sqrt{\frac{1}{3}\left(59\,711+55\,161\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{2304}-$$

$$\frac{\sqrt{\frac{1}{3}\left(-59\,711+55\,161\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{4608}+\frac{\sqrt{\frac{1}{3}\left(-59\,711+55\,161\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{4608}$$

Result (type 3, 140 leaves):

$$\frac{-\frac{12\left(768+1849x^2+1412x^4+611x^6+166x^8\right)}{x\left(3+2x^2+x^4\right)^2}+\frac{3\,i\left(332\,i+7\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\sqrt{2}}}-\frac{3\,i\left(-332\,i+7\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 262 leaves, 13 steps):

$$-\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}-$$

$$\frac{\sqrt{\frac{1}{3}\left(10\,004\,741+11\,240\,451\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{20\,736}+\frac{\sqrt{\frac{1}{3}\left(10\,004\,741+11\,240\,451\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{20\,736}+$$

$$\frac{\sqrt{\frac{1}{3}\left(-10\,004\,741+11\,240\,451\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{41\,472}-\frac{\sqrt{\frac{1}{3}\left(-10\,004\,741+11\,240\,451\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{41\,472}$$

Result (type 3, 139 leaves):

$$\frac{\frac{4\left(-2304+9024x^2+20\,090x^4+19\,939x^6+8644x^8+2369x^{10}\right)}{x^3\left(3+2x^2+x^4\right)^2}+\frac{\left(4738+127\,i\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127\,i\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\sqrt{2}}}}{20\,736}$$

Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$\frac{e\left(21 B c d^2+21 A c d e-5 a B e^2\right) x \sqrt{a+c x^4}}{21 c^2}+\frac{e^2 (3 B d+A e) x^3 \sqrt{a+c x^4}}{5 c}+$$
$$\frac{B e^3 x^5 \sqrt{a+c x^4}}{7 c}+\frac{\left(5 B c d^3+15 A c d^2 e-9 a B d e^2-3 a A e^3\right) x \sqrt{a+c x^4}}{5 c^{3/2}\left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{1}{5 c^{7/4} \sqrt{a+c x^4}}$$
$$a^{1/4}\left(5 B c d^3+15 A c d^2 e-9 a B d e^2-3 a A e^3\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+$$
$$\frac{1}{210 a^{1/4} c^{9/4} \sqrt{a+c x^4}}\left(105 A c^2 d^3+25 a^2 B e^3-105 a c d e (B d+A e)-63 a^{3/2} \sqrt{c} e^2 (3 B d+A e)+105 \sqrt{a} c^{3/2} d^2 (B d+3 A e)\right)$$
$$\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 323 leaves) :

$$\frac{1}{105 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^2 \sqrt{a+c x^4}}\left(-\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} e x \left(a+c x^4\right)\left(25 a B e^2-21 A c e \left(5 d+e x^2\right)-3 B c \left(35 d^2+21 d e x^2+5 e^2 x^4\right)\right)-\right.$$
$$21 \sqrt{a} \sqrt{c}\left(-5 B c d^3-15 A c d^2 e+9 a B d e^2+3 a A e^3\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+$$
$$\left(-105 i A c^2 d^3-25 i a^2 B e^3+105 i a c d e (B d+A e)+63 a^{3/2} \sqrt{c} e^2 (3 B d+A e)-105 \sqrt{a} c^{3/2} d^2 (B d+3 A e)\right)$$
$$\left.\sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right)\left(d+e x^2\right)^2}{\sqrt{a+c x^4}} \, dx$$

Optimal (type 4, 367 leaves, 12 steps) :

$$\frac{e\,(2\,B\,d+A\,e)\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{B\,e^2\,x^3\,\sqrt{a+c\,x^4}}{5\,c}+\frac{\left(5\,B\,c\,d^2+10\,A\,c\,d\,e-3\,a\,B\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{5\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\frac{a^{1/4}\,\left(5\,B\,c\,d^2+10\,A\,c\,d\,e-3\,a\,B\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{7/4}\,\sqrt{a+c\,x^4}}+\frac{1}{30\,a^{1/4}\,c^{7/4}\,\sqrt{a+c\,x^4}}$$

$$\left(15\,A\,c^{3/2}\,d^2-9\,a^{3/2}\,B\,e^2-5\,a\,\sqrt{c}\,e\,(2\,B\,d+A\,e)+15\,\sqrt{a}\,c\,d\,(B\,d+2\,A\,e)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 260 leaves):

$$\frac{1}{15\,\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,c^{3/2}\,\sqrt{a+c\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}\,e\,x\,\left(10\,B\,d+5\,A\,e+3\,B\,e\,x^2\right)\,\left(a+c\,x^4\right)-3\,\sqrt{a}\,\left(-5\,B\,c\,d^2-10\,A\,c\,d\,e+3\,a\,B\,e^2\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\left.\left(-15\,i\,A\,c^{3/2}\,d^2+9\,a^{3/2}\,B\,e^2+5\,i\,a\,\sqrt{c}\,e\,(2\,B\,d+A\,e)-15\,\sqrt{a}\,c\,d\,(B\,d+2\,A\,e)\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 277 leaves, 8 steps):

$$\frac{B\,e\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{(B\,d+A\,e)\,x\,\sqrt{a+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4}\,(B\,d+A\,e)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\,\sqrt{a+c\,x^4}}+$$

$$\frac{a^{1/4}\,\left(3\,\sqrt{c}\,(B\,d+A\,e)+\frac{3\,A\,c\,d-a\,B\,e}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{6\,c^{5/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 202 leaves):

$$\frac{1}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c\sqrt{a+c x^4}}\left(B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,e\,x\,(a+c x^4)+3\sqrt{a}\,\sqrt{c}\,(B d+A e)\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$
$$\left.i\left(-3 A c d+a B e+3 i\sqrt{a}\,\sqrt{c}\,(B d+A e)\right)\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 226 leaves, 3 steps):

$$\frac{B x\sqrt{a+c x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4} B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\sqrt{a+c x^4}}+$$
$$\frac{a^{1/4}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 c^{3/4}\sqrt{a+c x^4}}$$

Result (type 4, 131 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}\,\sqrt{a+c x^4}}\sqrt{1+\frac{c x^4}{a}}\left(\sqrt{a}\,B\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\left(\sqrt{a}\,B+i A\sqrt{c}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{\left(d+e x^2\right)\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 369 leaves, 3 steps):

$$-\frac{(B d-A e)\,\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{2\sqrt{d}\,\sqrt{e}\,\sqrt{c d^2+a e^2}}-\frac{\left(\sqrt{a}\,B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}c^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+c x^4}}+$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2(B d-A e)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4\,c^{1/4}d e\left(c d^2-a e^2\right)\sqrt{a+c x^4}\right)$$

Result (type 4, 138 leaves):

$$-\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d e\sqrt{a+c x^4}}\,i\sqrt{1+\frac{c x^4}{a}}\left(B d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+(-B d+A e)\,\text{EllipticPi}\left[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{\left(d+e x^2\right)^2\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 641 leaves, 6 steps):

$$\frac{\sqrt{c}\,(B d-A e)\,x\sqrt{a+c x^4}}{2 d\,(c d^2+a e^2)\,(\sqrt{a}+\sqrt{c}\,x^2)}-\frac{e\,(B d-A e)\,x\sqrt{a+c x^4}}{2 d\,(c d^2+a e^2)\,(d+e x^2)}-\frac{(B c d^3-3 A c d^2 e-a B d e^2-a A e^3)\,\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{4 d^{3/2}\sqrt{e}\,(c d^2+a e^2)^{3/2}}-$$

$$\frac{a^{1/4} c^{1/4}\,(B d-A e)\,(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 d\,(c d^2+a e^2)\sqrt{a+c x^4}}+$$

$$\frac{A c^{1/4}\,(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4} d\,(\sqrt{c}\,d-\sqrt{a}\,e)\sqrt{a+c x^4}}+$$

$$\left((\sqrt{c}\,d+\sqrt{a}\,e)\,(B c d^3-3 A c d^2 e-a B d e^2-a A e^3)\,(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticPi}\left[-\frac{(\sqrt{c}\,d-\sqrt{a}\,e)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^{1/4} c^{1/4} d^2 e\,(\sqrt{c}\,d-\sqrt{a}\,e)\,(c d^2+a e^2)\sqrt{a+c x^4}\right)$$

Result (type 4, 297 leaves) :

$$\frac{1}{2 d^2\sqrt{a+c x^4}}\left(\frac{d e\,(-B d+A e)\,x\,(a+c x^4)}{(c d^2+a e^2)\,(d+e x^2)}-\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,(c d^2 e+a e^3)}\,i\sqrt{1+\frac{c x^4}{a}}\right.$$

$$\left(i\sqrt{a}\,\sqrt{c}\,d e\,(B d-A e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{c}\,d\,(\sqrt{c}\,d-i\sqrt{a}\,e)\,(B d-A e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\left.\left.(-B c d^3+3 A c d^2 e+a B d e^2+a A e^3)\,\text{EllipticPi}\left[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{(d+e x^2)^3\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 875 leaves, 7 steps):

$$\frac{\sqrt{c}\,\left(5 B c d^3-9 A c d^2 e-a B d e^2-3 a A e^3\right) x \sqrt{a+c x^4}}{8 d^2 \left(c d^2+a e^2\right)^2 \left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{e (B d-A e) x \sqrt{a+c x^4}}{4 d \left(c d^2+a e^2\right) \left(d+e x^2\right)^2}-\frac{e \left(5 B c d^3-9 A c d^2 e-a B d e^2-3 a A e^3\right) x \sqrt{a+c x^4}}{8 d^2 \left(c d^2+a e^2\right)^2 \left(d+e x^2\right)}+$$

$$\frac{\left(3 A e \left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)-B \left(3 c^2 d^5-10 a c d^3 e^2-a^2 d e^4\right)\right) \text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c x^4}}\right]}{16 d^{5/2} \sqrt{e}\,\left(c d^2+a e^2\right)^{5/2}}-$$

$$\left(a^{1/4} c^{1/4} \left(5 B c d^3-9 A c d^2 e-a B d e^2-3 a A e^3\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(8 d^2 \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}\right)+$$

$$\left(c^{1/4} \left(4 A c d^2+\sqrt{a}\,\sqrt{c}\, d (B d-A e)+a e (B d+3 A e)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^{1/4} d^2 \left(\sqrt{c}\, d-\sqrt{a}\, e\right) \left(c d^2+a e^2\right) \sqrt{a+c x^4}\right)-$$

$$\left(\left(\sqrt{c}\, d+\sqrt{a}\, e\right) \left(3 A e \left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)-B \left(3 c^2 d^5-10 a c d^3 e^2-a^2 d e^4\right)\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4 \sqrt{a}\,\sqrt{c}\, d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/\left(32 a^{1/4} c^{1/4} d^3 e \left(\sqrt{c}\, d-\sqrt{a}\, e\right) \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}\right)$$

Result (type 4, 453 leaves):

$$\frac{1}{8\,d^3\,e\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}}\left(-\frac{d\,e^2\,x\,\left(a+c\,x^4\right)\,\left(2\,d\,\left(B\,d-A\,e\right)\,\left(c\,d^2+a\,e^2\right)+\left(5\,B\,c\,d^3-9\,A\,c\,d^2\,e-a\,B\,d\,e^2-3\,a\,A\,e^3\right)\,\left(d+e\,x^2\right)\right)}{\left(d+e\,x^2\right)^2}-\right.$$

$$\frac{1}{\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}}\,\mathrm{i}\,\sqrt{1+\frac{c\,x^4}{a}}\,\left(-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,d\,e\,\left(-5\,B\,c\,d^3+9\,A\,c\,d^2\,e+a\,B\,d\,e^2+3\,a\,A\,e^3\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\sqrt{c}\,d\,\left(\sqrt{c}\,d-\mathrm{i}\,\sqrt{a}\,e\right)\,\left(A\,e\,\left(-7\,c\,d^2+2\,\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,d\,e-3\,a\,e^2\right)+B\,d\,\left(3\,c\,d^2-2\,\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,d\,e-a\,e^2\right)\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left.\left.\left(3\,A\,e\,\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)+B\,\left(-3\,c^2\,d^5+10\,a\,c\,d^3\,e^2+a^2\,d\,e^4\right)\right)\,\text{EllipticPi}\left[-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^3}{\left(a+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 912 leaves, 12 steps) :

$$\frac{x\left(A c d\left(c d^2-3 a e^2\right)-a B e\left(3 c d^2-a e^2\right)+c\left(B c d^3+3 A c d^2 e-3 a B d e^2-a A e^3\right) x^2\right)}{2 a c^2 \sqrt{a+c x^4}}+\frac{B e^3 x \sqrt{a+c x^4}}{3 c^2}+\frac{e^2 (3 B d+A e) x \sqrt{a+c x^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c} x^2\right)}-$$

$$\frac{\left(B c d^3+3 A c d^2 e-3 a B d e^2-a A e^3\right) x \sqrt{a+c x^4}}{2 a c^{3/2}\left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{a^{1/4} e^2 (3 B d+A e)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{7/4} \sqrt{a+c x^4}}+$$

$$\frac{\left(B c d^3+3 A c d^2 e-3 a B d e^2-a A e^3\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{7/4} \sqrt{a+c x^4}}-$$

$$\frac{a^{3/4} B e^3\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 c^{9/4} \sqrt{a+c x^4}}+$$

$$\frac{a^{1/4} e^2 (3 B d+A e)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 c^{7/4} \sqrt{a+c x^4}}+$$

$$\frac{e\left(3 B c d^2+3 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{9/4} \sqrt{a+c x^4}}+$$

$$\frac{1}{4 a^{5/4} c^{9/4} \sqrt{a+c x^4}}\left(A c^2 d^3+a^2 B e^3-3 a c d e (B d+A e)+a^{3/2} \sqrt{c} e^2 (3 B d+A e)-\sqrt{a} c^{3/2} d^2 (B d+3 A e)\right)$$

$$\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 351 leaves) :

$$\frac{1}{6\,a\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c^2\,\sqrt{a+c\,x^4}}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\,\left(3\,A\,c\,\left(-a\,e^2\,\left(3\,d+e\,x^2\right)+c\,d^2\,\left(d+3\,e\,x^2\right)\right)+B\,\left(5\,a^2\,e^3+3\,c^2\,d^3\,x^2+a\,c\,e\,\left(-9\,d^2-9\,d\,e\,x^2+2\,e^2\,x^4\right)\right)\right)+\right.$$

$$3\,\sqrt{a}\,\sqrt{c}\,\left(-B\,c\,d^3-3\,A\,c\,d^2\,e+9\,a\,B\,d\,e^2+3\,a\,A\,e^3\right)\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left(-3\,i\,A\,c^2\,d^3+5\,i\,a^2\,B\,e^3-9\,i\,a\,c\,d\,e\,\left(B\,d+A\,e\right)-9\,a^{3/2}\,\sqrt{c}\,e^2\,\left(3\,B\,d+A\,e\right)+3\,\sqrt{a}\,c^{3/2}\,d^2\,\left(B\,d+3\,A\,e\right)\right)$$

$$\left.\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 9: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^2}{\left(a+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 694 leaves, 10 steps) :

$$\frac{x\left(A c d^2-2 a B d e-a A e^2+\left(B c d^2+2 A c d e-a B e^2\right) x^2\right)}{2 a c \sqrt{a+c x^4}}+\frac{B e^2 x \sqrt{a+c x^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-$$

$$\frac{\left(B c d^2+2 A c d e-a B e^2\right) x \sqrt{a+c x^4}}{2 a c^{3/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{a^{1/4} B e^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{7/4}\sqrt{a+c x^4}}+$$

$$\frac{\left(B c d^2+2 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} c^{7/4}\sqrt{a+c x^4}}+$$

$$\frac{a^{1/4} B e^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 c^{7/4}\sqrt{a+c x^4}}+$$

$$\frac{e\,(2 B d+A e)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4} c^{5/4}\sqrt{a+c x^4}}-\frac{1}{4 a^{3/4} c^{7/4}\sqrt{a+c x^4}}$$

$$\left(B c d^2+2 A c d e-a B e^2-\frac{\sqrt{c}\left(A c d^2-2 a B d e-a A e^2\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 282 leaves) :

$$\frac{1}{2 a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ c^{3/2}\sqrt{a+c x^4}}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c}\ x\left(-a A e^2+B c d^2 x^2-a B e\left(2 d+e x^2\right)+A c d\left(d+2 e x^2\right)\right)+\right.$$

$$\sqrt{a}\left(-B c d^2-2 A c d e+3 a B e^2\right)\sqrt{1+\frac{c x^4}{a}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]-$$

$$\left.\left(i A c^{3/2} d^2+3 a^{3/2} B e^2+i a\sqrt{c}\ e\,(2 B d+A e)-\sqrt{a}\ c d\,(B d+2 A e)\right)\sqrt{1+\frac{c x^4}{a}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)}{\left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 395 leaves, 7 steps):

$$\frac{x \left(A c d-a B e+c \left(B d+A e\right) x^2\right)}{2 a c \sqrt{a+c x^4}}-\frac{\left(B d+A e\right) x \sqrt{a+c x^4}}{2 a \sqrt{c} \left(\sqrt{a}+\sqrt{c} x^2\right)}+$$
$$\frac{\left(B d+A e\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a+c x^4}}+\frac{B e \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{5/4} \sqrt{a+c x^4}}+$$
$$\frac{\left(A c d-a B e-\sqrt{a} \sqrt{c} \left(B d+A e\right)\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{5/4} \sqrt{a+c x^4}}$$

Result (type 4, 218 leaves):

$$\frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a+c x^4}}\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \left(-a B e+B c d x^2+A c \left(d+e x^2\right)\right)-\sqrt{a} \sqrt{c} \left(B d+A e\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]+\right.$$
$$\left.\left(-i A c d-i a B e+\sqrt{a} \sqrt{c} \left(B d+A e\right)\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]\right)$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 262 leaves, 4 steps):

$$\frac{x\left(A+B x^2\right)}{2 a \sqrt{a+c x^4}}-\frac{B x \sqrt{a+c x^4}}{2 a \sqrt{c}\left(\sqrt{a}+\sqrt{c} x^2\right)}+\frac{B\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a+c x^4}}-$$

$$\frac{\left(\sqrt{a} B-A \sqrt{c}\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 182 leaves) :

$$\frac{1}{2 a^{3/2}\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{a+c x^4}} i \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c}\, x\left(A+B x^2\right)-\right.$$

$$\left.\sqrt{a} B \sqrt{1+\frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+\left(\sqrt{a} B-i A \sqrt{c}\right) \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(d+e x^2\right)\left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 732 leaves, 9 steps) :

$$\frac{x\left(A c d+a B e+c\left(B d-A e\right) x^2\right)}{2 a\left(c d^2+a e^2\right)\sqrt{a+c x^4}}-\frac{\sqrt{c}\left(B d-A e\right) x\sqrt{a+c x^4}}{2 a\left(c d^2+a e^2\right)\left(\sqrt{a}+\sqrt{c} x^2\right)}-$$

$$\frac{e^{3/2}\left(B d-A e\right)\text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d}\sqrt{e}\sqrt{a+c x^4}}\right]}{2\sqrt{d}\left(c d^2+a e^2\right)^{3/2}}+\frac{c^{1/4}\left(B d-A e\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4}\left(c d^2+a e^2\right)\sqrt{a+c x^4}}-$$

$$\frac{c^{1/4} e\left(B d-A e\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4}\left(\sqrt{c} d-\sqrt{a} e\right)\left(c d^2+a e^2\right)\sqrt{a+c x^4}}+$$

$$\frac{\left(A c d+a B e-\sqrt{a}\sqrt{c}\left(B d-A e\right)\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{4 a^{5/4} c^{1/4}\left(c d^2+a e^2\right)\sqrt{a+c x^4}}+$$

$$\left(a^{3/4} e\left(\frac{\sqrt{c}\, d}{\sqrt{a}}+e\right)^2\left(B d-A e\right)\left(\sqrt{a}+\sqrt{c} x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4\sqrt{a}\sqrt{c}\, d e},2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4 c^{1/4} d\left(c^2 d^4-a^2 e^4\right)\sqrt{a+c x^4}\right)$$

Result (type 4, 432 leaves) :

$$\frac{1}{2 a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d\left(c d^2+a e^2\right)\sqrt{a+c x^4}}\left(A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, c d^2 x+a B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d e x+\right.$$

$$B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, c d^2 x^3-A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, c d e x^3-\sqrt{a}\sqrt{c}\, d\left(B d-A e\right)\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left(\sqrt{a}\, B-i A\sqrt{c}\right) d\left(\sqrt{c}\, d-i\sqrt{a}\, e\right)\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+2 i a B d e\sqrt{1+\frac{c x^4}{a}}$$

$$\left.\text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-2 i a A e^2\sqrt{1+\frac{c x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\left(d+e x^2\right)^2 \left(a+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 1494 leaves, 15 steps) :

$$\frac{c x \left(A c d^2+2 a B d e-a A e^2+\left(B c d^2-2 A c d e-a B e^2\right) x^2\right)}{2 a \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}}+\frac{\sqrt{c}\, e^2 (B d-A e) x \sqrt{a+c x^4}}{2 d \left(c d^2+a e^2\right)^2 \left(\sqrt{a}+\sqrt{c} x^2\right)}-$$

$$\frac{\sqrt{c} \left(B c d^2-2 A c d e-a B e^2\right) x \sqrt{a+c x^4}}{2 a \left(c d^2+a e^2\right)^2 \left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{e^3 (B d-A e) x \sqrt{a+c x^4}}{2 d \left(c d^2+a e^2\right)^2 \left(d+e x^2\right)}-\frac{e^{3/2} (B d-A e) \left(3 c d^2+a e^2\right) \text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d} \sqrt{e} \sqrt{a+c x^4}}\right]}{4 d^{3/2} \left(c d^2+a e^2\right)^{5/2}}-$$

$$\frac{e^{3/2} \left(B c d^2-2 A c d e-a B e^2\right) \text{ArcTan}\left[\frac{\sqrt{c d^2+a e^2}\, x}{\sqrt{d} \sqrt{e} \sqrt{a+c x^4}}\right]}{2 \sqrt{d} \left(c d^2+a e^2\right)^{5/2}}-\frac{a^{1/4} c^{1/4} e^2 (B d-A e) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 d \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}}+$$

$$\frac{c^{1/4} \left(B c d^2-2 A c d e-a B e^2\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}}-$$

$$\frac{c^{1/4} e (B d-A e) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} d \left(\sqrt{c} d-\sqrt{a} e\right) \left(c d^2+a e^2\right) \sqrt{a+c x^4}}-$$

$$\frac{c^{1/4} e \left(B c d^2-2 A c d e-a B e^2\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} \left(\sqrt{c} d-\sqrt{a} e\right) \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}}-$$

$$\left(c^{1/4} \left(B c d^2-2 A c d e-a B e^2-\frac{\sqrt{c} \left(A c d^2+2 a B d e-a A e^2\right)}{\sqrt{a}}\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(4 a^{3/4} \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}\right)+$$

$$\left(e\left(\sqrt{c}\,d+\sqrt{a}\,e\right)(B d-A e)\left(3 c d^2+a e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^{1/4} c^{1/4} d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c d^2+a e^2\right)^2\sqrt{a+c x^4}\right)+$$

$$\left(a^{3/4} e\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(B c d^2-2 A c d e-a B e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e},2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4 c^{1/4} d\left(c d^2-a e^2\right)\left(c d^2+a e^2\right)^2\sqrt{a+c x^4}\right)$$

Result (type 4, 427 leaves) :

$$\frac{1}{2 a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(c d^3+a d e^2\right)^2\left(d+e x^2\right)\sqrt{a+c x^4}}$$

$$\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\left(a e^3(-B d+A e)\,x\left(a+c x^4\right)+c d x\left(d+e x^2\right)\left(-a A e^2+B c d^2 x^2+A c d\left(d-2 e x^2\right)+a B e\left(2 d-e x^2\right)\right)\right)-\right.$$

$$\left(d+e x^2\right)\sqrt{1+\frac{c x^4}{a}}\left(-\sqrt{a}\,\sqrt{c}\,d\left(-B c d^3+2 A c d^2 e+2 a B d e^2-a A e^3\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$i\left(\sqrt{c}\,d\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(A c d^2+i\sqrt{a}\,\sqrt{c}\,d(B d-A e)+a e(2 B d-A e)\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\left.\left.\left.a e\left(-5 B c d^3+7 A c d^2 e+a B d e^2+a A e^3\right)\text{EllipticPi}\left[-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\right)$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{\left(d+e x^2\right)^3\left(a+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 2452 leaves, 22 steps) :

$$\frac{c\,x\left(A\,c\,d\left(c\,d^2-3\,a\,e^2\right)+a\,B\,e\left(3\,c\,d^2-a\,e^2\right)+c\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)x^2\right)}{2\,a\left(c\,d^2+a\,e^2\right)^3\sqrt{a+c\,x^4}}+$$

$$\frac{3\sqrt{c}\,e^2\left(B\,d-A\,e\right)\left(3\,c\,d^2+a\,e^2\right)x\sqrt{a+c\,x^4}}{8\,d^2\left(c\,d^2+a\,e^2\right)^3\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt{c}\,e^2\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)x\sqrt{a+c\,x^4}}{2\,d\left(c\,d^2+a\,e^2\right)^3\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\frac{c^{3/2}\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)x\sqrt{a+c\,x^4}}{2\,a\left(c\,d^2+a\,e^2\right)^3\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{e^3\left(B\,d-A\,e\right)x\sqrt{a+c\,x^4}}{4\,d\left(c\,d^2+a\,e^2\right)^2\left(d+e\,x^2\right)^2}-\frac{3\,e^3\left(B\,d-A\,e\right)\left(3\,c\,d^2+a\,e^2\right)x\sqrt{a+c\,x^4}}{8\,d^2\left(c\,d^2+a\,e^2\right)^3\left(d+e\,x^2\right)}-$$

$$\frac{e^3\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)x\sqrt{a+c\,x^4}}{2\,d\left(c\,d^2+a\,e^2\right)^3\left(d+e\,x^2\right)}-\frac{e^{3/2}\left(3\,c\,d^2+a\,e^2\right)\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{4\,d^{3/2}\left(c\,d^2+a\,e^2\right)^{7/2}}-$$

$$\frac{c\,e^{3/2}\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{d}\left(c\,d^2+a\,e^2\right)^{7/2}}-\frac{3\,e^{3/2}\left(B\,d-A\,e\right)\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{16\,d^{5/2}\left(c\,d^2+a\,e^2\right)^{7/2}}-$$

$$\frac{3\,a^{1/4}\,c^{1/4}\,e^2\left(B\,d-A\,e\right)\left(3\,c\,d^2+a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{8\,d^2\left(c\,d^2+a\,e^2\right)^3\sqrt{a+c\,x^4}}-$$

$$\frac{a^{1/4}\,c^{1/4}\,e^2\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,d\left(c\,d^2+a\,e^2\right)^3\sqrt{a+c\,x^4}}+$$

$$\frac{c^{5/4}\left(B\,c\,d^3-3\,A\,c\,d^2\,e-3\,a\,B\,d\,e^2+a\,A\,e^3\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\left(c\,d^2+a\,e^2\right)^3\sqrt{a+c\,x^4}}-$$

$$\left(c^{1/4}\,e\left(B\,d-A\,e\right)\left(4\,c\,d^2-\sqrt{a}\,\sqrt{c}\,d\,e+3\,a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}\right)-\frac{c^{1/4}\,e\left(B\,c\,d^2-2\,A\,c\,d\,e-a\,B\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,d\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(c\,d^2+a\,e^2\right)^2\sqrt{a+c\,x^4}}-$$

$$\left(c^{5/4} e \left(B c d^3 - 3 A c d^2 e - 3 a B d e^2 + a A e^3\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(2 a^{1/4} \left(\sqrt{c} d - \sqrt{a} e\right) \left(c d^2 + a e^2\right)^3 \sqrt{a + c x^4}\right) + \left(c^{3/4} \left(A c^2 d^3 - a^2 B e^3 - \sqrt{a} c^{3/2} d^2 (B d - 3 A e) + 3 a c d e (B d - A e) + a^{3/2} \sqrt{c} e^2 (3 B d - A e)\right)\right.$$

$$\left.\left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(4 a^{5/4} \left(c d^2 + a e^2\right)^3 \sqrt{a + c x^4}\right) +$$

$$\left(e \left(\sqrt{c} d + \sqrt{a} e\right) \left(3 c d^2 + a e^2\right) \left(B c d^2 - 2 A c d e - a B e^2\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c} d - \sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} c^{1/4} d^2 \left(\sqrt{c} d - \sqrt{a} e\right) \left(c d^2 + a e^2\right)^3 \sqrt{a + c x^4}\right) +$$

$$\left(a^{3/4} c^{3/4} e \left(\frac{\sqrt{c} d}{\sqrt{a}} + e\right)^2 \left(B c d^3 - 3 A c d^2 e - 3 a B d e^2 + a A e^3\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c} d - \sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(4 d \left(c d^2 - a e^2\right) \left(c d^2 + a e^2\right)^3 \sqrt{a + c x^4}\right) +$$

$$\left(3 e \left(\sqrt{c} d + \sqrt{a} e\right) (B d - A e) \left(5 c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c} d - \sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(32 a^{1/4} c^{1/4} d^3 \left(\sqrt{c} d - \sqrt{a} e\right) \left(c d^2 + a e^2\right)^3 \sqrt{a + c x^4}\right)$$

Result (type 4, 630 leaves) :

Problem 15: Unable to integrate problem.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^q}{a+c x^4} \, dx$$

Optimal (type 6, 169 leaves, 6 steps):

$$\frac{\left(A-\frac{\sqrt{-a}\, B}{\sqrt{c}}\right) x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}\, x^2}{\sqrt{-a}},-\frac{e x^2}{d}\right]}{2 a}+\frac{\left(A+\frac{\sqrt{-a}\, B}{\sqrt{c}}\right) x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} \text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}\, x^2}{\sqrt{-a}},-\frac{e x^2}{d}\right]}{2 a}$$

Result (type 8, 28 leaves):

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^q}{a+c x^4} \, dx$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+x^2}{\left(1+x^2\right) \sqrt{2+3 x^2+x^4}} \, dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\frac{\sqrt{2} \left(2+x^2\right) \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{\frac{2+x^2}{1+x^2}} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{2+3 x^2+x^4}}\left(2 x+x^3+i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B x^2\right) \left(d+e x^2\right)^3}{\sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 755 leaves, 6 steps):

$$\frac{e\left(7Ace\left(15cd-4be\right)+B\left(105c^2d^2+24b^2e^2-ce\left(84bd+25ae\right)\right)\right)x\sqrt{a+bx^2+cx^4}}{105c^3}+$$

$$\frac{e^2\left(21Bcd-6bBe+7Ace\right)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}+\frac{1}{105c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

$$\left(7Ace\left(45c^2d^2+8b^2e^2-3ce\left(10bd+3ae\right)\right)+B\left(105c^3d^3-48b^3e^3-21c^2de\left(10bd+9ae\right)+8bce^2\left(21bd+13ae\right)\right)\right)x\sqrt{a+bx^2+cx^4}-$$

$$\frac{1}{105c^{15/4}\sqrt{a+bx^2+cx^4}}$$

$$a^{1/4}\left(7Ace\left(45c^2d^2+8b^2e^2-3ce\left(10bd+3ae\right)\right)+B\left(105c^3d^3-48b^3e^3-21c^2de\left(10bd+9ae\right)+8bce^2\left(21bd+13ae\right)\right)\right)$$

$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]+\frac{1}{210c^{15/4}\sqrt{a+bx^2+cx^4}}$$

$$a^{1/4}\Bigg(7Ace\left(45c^2d^2+8b^2e^2-3ce\left(10bd+3ae\right)\right)+B\left(105c^3d^3-48b^3e^3-21c^2de\left(10bd+9ae\right)+8bce^2\left(21bd+13ae\right)\right)+$$

$$\frac{\sqrt{c}\left(7Ac\left(15c^2d^3-15acde^2+4abe^3\right)-aBe\left(105c^2d^2+24b^2e^2-ce\left(84bd+25ae\right)\right)\right)}{\sqrt{a}}\Bigg)$$

$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 4473 leaves) :

$$\sqrt{a+bx^2+cx^4}\left(-\frac{e\left(-105Bc^2d^2+84bBcde-105Ac^2de-24b^2Be^2+28Abce^2+25aBce^2\right)x}{105c^3}+\frac{e^2\left(21Bcd-6bBe+7Ace\right)x^3}{35c^2}+\frac{Be^3x^5}{7c}\right)+$$

$$\frac{1}{105c^3}\Bigg(\Bigg(\Bigg(105\,i\,Bc^2\left(-b+\sqrt{b^2-4ac}\right)d^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}$$

$$\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Bigg)\Bigg/\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\Bigg(105\,i\,bBc\left(-b+\sqrt{b^2-4ac}\right)d^2e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\right.\right.$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/$$

$$\Big(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)+\Big(315\,\mathrm{i}\,A\,c^2\,\Big(-b+\sqrt{b^2-4\,a\,c}\Big)\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/\Big(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)+$$

$$\Big(42\,\mathrm{i}\,\sqrt{2}\,b^2\,B\,\Big(-b+\sqrt{b^2-4\,a\,c}\Big)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/\Big(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-$$

$$\Big(105\,\mathrm{i}\,A\,b\,c\,\Big(-b+\sqrt{b^2-4\,a\,c}\Big)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/$$

$$\Big(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-\Big(189\,\mathrm{i}\,a\,B\,c\,\Big(-b+\sqrt{b^2-4\,a\,c}\Big)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/\Big(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)+$$

$$\left(14\,i\,\sqrt{2}\,A\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(26\,i\,\sqrt{2}\,a\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(12\,i\,\sqrt{2}\,b^3\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(63\,i\,a\,A\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(105\,i\,A\,c^3\,d^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(105\, \mathbb{i}\, a\, B\, c^2\, d^2\, e \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$
$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) -$$

$$\left(42\, \mathbb{i} \sqrt{2}\, a\, b\, B\, c\, d\, e^2 \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) +$$

$$\left(105\, \mathbb{i}\, a\, A\, c^2\, d\, e^2 \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$
$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) +$$

$$\left(12\, \mathbb{i} \sqrt{2}\, a\, b^2\, B\, e^3 \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) -$$

$$\left(14\, \mathbb{i} \sqrt{2}\, a\, A\, b\, c\, e^3 \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) -$$

$$\left(25\, \mathbb{i}\, a^2\, B\, c\, e^3 \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 528 leaves, 5 steps):

Result (type 4, 2613 leaves):

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 368 leaves, 4 steps):

$$\frac{B\,e\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c}+\frac{\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{3\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{3\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$a^{1/4}\,\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]+\frac{1}{6\,c^{7/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$a^{1/4}\,\left(3\,B\,c\,d-2\,b\,B\,e+3\,A\,c\,e+\frac{\sqrt{c}\,\left(3\,A\,c\,d-a\,B\,e\right)}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 521 leaves):

$$\frac{1}{12\,c^2\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,B\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;e\,x\,\left(a+b\,x^2+c\,x^4\right)-\mathrm{i}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(-3\,B\,c\,d+2\,b\,B\,e-3\,A\,c\,e\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$\mathrm{i}\,\left(-2\,b^2\,B\,e-c\,\left(6\,A\,c\,d+3\,B\,\sqrt{b^2-4\,a\,c}\;d-2\,a\,B\,e+3\,A\,\sqrt{b^2-4\,a\,c}\;e\right)+b\,\left(3\,B\,c\,d+3\,A\,c\,e+2\,B\,\sqrt{b^2-4\,a\,c}\;e\right)\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{B\,x\,\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{a^{1/4}\,B\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{a^{1/4}\,\left(B+\frac{A\,\sqrt{c}}{\sqrt{a}}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 302 leaves):

$$\left(\mathrm{i}\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\;\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\left(B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left.\left.\left(b\,B-2\,A\,c-B\,\sqrt{b^2-4\,a\,c}\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Bigg/\left(2\,\sqrt{2}\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{(d + e x^2) \sqrt{a + b x^2 + c x^4}} \, dx$$

Optimal (type 4, 436 leaves, 3 steps):

$$-\frac{(B d - A e) \operatorname{ArcTan}\left[\frac{\sqrt{c d^2 - b d e + a e^2} \, x}{\sqrt{d} \sqrt{e} \sqrt{a + b x^2 + c x^4}}\right]}{2 \sqrt{d} \sqrt{e} \sqrt{c d^2 - b d e + a e^2}} - \frac{\left(\sqrt{a} B - A \sqrt{c}\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]}{2 a^{1/4} c^{1/4} \left(\sqrt{c} d - \sqrt{a} e\right) \sqrt{a + b x^2 + c x^4}} +$$

$$\left(a^{3/4} \left(\frac{\sqrt{c} d}{\sqrt{a}} + e\right)^2 (B d - A e) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{c} d - \sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(4 c^{1/4} d e \left(c d^2 - a e^2\right) \sqrt{a + b x^2 + c x^4}\right)$$

Result (type 4, 298 leaves):

$$-\left(\left(i \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left(B d \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] + (-B d + A e)\right.\right.\right.$$

$$\left.\left.\left.\operatorname{EllipticPi}\left[\frac{\left(b + \sqrt{b^2 - 4 a c}\right) e}{2 c d}, i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right)\right) \Big/ \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} d e \sqrt{a + b x^2 + c x^4}\right)\right)$$

Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B x^2}{(d + e x^2)^2 \sqrt{a + b x^2 + c x^4}} \, dx$$

Optimal (type 4, 782 leaves, 6 steps):

$$\frac{\sqrt{c}\,(B\,d-A\,e)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{e\,(B\,d-A\,e)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)\,(d+e\,x^2)}-\frac{\left(B\,(c\,d^3-a\,d\,e^2)-A\,e\,(3\,c\,d^2-e\,(2\,b\,d-a\,e))\right)\,\mathrm{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{4\,d^{3/2}\,\sqrt{e}\,(c\,d^2-b\,d\,e+a\,e^2)^{3/2}}-$$

$$\frac{a^{1/4}\,c^{1/4}\,(B\,d-A\,e)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{A\,c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,a^{1/4}\,d\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\Bigg(\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(B\,(c\,d^3-a\,d\,e^2)-A\,e\,(3\,c\,d^2-e\,(2\,b\,d-a\,e))\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}$$

$$\mathrm{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\Bigg)\Big/\left(8\,a^{1/4}\,c^{1/4}\,d^2\,e\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,(c\,d^2-b\,d\,e+a\,e^2)\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 2187 leaves):

$$-\frac{e\,(B\,d-A\,e)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)\,(d+e\,x^2)}+\frac{1}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)}$$

$$\Bigg(\Bigg(\mathrm{i}\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Big/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(\mathrm{i}\,A$$

$$\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Big/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\frac{\mathrm{i}\, A\, c\, d \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]}{\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}} -$$

$$\left(\mathrm{i}\, B\, c\, d^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ e \sqrt{a+b x^2+c x^4}\right) - \left(3\, \mathrm{i}\, A\, c\, d \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) + \left(\mathrm{i}\, B\, c\, d^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ e \sqrt{a+b x^2+c x^4}\right) + \left(\mathrm{i} \sqrt{2}\, A\, b\, e \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(\mathrm{i}\, a\, B\, e \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right],\right.\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(\mathrm{i}\, a\, A\, e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4 a c}\right) e}{2 c d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\right],\right.\right.$$

$$\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,d\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)^3\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 1125 leaves, 7 steps) :

$$-\frac{\sqrt{c}\left(3 A e\left(3 c d^2-e(2 b d-a e)\right)-B d\left(5 c d^2-e(2 b d+a e)\right)\right) x \sqrt{a+b x^2+c x^4}}{8 d^2\left(c d^2-b d e+a e^2\right)^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-$$

$$\frac{e(B d-A e) x \sqrt{a+b x^2+c x^4}}{4 d\left(c d^2-b d e+a e^2\right)\left(d+e x^2\right)^2}+\frac{e\left(3 A e\left(3 c d^2-e(2 b d-a e)\right)-B d\left(5 c d^2-e(2 b d+a e)\right)\right) x \sqrt{a+b x^2+c x^4}}{8 d^2\left(c d^2-b d e+a e^2\right)^2\left(d+e x^2\right)}-$$

$$\left(\left(B d\left(3 c^2 d^4-10 a c d^2 e^2+a e^3(4 b d-a e)\right)-A e\left(15 c^2 d^4-2 c d^2 e(10 b d-3 a e)+e^2\left(8 b^2 d^2-8 a b d e+3 a^2 e^2\right)\right)\right)\right.$$

$$\left.\text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2}\, x}{\sqrt{d} \sqrt{e} \sqrt{a+b x^2+c x^4}}\right]\right) \Big/ \left(16 d^{5/2} \sqrt{e}\left(c d^2-b d e+a e^2\right)^{5/2}\right)+$$

$$\left(a^{1/4} c^{1/4}\left(3 A e\left(3 c d^2-e(2 b d-a e)\right)-B d\left(5 c d^2-e(2 b d+a e)\right)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(8 d^2\left(c d^2-b d e+a e^2\right)^2 \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(c^{1/4}\left(\sqrt{a} \sqrt{c}\, d(B d-A e)+a e(B d+3 A e)+4 A d(c d-b e)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(8 a^{1/4} d^2\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\left(c d^2-b d e+a e^2\right) \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(\left(\sqrt{c}\, d+\sqrt{a}\, e\right)\left(B d\left(3 c^2 d^4-10 a c d^2 e^2+a e^3(4 b d-a e)\right)-A e\left(15 c^2 d^4-2 c d^2 e(10 b d-3 a e)+e^2\left(8 b^2 d^2-8 a b d e+3 a^2 e^2\right)\right)\right)\right.$$

$$\left.\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4 \sqrt{a} \sqrt{c}\, d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(32 a^{1/4} c^{1/4} d^3 e\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\left(c d^2-b d e+a e^2\right)^2 \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 5205 leaves) :

$$\sqrt{a+b x^2+c x^4}\left(-\frac{e(B d-A e) x}{4 d\left(c d^2-b d e+a e^2\right)\left(d+e x^2\right)^2}-\frac{e\left(5 B c d^3-2 b B d^2 e-9 A c d^2 e+6 A b d e^2-a B d e^2-3 a A e^3\right) x}{8 d^2\left(c d^2-b d e+a e^2\right)^2\left(d+e x^2\right)}\right)+$$

$$\frac{1}{8\,d^2\,(c\,d^2-b\,d\,e+a\,e^2)^2}\Bigg(\Bigg(5\,i\,B\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\mathrm{EllipticE}\Big[$$

$$i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\mathrm{EllipticF}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(i\,b\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\mathrm{EllipticE}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\mathrm{EllipticF}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(9\,i\,A\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\mathrm{EllipticE}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\mathrm{EllipticF}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\Bigg(3\,i\,A\,b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\mathrm{EllipticE}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\mathrm{EllipticF}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(i\,a\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\mathrm{EllipticE}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\mathrm{EllipticF}\Big[i\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\,\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(3\,i\,a\,A\left(-b+\sqrt{b^2-4ac}\right)e^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\Big/\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(7\,i\,A\,c^2d^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(3\,i\,B\,c^2d^4\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,e\sqrt{a+bx^2+cx^4}\right)-$$

$$\left(2\,i\sqrt{2}\,A\,b\,c\,d^2e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(3\,i\,a\,B\,c\,d^2e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+$$

$$\left(i\,a\,A\,c\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(15\,\mathrm{i}\,A\,c^2\,d^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\left(3\,\mathrm{i}\,B\,c^2\,d^4\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,e\sqrt{a+bx^2+cx^4}\right)+\left(10\,\mathrm{i}\sqrt{2}\,A\,b\,c\,d^2\,e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(5\,\mathrm{i}\sqrt{2}\,a\,B\,c\,d^2\,e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(4\,\mathrm{i}\sqrt{2}\,A\,b^2\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\left(2\,\mathrm{i}\sqrt{2}\,a\,b\,B\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\Bigg(3\,i\sqrt{2}\,aAcde^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\Bigg(4\,i\sqrt{2}\,aAbe^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\ i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\Bigg(i\,a^2Be^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\right.$$

$$\left.i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-$$

$$\Bigg(3\,i\,a^2Ae^4\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\right.$$

$$\left.i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right)\Bigg)$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 859 leaves, 5 steps) :

$$\left(x\left(A\,c\left(b^2\,c\,d^3-2\,a\,c\,d\left(c\,d^2-3\,a\,e^2\right)-a\,b\,e\left(3\,c\,d^2+a\,e^2\right)\right)+a\,B\left(a\,b^2\,e^3+2\,a\,c\,e\left(3\,c\,d^2-a\,e^2\right)-b\,c\,d\left(c\,d^2+3\,a\,e^2\right)\right)-\left(a\,B\left(2\,c\,d-b\,e\right)\left(c^2\,d^2+b^2\,e^2-c\,e\left(b\,d+3\,a\,e\right)\right)+A\,c\left(a\,b^2\,e^3+2\,a\,c\,e\left(3\,c\,d^2-a\,e^2\right)-b\,c\,d\left(c\,d^2+3\,a\,e^2\right)\right)\right)x^2\right)\right)\Big/\left(a\,c^2\left(b^2-4\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}\right)+\frac{B\,e^3\,x\sqrt{a+b\,x^2+c\,x^4}}{3\,c^2}+$$

$$\left(\left(a\,B\left(6\,c^3\,d^3-8\,b^3\,e^3-9\,c^2\,d\,e\left(b\,d+6\,a\,e\right)+b\,c\,e^2\left(18\,b\,d+29\,a\,e\right)\right)+3\,A\,c\left(2\,a\,b^2\,e^3+6\,a\,c\,e\left(c\,d^2-a\,e^2\right)-b\,c\,d\left(c\,d^2+3\,a\,e^2\right)\right)\right)x\sqrt{a+b\,x^2+c\,x^4}\right)\Big/\left(3\,a\,c^{5/2}\left(b^2-4\,a\,c\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right)-$$

$$\left(\left(a\,B\left(6\,c^3\,d^3-8\,b^3\,e^3-9\,c^2\,d\,e\left(b\,d+6\,a\,e\right)+b\,c\,e^2\left(18\,b\,d+29\,a\,e\right)\right)+3\,A\,c\left(2\,a\,b^2\,e^3+6\,a\,c\,e\left(c\,d^2-a\,e^2\right)-b\,c\,d\left(c\,d^2+3\,a\,e^2\right)\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(3\,a^{3/4}\,c^{11/4}\left(b^2-4\,a\,c\right)\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\left(3\,A\,c^3\,d^3-5\,a^2\,B\,c\,e^3-3\sqrt{a}\,c^{5/2}\,d^2\left(B\,d+3\,A\,e\right)+a\,e\left(3\,c\,d-2\,b\,e\right)\left(3\,B\,c\,d-4\,b\,B\,e+3\,A\,c\,e\right)+3\,a^{3/2}\sqrt{c}\,e^2\left(9\,B\,c\,d-4\,b\,B\,e+3\,A\,c\,e\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(6\,a^{3/4}\left(b-2\sqrt{a}\,\sqrt{c}\right)c^{11/4}\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 5432 leaves) :

$$\sqrt{a+b\,x^2+c\,x^4}\left(\frac{B\,e^3\,x}{3\,c^2}+\frac{1}{a\,c^2\left(-b^2+4\,a\,c\right)\left(a+b\,x^2+c\,x^4\right)}\left(-A\,b^2\,c^2\,d^3\,x+a\,b\,B\,c^2\,d^3\,x+2\,a\,A\,c^3\,d^3\,x+3\,a\,A\,b\,c^2\,d^2\,e\,x-6\,a^2\,B\,c^2\,d^2\,e\,x+3\,a^2\,b\,B\,c\,d\,e^2\,x-6\,a^2\,A\,c^2\,d\,e^2\,x-a^2\,b^2\,B\,e^3\,x+a^2\,A\,b\,c\,e^3\,x+2\,a^3\,B\,c\,e^3\,x-A\,b\,c^3\,d^3\,x^3+2\,a\,B\,c^3\,d^3\,x^3-3\,a\,b\,B\,c^2\,d^2\,e\,x^3+6\,a\,A\,c^3\,d^2\,e\,x^3+3\,a\,b^2\,B\,c\,d\,e^2\,x^3-3\,a\,A\,b\,c^2\,d\,e^2\,x^3-6\,a^2\,B\,c^2\,d\,e^2\,x^3-a\,b^3\,B\,e^3\,x^3+a\,A\,b^2\,c\,e^3\,x^3+3\,a^2\,b\,B\,c\,e^3\,x^3-2\,a^2\,A\,c^2\,e^3\,x^3\right)\right)-$$

$$\frac{1}{3\,a\,c^2\left(-b^2+4\,a\,c\right)}\left(-\left(\left(3\,i\,A\,b\,c^2\left(-b+\sqrt{b^2-4\,a\,c}\right)d^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\right)+$$

$$\left(3\,\mathbb{i}\,a\,B\,c^2\left(-b+\sqrt{b^2-4\,a\,c}\right)d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.\right.\right.$$

$$\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\left(9\,\mathbb{i}\,a\,b\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(9\,\mathbb{i}\,a\,A\,c^2\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\right.\right.\right.$$

$$\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\left(9\,\mathbb{i}\,a\,b^2\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(9\,i\,a\,A\,b\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(27\,i\,a^2\,B\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(3\,i\,a\,A\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\Bigg(29\,i\,a^2\,b\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(2\,i\,\sqrt{2}\,a\,b^3\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/$$

$$\Big(c\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-\Big(9\,\mathrm{i}\,a^2\,A\,c\,\Big(-b+\sqrt{b^2-4\,a\,c}\Big)\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Big(\mathrm{EllipticE}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big)\Big/\Big(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-$$

$$\Big(3\,\mathrm{i}\,a\,b\,B\,c^2\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)+$$

$$\Big(3\,\mathrm{i}\,\sqrt{2}\,a\,A\,c^3\,d^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-$$

$$\Big(9\,\mathrm{i}\,a\,A\,b\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)+$$

$$\Big(9\,\mathrm{i}\,\sqrt{2}\,a^2\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\mathrm{EllipticF}\Big[\mathrm{i}\,\mathrm{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Big)\Big/$$

$$\Big(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\Big)-$$

$$\left(9\,\mathbb{i}\,a^2\,b\,B\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(9\,\mathbb{i}\,\sqrt{2}\,a^2\,A\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(2\,\mathbb{i}\,\sqrt{2}\,a^2\,b^2\,B\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$
$$\left(3\,\mathbb{i}\,a^2\,A\,b\,c\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$
$$\left(5\,\mathbb{i}\,\sqrt{2}\,a^3\,B\,c\,e^3\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Bigg/$$
$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^2}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 628 leaves, 4 steps):

$$-\left(\left(x\left(a B\left(b c d^2-4 a c d e+a b e^2\right)-A c\left(b^2 d^2-2 a b d e-2 a\left(c d^2-a e^2\right)\right)-\left(A c\left(b c d^2-4 a c d e+a b e^2\right)-a B\left(2 c^2 d^2+b^2 e^2-2 c e(b d+a e)\right)\right) x^2\right)\right)\Big/\left(a c\left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right)\right)-\frac{\left(A c\left(b c d^2-4 a c d e+a b e^2\right)-2 a B\left(c^2 d^2+b^2 e^2-c e(b d+3 a e)\right)\right) x \sqrt{a+b x^2+c x^4}}{a c^{3/2}\left(b^2-4 a c\right)\left(\sqrt{a}+\sqrt{c} x^2\right)}+\left(\left(A c\left(b c d^2-4 a c d e+a b e^2\right)-2 a B\left(c^2 d^2+b^2 e^2-c e(b d+3 a e)\right)\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right)\Big/\left(a^{3/4} c^{7/4}\left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right)-\left(\left(A c^2 d^2+3 a^{3/2} B \sqrt{c} e^2-\sqrt{a} c^{3/2} d(B d+2 A e)+a e(2 B c d-2 b B e+A c e)\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right)\Big/\left(2 a^{3/4}\left(b-2 \sqrt{a} \sqrt{c}\right) c^{7/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 3464 leaves):

$$\left(-A b^2 c d^2 x+a b B c d^2 x+2 a A c^2 d^2 x+2 a A b c d e x-4 a^2 B c d e x+a^2 b B e^2 x-2 a^2 A c e^2 x-A b c^2 d^2 x^3+2 a B c^2 d^2 x^3-2 a b B c d e x^3+4 a A c^2 d e x^3+a b^2 B e^2 x^3-a A b c e^2 x^3-2 a^2 B c e^2 x^3\right)\Big/\left(a c\left(-b^2+4 a c\right) \sqrt{a+b x^2+c x^4}\right)-\frac{1}{a c\left(-b^2+4 a c\right)}\left(-\left(\left(\left(i A b c\left(-b+\sqrt{b^2-4 a c}\right) d^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\Big/\left(2 \sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right)\right)+\right.\right.$$

$$\Bigg(i\,a\,B\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(i\,a\,b\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Big/\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(i\,\sqrt{2}\,a\,A\,c\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,d\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(i\,a\,A\,b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Big/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(3\,i\,a^2\,B\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],$$

$$\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\Big],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+\left(\mathbb{i}\,a\,b^2\,B\left(-b+\sqrt{b^2-4\,a\,c}\right)e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{2}\,c\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathbb{i}\,a\,b\,B\,c\,d^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\mathbb{i}\sqrt{2}\,a\,A\,c^2\,d^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathbb{i}\sqrt{2}\,a\,A\,b\,c\,d\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(2\,\mathbb{i}\sqrt{2}\,a^2\,B\,c\,d\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\mathbb{i}\,a^2\,b\,B\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(i\,\sqrt{2}\,a^2\,A\,c\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 481 leaves, 4 steps):

$$-\frac{x\,\left(a\,B\,\left(b\,d-2\,a\,e\right)-A\,\left(b^2\,d-2\,a\,c\,d-a\,b\,e\right)-\left(A\,c\,\left(b\,d-2\,a\,e\right)-a\,B\,\left(2\,c\,d-b\,e\right)\right)\,x^2\right)}{a\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{\left(A\,c\,\left(b\,d-2\,a\,e\right)-a\,B\,\left(2\,c\,d-b\,e\right)\right)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{a\,\sqrt{c}\,\left(b^2-4\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\left(\left(A\,c\,\left(b\,d-2\,a\,e\right)-a\,B\,\left(2\,c\,d-b\,e\right)\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(a^{3/4}\,c^{3/4}\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(\left(\sqrt{a}\,B-A\,\sqrt{c}\right)\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2\,a^{3/4}\,\left(b-2\,\sqrt{a}\,\sqrt{c}\right)\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 597 leaves):

$$\frac{1}{4\,a\,c\,\left(-b^2+4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\left(a\,B\,\left(-2\,a\,e+2\,c\,d\,x^2+b\,\left(d-e\,x^2\right)\right)+A\,\left(-b^2\,d+b\,\left(a\,e-c\,d\,x^2\right)+2\,a\,c\,\left(d+e\,x^2\right)\right)\right)+\right.$$

$$i\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(A\,c\,\left(b\,d-2\,a\,e\right)+a\,B\,\left(-2\,c\,d+b\,e\right)\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$i\,\left(A\,c\,\left(-b^2\,d+4\,a\,c\,d+b\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,\sqrt{b^2-4\,a\,c}\,e\right)+a\,B\,\left(b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e+c\,\left(-2\,\sqrt{b^2-4\,a\,c}\,d+4\,a\,e\right)\right)\right)$$

$$\left.\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 398 leaves, 4 steps):

$$\frac{x\,\left(A\,b^2-a\,b\,B-2\,a\,A\,c+\left(A\,b-2\,a\,B\right)\,c\,x^2\right)}{a\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}-\frac{\left(A\,b-2\,a\,B\right)\,\sqrt{c}\,x\,\sqrt{a+b\,x^2+c\,x^4}}{a\,\left(b^2-4\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

$$\frac{\left(A\,b-2\,a\,B\right)\,c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{a^{3/4}\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{\left(\sqrt{a}\,B-A\,\sqrt{c}\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,a^{3/4}\,\left(b-2\,\sqrt{a}\,\sqrt{c}\right)\,c^{1/4}\,\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 497 leaves):

$$-\frac{1}{4\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\left(a\,B\,\left(b+2\,c\,x^2\right)-A\,\left(b^2-2\,a\,c+b\,c\,x^2\right)\right)+\mathrm{i}\,\left(A\,b-2\,a\,B\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$\mathrm{i}\,\left(-2\,a\,B\,\sqrt{b^2-4\,a\,c}+A\,\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 867 leaves, 9 steps):

$$-\frac{x\left(a\,b\,c\,(B\,d-A\,e)-\left(b^2-2\,a\,c\right)\left(A\,c\,d-A\,b\,e+a\,B\,e\right)+c\left(a\,B\,(2\,c\,d-b\,e)-A\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)\right)x^2\right)}{a\left(b^2-4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\frac{\sqrt{c}\,\left(a\,B\,(2\,c\,d-b\,e)-A\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)\right)x\sqrt{a+b\,x^2+c\,x^4}}{a\left(b^2-4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{e^{3/2}\,(B\,d-A\,e)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\sqrt{d}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{3/2}}-$$

$$\Bigg(c^{1/4}\left(a\,B\,(2\,c\,d-b\,e)-A\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\Bigg)\Big/$$

$$\left(a^{3/4}\left(b^2-4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\frac{\left(\sqrt{a}\,B-A\,\sqrt{c}\right)c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}-\frac{b}{4\,\sqrt{a}\,\sqrt{c}}\right]}{2\,a^{3/4}\left(b-2\,\sqrt{a}\,\sqrt{c}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+b\,x^2+c\,x^4}}+$$

$$\Bigg(a^{3/4}\,e\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2(B\,d-A\,e)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\Bigg)\Big/$$

$$\left(4\,c^{1/4}\,d\left(c\,d^2-a\,e^2\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 3361 leaves):

$$\left(-A\,b^2\,c\,d\,x+a\,b\,B\,c\,d\,x+2\,a\,A\,c^2\,d\,x+A\,b^3\,e\,x-a\,b^2\,B\,e\,x-3\,a\,A\,b\,c\,e\,x+2\,a^2\,B\,c\,e\,x-A\,b\,c^2\,d\,x^3+2\,a\,B\,c^2\,d\,x^3+A\,b^2\,c\,e\,x^3-a\,b\,B\,c\,e\,x^3-2\,a\,A\,c^2\,e\,x^3\right)\Big/$$

$$\left(a\left(-b^2+4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\frac{1}{a\left(-b^2+4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)}\Bigg(-\Bigg(\Bigg(\Bigg(\mathrm{i}\,A\,b\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Big/\left(2\,\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)\Bigg)+$$

$$\left(\mathbb{i}\, a\, B\, c\, \left(-b+\sqrt{b^2-4\,a\,c}\right) d \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right],\right.\right.\right.$$

$$\left.\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right] - \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) + \left(\mathbb{i}\, A\, b^2 \left(-b+\sqrt{b^2-4\,a\,c}\right) e \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right) \Big/ \left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) -$$

$$\left(\mathbb{i}\, a\, b\, B \left(-b+\sqrt{b^2-4\,a\,c}\right) e \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}} \left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right],\right.\right.\right.$$

$$\left.\left.\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right] - \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) - \left(\mathbb{i}\, a\, A\, c \left(-b+\sqrt{b^2-4\,a\,c}\right) e \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right) \Big/ \left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) -$$

$$\left(\mathbb{i}\, a\, b\, B\, c\, d \sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}} \sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\ \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\, x\right], \frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right) +$$

$$\left(i \sqrt{2}\, a\, A\, c^2\, d \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{a+b\, x^2+c\, x^4}\right) -$$

$$\left(i\, a\, A\, b\, c\, e \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{a+b\, x^2+c\, x^4}\right) +$$

$$\left(i \sqrt{2}\, a^2\, B\, c\, e \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{a+b\, x^2+c\, x^4}\right) + \left(i\, a\, b^2\, B\, e \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\, a\, c}\right) e}{2\, c\, d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{a+b\, x^2+c\, x^4}\right) - \left(2\, i \sqrt{2}\, a^2\, B\, c\, e \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\, a\, c}\right) e}{2\, c\, d}, i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{a+b\, x^2+c\, x^4}\right) -$$

$$\left(i\, a\, A\, b^2\, e^2 \sqrt{1-\frac{2\, c\, x^2}{-b-\sqrt{b^2-4\, a\, c}}} \sqrt{1-\frac{2\, c\, x^2}{-b+\sqrt{b^2-4\, a\, c}}}\ \text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\, a\, c}\right) e}{2\, c\, d},\right.\right.$$

$$\left.\left. i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ x\right], \frac{-b-\sqrt{b^2-4\, a\, c}}{-b+\sqrt{b^2-4\, a\, c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4\, a\, c}}}\ d \sqrt{a+b\, x^2+c\, x^4}\right) +$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a^2\,A\,c\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},\right.\right.$$
$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,d\,\sqrt{a+b\,x^2+c\,x^4}\right)\right)$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B\,x^2}{\left(d+e\,x^2\right)^2\,\left(a+b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 1301 leaves, 15 steps):

$$\Big(x\,\big(a\,b\,c\,(A\,e\,(2\,c\,d-b\,e)-B\,(c\,d^2-a\,e^2))+(b^2-2\,a\,c)\,(a\,B\,e\,(2\,c\,d-b\,e)+A\,(c^2\,d^2+b^2\,e^2-c\,e\,(2\,b\,d+a\,e)))-$$

$$c\,(a\,B\,(2\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(b\,d+a\,e))+A\,(2\,b^2\,c\,d\,e-4\,a\,c^2\,d\,e-b^3\,e^2-b\,c\,(c\,d^2-3\,a\,e^2)))\,x^2\big)\Big)\Big/$$

$$\Big(a\,(b^2-4\,a\,c)\,(c\,d^2-b\,d\,e+a\,e^2)^2\,\sqrt{a+b\,x^2+c\,x^4}\Big)+\Big(\sqrt{c}\,\big(a\,B\,d\,(-4\,c^2\,d^2-3\,b^2\,e^2+4\,c\,e\,(b\,d+2\,a\,e))+$$

$$A\,(2\,b^3\,d\,e^2+2\,b\,c\,d\,(c\,d^2-3\,a\,e^2)-4\,a\,c\,e\,(-2\,c\,d^2+a\,e^2)+b^2\,(-4\,c\,d^2\,e+a\,e^3))\big)\,x\,\sqrt{a+b\,x^2+c\,x^4}\Big)\Big/$$

$$\Big(2\,a\,(-b^2+4\,a\,c)\,d\,(c\,d^2+e\,(-b\,d+a\,e))^2\,(\sqrt{a}+\sqrt{c}\,x^2)\Big)-\frac{e^3\,(B\,d-A\,e)\,x\,\sqrt{a+b\,x^2+c\,x^4}}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)^2\,(d+e\,x^2)}+$$

$$\frac{e^{3/2}\,(A\,e\,(7\,c\,d^2-e\,(4\,b\,d-a\,e))-B\,d\,(5\,c\,d^2-e\,(2\,b\,d+a\,e)))\,\text{ArcTan}\Big[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\Big]}{4\,d^{3/2}\,(c\,d^2-b\,d\,e+a\,e^2)^{5/2}}-$$

$$\Big(c^{1/4}\,\big(a\,B\,d\,(4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(b\,d+2\,a\,e))-A\,(2\,b^3\,d\,e^2+2\,b\,c\,d\,(c\,d^2-3\,a\,e^2)-4\,a\,c\,e\,(-2\,c\,d^2+a\,e^2)+b^2\,(-4\,c\,d^2\,e+a\,e^3))\big)\,(\sqrt{a}+\sqrt{c}\,x^2)$$

$$\sqrt{\frac{a+b\,x^2+c\,x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,\text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{c^{1/4}\,x}{a^{1/4}}\Big],\frac{1}{2}-\frac{b}{4\,\sqrt{a}\,\sqrt{c}}\Big]\Big)\Big/\Big(2\,a^{3/4}\,(b^2-4\,a\,c)\,d\,(c\,d^2+e\,(-b\,d+a\,e))^2\,\sqrt{a+b\,x^2+c\,x^4}\Big)+$$

$$\Big(c^{1/4}\,\big(a\,\sqrt{c}\,e\,(B\,d-2\,A\,e)+\sqrt{a}\,(B\,d-A\,e)\,(c\,d-b\,e)+A\,\sqrt{c}\,d\,(-c\,d+b\,e)\big)\,(\sqrt{a}+\sqrt{c}\,x^2)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}$$

$$\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{c^{1/4}\,x}{a^{1/4}}\Big],\frac{1}{2}-\frac{b}{4\,\sqrt{a}\,\sqrt{c}}\Big]\Big)\Big/\Big(2\,a^{3/4}\,(b-2\,\sqrt{a}\,\sqrt{c})\,d\,(-\sqrt{c}\,d+\sqrt{a}\,e)\,(-c\,d^2+e\,(b\,d-a\,e))\,\sqrt{a+b\,x^2+c\,x^4}\Big)-$$

$$\Big(e\,(\sqrt{c}\,d+\sqrt{a}\,e)\,(A\,e\,(7\,c\,d^2-e\,(4\,b\,d-a\,e))-B\,d\,(5\,c\,d^2-e\,(2\,b\,d+a\,e)))\,(\sqrt{a}+\sqrt{c}\,x^2)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}$$

$$\text{EllipticPi}\Big[-\frac{(\sqrt{c}\,d-\sqrt{a}\,e)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\Big[\frac{c^{1/4}\,x}{a^{1/4}}\Big],\frac{1}{4}\Big(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\Big)\Big]\Big)\Big/\Big(8\,a^{1/4}\,c^{1/4}\,d^2\,(\sqrt{c}\,d-\sqrt{a}\,e)\,(c\,d^2-b\,d\,e+a\,e^2)^2\,\sqrt{a+b\,x^2+c\,x^4}\Big)$$

Result (type 4, 8031 leaves):

$$\sqrt{a+b\,x^2+c\,x^4}$$

$$\Big(-\frac{e^3\,(B\,d-A\,e)\,x}{2\,d\,(c\,d^2-b\,d\,e+a\,e^2)^2\,(d+e\,x^2)}+\frac{1}{a\,(-b^2+4\,a\,c)\,(c\,d^2-b\,d\,e+a\,e^2)^2\,(a+b\,x^2+c\,x^4)}\,(-A\,b^2\,c^2\,d^2\,x+a\,b\,B\,c^2\,d^2\,x+2\,a\,A\,c^3\,d^2\,x+2\,A\,b^3\,c\,d\,e\,x-$$

$$2 a b^2 B c d e x - 6 a A b c^2 d e x + 4 a^2 B c^2 d e x - A b^4 e^2 x + a b^3 B e^2 x + 4 a A b^2 c e^2 x - 3 a^2 b B c e^2 x - 2 a^2 A c^2 e^2 x - A b c^3 d^2 x^3 +$$

$$2 a B c^3 d^2 x^3 + 2 A b^2 c^2 d e x^3 - 2 a b B c^2 d e x^3 - 4 a A c^3 d e x^3 - A b^3 c e^2 x^3 + a b^2 B c e^2 x^3 + 3 a A b c^2 e^2 x^3 - 2 a^2 B c^2 e^2 x^3 \Big) \Bigg) -$$

$$\frac{1}{2 a \left(-b^2 + 4 a c\right) d \left(c d^2 - b d e + a e^2\right)^2} \Bigg(- \Bigg(\Bigg(\Bigg(\mathbb{i}\, A b c^2 \left(-b + \sqrt{b^2 - 4 a c}\right) d^3 \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}}$$

$$\Bigg(\mathrm{EllipticE}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] -$$

$$\mathrm{EllipticF}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] \Bigg) \Bigg) \Bigg/ \left(\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right) \Bigg) +$$

$$\Bigg(\mathbb{i} \sqrt{2}\, a B c^2 \left(-b + \sqrt{b^2 - 4 a c}\right) d^3 \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \Bigg(\mathrm{EllipticE}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right],\right.$$

$$\left.\frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] - \mathrm{EllipticF}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] \Bigg) \Bigg) \Bigg/ \left(\sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right) +$$

$$\Bigg(\mathbb{i} \sqrt{2}\, A b^2 c \left(-b + \sqrt{b^2 - 4 a c}\right) d^2 e \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \Bigg(\mathrm{EllipticE}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right],\right.$$

$$\left.\frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] - \mathrm{EllipticF}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] \Bigg) \Bigg) \Bigg/ \left(\sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right) -$$

$$\Bigg(\mathbb{i} \sqrt{2}\, a b B c \left(-b + \sqrt{b^2 - 4 a c}\right) d^2 e \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \Bigg(\mathrm{EllipticE}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right],\right.$$

$$\left.\frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] - \mathrm{EllipticF}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] \Bigg) \Bigg) \Bigg/ \left(\sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right) -$$

$$\Bigg(2\, \mathbb{i} \sqrt{2}\, a A c^2 \left(-b + \sqrt{b^2 - 4 a c}\right) d^2 e \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \Bigg(\mathrm{EllipticE}\left[\mathbb{i}\, \mathrm{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right],\right.$$

$$\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big] - \text{EllipticF}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big) \Big/ \Big(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\Big) -$$

$$\Big(\mathrm{i}\ A\, b^3 \left(-b+\sqrt{b^2-4 a c}\right) d\, e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big],$$

$$\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big] - \text{EllipticF}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big) \Big/$$

$$\Big(\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\Big) + \Big(3\ \mathrm{i}\ a\, b^2\, B \left(-b+\sqrt{b^2-4 a c}\right) d\, e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}$$

$$\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big] -$$

$$\text{EllipticF}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big) \Big/ \Big(2 \sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\Big) +$$

$$\Big(3\ \mathrm{i}\ a\, A\, b\, c \left(-b+\sqrt{b^2-4 a c}\right) d\, e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big],$$

$$\frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big] - \text{EllipticF}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big) \Big/$$

$$\Big(\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\Big) - \Big(2\ \mathrm{i} \sqrt{2}\ a^2\, B\, c \left(-b+\sqrt{b^2-4 a c}\right) d\, e^2 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}}$$

$$\sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big] -$$

$$\text{EllipticF}\Big[\mathrm{i}\ \text{ArcSinh}\Big[\sqrt{2}\ \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ x\Big], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\Big]\Big)\Big) \Big/ \Big(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\ \sqrt{a+b x^2+c x^4}\Big) -$$

$$\left(i\, a\, A\, b^2 \left(-b+\sqrt{b^2-4 a c}\right) e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] - \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) + \left(i \sqrt{2}\, a^2 A\, c \left(-b+\sqrt{b^2-4 a c}\right) e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] - \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(i \sqrt{2}\, a\, b\, B\, c^2 d^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) +$$

$$\left(2\, i \sqrt{2}\, a\, A\, c^3 d^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(i\, a\, b^2 B\, c\, d^2 e \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\left(2\,i\,\sqrt{2}\,a\,A\,b\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(6\,i\,\sqrt{2}\,a^2\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(3\,i\,a\,A\,b^2\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(i\,\sqrt{2}\,a^2\,b\,B\,c\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(4\,i\,\sqrt{2}\,a^2\,A\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\left(5\,i\,a\,b^2\,B\,c\,d^2\,e\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,d},\,i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(10\,\mathrm{i}\,\sqrt{2}\,a^2\,B\,c^2\,d^2\,e\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\mathtt{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(\mathrm{i}\,\sqrt{2}\,a\,b^3\,B\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\mathtt{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)-\left(7\,\mathrm{i}\,a\,A\,b^2\,c\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\mathtt{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\left(4\,\mathrm{i}\,\sqrt{2}\,a^2\,b\,B\,c\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\mathtt{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\left(14\,\mathrm{i}\,\sqrt{2}\,a^2\,A\,c^2\,d\,e^2\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\left.\mathtt{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right],\,\frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)+\left(2\,\mathrm{i}\,\sqrt{2}\,a\,A\,b^3\,e^3\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd}, i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right) - (i\,a^2 b^2 B e^3 \sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd}, i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right) - (8\,i\sqrt{2}\,a^2 A b c e^3 \sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd}, i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right) + (2\,i\sqrt{2}\,a^3 B c e^3 \sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}$$

$$\text{EllipticPi}\left[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd}, i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right) -$$

$$(i\,a^2 A b^2 e^4 \sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},$$

$$i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}]) \Big/ \left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right) +$$

$$(2\,i\sqrt{2}\,a^3 A c e^4 \sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}[-\frac{\left(-b-\sqrt{b^2-4ac}\right)e}{2cd},$$

$$i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,x\right], \frac{-b-\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}]) \Big/ \left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right))$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 273 leaves, 1 step):

$$-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\sqrt{d}\,\sqrt{e}\,\sqrt{c\,d^2-b\,d\,e+a\,e^2}}+\left(\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(4\,a^{1/4}\,c^{1/4}\,d\,e\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 310 leaves):

$$-\left(\left(i\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\left(\sqrt{c}\,d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\left(-\sqrt{c}\,d+\sqrt{a}\,e\right)\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,d\,e\sqrt{a+b\,x^2+c\,x^4}\right)\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\left(d+e\,x^2\right)\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 271 leaves, 1 step):

$$-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)\text{ArcTan}\left[\frac{\sqrt{c\,d^2-b\,d\,e+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\sqrt{d}\,\sqrt{e}\,\sqrt{c\,d^2-b\,d\,e+a\,e^2}}+\frac{\left(\sqrt{\frac{c}{a}}\,d+e\right)\left(1+\sqrt{\frac{c}{a}}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{a\left(1+\sqrt{\frac{c}{a}}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)^2}{4\sqrt{\frac{c}{a}}\,d\,e},2\,\text{ArcTan}\left[\left(\frac{c}{a}\right)^{1/4}x\right],\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right]}{4\left(\frac{c}{a}\right)^{1/4}d\,e\sqrt{a+b\,x^2+c\,x^4}}$$

Result (type 4, 312 leaves):

$$-\left(\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{\frac{c}{a}}\,d\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+\left(-\sqrt{\frac{c}{a}}\,d+e\right)\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\right)\Big/\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d\,e\sqrt{a+bx^2+cx^4}\right)\right)$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{946+315x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 106 leaves, 4 steps):

$$\frac{631\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{2525\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 74 leaves):

$$-\frac{\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\left(441\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+505\,\text{EllipticPi}\left[\frac{10}{7},\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)}{7\sqrt{2+3x^2+x^4}}$$

Problem 33: Unable to integrate problem.

$$\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Optimal (type 6, 218 leaves, 6 steps):

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right]}{b-\sqrt{b^2-4ac}}+$$

$$\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right]}{b+\sqrt{b^2-4ac}}$$

Result (type 8, 33 leaves):

$$\int \frac{\left(A+B\,x^2\right)\,\left(d+e\,x^2\right)^q}{a+b\,x^2+c\,x^4}\,dx$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,\left(1+2\,x^2\right)}{\sqrt{1+x^2}\,\left(1+x^2+x^4\right)}\,dx$$

Optimal (type 3, 106 leaves, 11 steps):

$$-\frac{1}{2}\,\text{ArcTan}\left[\sqrt{3}-2\,\sqrt{1+x^2}\right]+\frac{1}{2}\,\text{ArcTan}\left[\sqrt{3}+2\,\sqrt{1+x^2}\right]+\frac{1}{4}\,\sqrt{3}\,\text{Log}\left[2+x^2-\sqrt{3}\,\sqrt{1+x^2}\right]-\frac{1}{4}\,\sqrt{3}\,\text{Log}\left[2+x^2+\sqrt{3}\,\sqrt{1+x^2}\right]$$

Result (type 3, 103 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{1+x^2}}{\sqrt{-1-i\,\sqrt{3}}}\right]}{\sqrt{\frac{1}{2}\,\left(-1-i\,\sqrt{3}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{1+x^2}}{\sqrt{-1+i\,\sqrt{3}}}\right]}{\sqrt{\frac{1}{2}\,\left(-1+i\,\sqrt{3}\right)}}$$

Problem 35: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b\,x^2+c\,x^4}}{a\,d-c\,d\,x^4}\,dx$$

Optimal (type 3, 145 leaves, 4 steps):

$$-\frac{\sqrt{b-2\,\sqrt{a}\,\sqrt{c}}\,\text{ArcTanh}\left[\frac{\sqrt{b-2\,\sqrt{a}\,\sqrt{c}}\,x}{\sqrt{a+b\,x^2+c\,x^4}}\right]}{4\,\sqrt{a}\,\sqrt{c}\,d}+\frac{\sqrt{b+2\,\sqrt{a}\,\sqrt{c}}\,\text{ArcTanh}\left[\frac{\sqrt{b+2\,\sqrt{a}\,\sqrt{c}}\,x}{\sqrt{a+b\,x^2+c\,x^4}}\right]}{4\,\sqrt{a}\,\sqrt{c}\,d}$$

Result (type 4, 441 leaves):

$$\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(2\sqrt{a}\sqrt{c}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-\right.\right.$$

$$\left(b+2\sqrt{a}\sqrt{c}\right)\text{EllipticPi}\left[\frac{-b-\sqrt{b^2-4ac}}{2\sqrt{a}\sqrt{c}},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+$$

$$\left.\left.\left(b-2\sqrt{a}\sqrt{c}\right)\text{EllipticPi}\left[\frac{b+\sqrt{b^2-4ac}}{2\sqrt{a}\sqrt{c}},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}\right)$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx$$

Optimal (type 3, 239 leaves, 1 step):

$$-\frac{\sqrt{b+\sqrt{b^2+4ac}}\;\text{ArcTan}\left[\frac{\sqrt{b+\sqrt{b^2+4ac}}\;x\left(b-\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right]}{2\sqrt{2}\sqrt{a}\sqrt{c}\,d}+\frac{\sqrt{-b+\sqrt{b^2+4ac}}\;\text{ArcTanh}\left[\frac{\sqrt{-b+\sqrt{b^2+4ac}}\;x\left(b+\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right]}{2\sqrt{2}\sqrt{a}\sqrt{c}\,d}$$

Result (type 4, 432 leaves):

$$\frac{1}{4\sqrt{a}\sqrt{c}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,d\sqrt{a+bx^2-cx^4}}$$

$$\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(2\,\mathrm{i}\sqrt{a}\sqrt{c}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]+\right.$$

$$\left(b-2\,\mathrm{i}\sqrt{a}\sqrt{c}\right)\text{EllipticPi}\left[-\frac{\mathrm{i}\left(b+\sqrt{b^2+4ac}\right)}{2\sqrt{a}\sqrt{c}},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]-$$

$$\left.\left(b+2\,\mathrm{i}\sqrt{a}\sqrt{c}\right)\text{EllipticPi}\left[\frac{\mathrm{i}\left(b+\sqrt{b^2+4ac}\right)}{2\sqrt{a}\sqrt{c}},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)$$

Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e\,x)\,\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 405 leaves, 7 steps):

$$\frac{e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-a\,e^4}\,x}{d\,e\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{-c\,d^4-a\,e^4}}-\frac{e\,\text{ArcTanh}\left[\frac{a\,e^2+c\,d^2\,x^2}{\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{c\,d^4+a\,e^4}}+\frac{c^{1/4}\,d\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\,\sqrt{a+c\,x^4}}-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d^2\,e^2},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c^{1/4}\,d\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\,\sqrt{a+c\,x^4}}$$

Result (type 4, 200 leaves):

$$\left(\sqrt{1+\frac{c\,x^4}{a}}\,\left(-2\,(-1)^{1/4}\,a^{1/4}\,\sqrt{1+\frac{c\,d^4}{a\,e^4}}\,e\,\text{EllipticPi}\left[\frac{i\,\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\text{ArcSin}\left[\frac{(-1)^{3/4}\,c^{1/4}\,x}{a^{1/4}}\right],-1\right]+c^{1/4}\,d\,\text{Log}\left[\frac{-d^2+e^2\,x^2}{c\,d^2\,x^2+a\,e^2\,\left(1+\sqrt{1+\frac{c\,d^4}{a\,e^4}}\,\sqrt{1+\frac{c\,x^4}{a}}\right)}\right]\right)\right)\Big/\left(2\,c^{1/4}\,d\,\sqrt{1+\frac{c\,d^4}{a\,e^4}}\,e\,\sqrt{a+c\,x^4}\right)$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e\,x)^2\,\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$-\frac{e^3\sqrt{a+c\,x^4}}{\left(c\,d^4+a\,e^4\right)(d+e\,x)}+\frac{\sqrt{c}\,e^2\,x\,\sqrt{a+c\,x^4}}{\left(c\,d^4+a\,e^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{c\,d^3\,e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-a\,e^4}\,x}{d\,e\,\sqrt{a+c\,x^4}}\right]}{\left(-c\,d^4-a\,e^4\right)^{3/2}}-\frac{c\,d^3\,e\,\text{ArcTanh}\left[\frac{a\,e^2+c\,d^2\,x^2}{\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}}\right]}{\left(c\,d^4+a\,e^4\right)^{3/2}}-$$
$$\frac{a^{1/4}\,c^{1/4}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{\left(c\,d^4+a\,e^4\right)\sqrt{a+c\,x^4}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+c\,x^4}}-$$
$$\frac{c^{3/4}\,d^2\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d^2\,e^2},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(c\,d^4+a\,e^4\right)\sqrt{a+c\,x^4}}$$

Result (type 4, 462 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(c\,d^4+a\,e^4\right)^{3/2}(d+e\,x)\sqrt{a+c\,x^4}}\left(\sqrt{a}\,\sqrt{c}\,e^2\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$
$$i\sqrt{c}\left(\sqrt{c}\,d^2+i\sqrt{a}\,e^2\right)\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-$$
$$\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(e^3\sqrt{c\,d^4+a\,e^4}\left(a+c\,x^4\right)+2\,(-1)^{1/4}\,a^{1/4}\,c^{3/4}\,d^2\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\text{ArcSin}\left[\frac{(-1)^{3/4}\,c^{1/4}\,x}{a^{1/4}}\right],-1\right]-\right.$$
$$\left.\left.c\,d^3\,e\,(d+e\,x)\sqrt{a+c\,x^4}\,\text{Log}\left[-d^2+e^2\,x^2\right]+c\,d^3\,e\,(d+e\,x)\sqrt{a+c\,x^4}\,\text{Log}\left[a\,e^2+c\,d^2\,x^2+\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}\right]\right)\right)$$

Problem 3: Unable to integrate problem.

$$\int \frac{1}{(d+e\,x)\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 518 leaves, 7 steps):

$$\frac{e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-b\,d^2\,e^2-a\,e^4}\;x}{d\,e\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\sqrt{-c\,d^4-b\,d^2\,e^2-a\,e^4}}-\frac{e\,\text{ArcTanh}\left[\frac{b\,d^2+2\,a\,e^2+\left(2\,c\,d^2+b\,e^2\right)\,x^2}{2\,\sqrt{c\,d^4+b\,d^2\,e^2+a\,e^4}\;\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\sqrt{c\,d^4+b\,d^2\,e^2+a\,e^4}}+$$

$$\frac{c^{1/4}\,d\,\left(\sqrt{a}+\sqrt{c}\;x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{2\,a^{1/4}\,\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\,\sqrt{a+b\,x^2+c\,x^4}}-$$

$$\left(\left(\sqrt{c}\;d^2-\sqrt{a}\;e^2\right)\,\left(\sqrt{a}+\sqrt{c}\;x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)^2}{4\,\sqrt{a}\,\sqrt{c}\;d^2\,e^2},\,2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(4\,a^{1/4}\,c^{1/4}\,d\,\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 8, 26 leaves):

$$\int \frac{1}{(d+e\,x)\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Problem 4: Unable to integrate problem.

$$\int \frac{1}{(d+e\,x)^2\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 822 leaves, 11 steps):

$$-\frac{e^3\sqrt{a+b x^2+c x^4}}{\left(c d^4+b d^2 e^2+a e^4\right)(d+e x)}+\frac{\sqrt{c}\, e^2 x\sqrt{a+b x^2+c x^4}}{\left(c d^4+b d^2 e^2+a e^4\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{d e\left(2 c d^2+b e^2\right)\mathrm{ArcTan}\left[\frac{\sqrt{-c d^4-b d^2 e^2-a e^4}\, x}{d e\sqrt{a+b x^2+c x^4}}\right]}{2\left(-c d^4-b d^2 e^2-a e^4\right)^{3/2}}-$$

$$\frac{d e\left(2 c d^2+b e^2\right)\mathrm{ArcTanh}\left[\frac{b d^2+2 a e^2+\left(2 c d^2+b e^2\right) x^2}{2\sqrt{c d^4+b d^2 e^2+a e^4}\sqrt{a+b x^2+c x^4}}\right]}{2\left(c d^4+b d^2 e^2+a e^4\right)^{3/2}}-\frac{a^{1/4} c^{1/4} e^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{\left(c d^4+b d^2 e^2+a e^4\right)\sqrt{a+b x^2+c x^4}}+$$

$$\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{2 a^{1/4}\left(\sqrt{c}\, d^2+\sqrt{a}\, e^2\right)\sqrt{a+b x^2+c x^4}}-$$

$$\left(\left(\sqrt{c}\, d^2-\sqrt{a}\, e^2\right)\left(2 c d^2+b e^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\mathrm{EllipticPi}\left[\frac{\left(\sqrt{c}\, d^2+\sqrt{a}\, e^2\right)^2}{4\sqrt{a}\sqrt{c}\, d^2 e^2},2\,\mathrm{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$

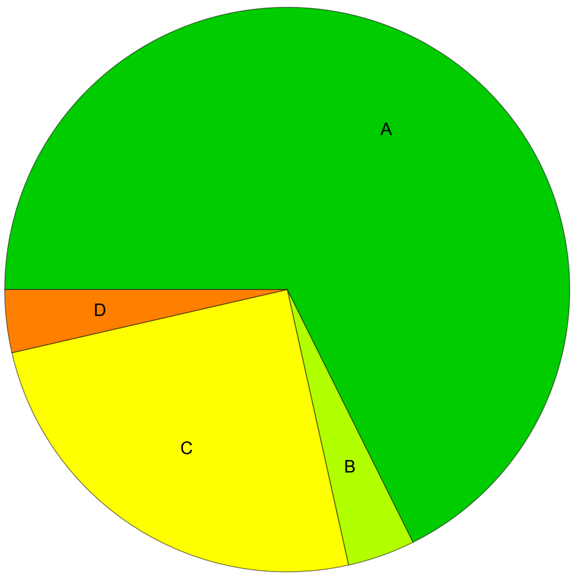
$$\left(4 a^{1/4} c^{1/4}\left(\sqrt{c}\, d^2+\sqrt{a}\, e^2\right)\left(c d^4+b d^2 e^2+a e^4\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 8, 26 leaves):

$$\int\frac{1}{(d+e x)^2\sqrt{a+b x^2+c x^4}}\,dx$$

Summary of Integration Test Results

2254 integration problems



A - 1525 optimal antiderivatives

B - 87 more than twice size of optimal antiderivatives

C - 561 unnecessarily complex antiderivatives

D - 81 unable to integrate problems

E - 0 integration timeouts